PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

Letters to the Editor

PUBLICATION of brief reports of important discoveries in physics may be secured by addressing them to this department. The closing date for this department is five weeks prior to the date of issue. No proof will be sent to the authors. The Board of Editors does not hold itself responsible for the opinions expressed by the correspondents. Communications should not exceed 600 words in length and should be submitted in duplicate.

Proposed Negative-Mass Microwave Amplifier

HERBERT KRÖMER*
RCA Laboratories, Princeton, New Jersey
(Received November 11, 1957)

AT high kinetic energies, electrons and holes in semiconductors have negative effective masses. In an electric field the current contribution from negative-mass carriers is opposite to the electric field, and a semiconductor containing a sufficient number of such carriers would have a negative resistance. It could thus be used as an active element in an oscillator or amplifier,[1] up to frequencies of the order of the reciprocal collision time of the carriers, i.e., up to about 1000 kMc/sec.

Any attempt to accelerate normal carriers up into the negative-mass energy range faces the difficulty that optical-phonon scattering and avalanche ionization will, in most cases, prevent the carriers from reaching that range.

We want to point out that negative masses occur at energies close to the band edge if the energy contours are re-entrant there, as is the case, for example, for the heavy holes in germanium. There, the energy contours are re-entrant along the six [100] directions of k space[2] (Fig. 1). Along these directions, then, the *transverse* effective masses are negative, i.e., along the k_x axis,

$$m_y=\hbar^2\Big/\frac{\partial^2\epsilon}{\partial k_y{}^2}=m_z=\hbar^2\Big/\frac{\partial^2\epsilon}{\partial k_z{}^2}<0.$$

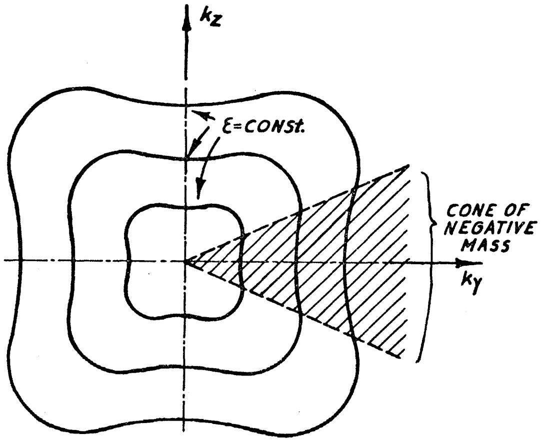

FIG. 1. Schematic energy contours for heavy holes in Ge.

From the data of reference 2, one calculates

$$m_y=m_z\approx -0.3m_0$$

for germanium.

A microwave amplifier, therefore, is proposed, consisting of a p-type Ge crystal with a strong dc bias field, applied in the [100] direction, and the electrical rf field of a resonant cavity or a wave guide perpendicular to it; the bias field shifts the total hole population away from $k=0$ and toward the inside of the conical region of negative transverse mass. The rf field will then be amplified if the majority of the holes are brought inside this cone.

In order to make this possible, scattering of the holes out of the negative-mass cone must be avoided. A totally inelastic type of scattering is desirable where the holes after each collision return to nearly $k=0$. If the collision cross section is large enough, optical phonon scattering is of this type. Shockley's interpretation[3] of the high-field mobility data[4] in Ge suggests that this might be the case in Ge at biasing fields of a few thousand volts/cm.

This amplifier principle is not restricted to germanium, but should hold for all semiconductors with re-entrant energy contours and high optical-phonon cross sections, such as p-type Si and III–V compounds.

The detailed theory will be presented elsewhere shortly.

Note added in proof.—Preliminary experiments with Ge have not shown the effect. This is interpreted as being the result of Ge's not having a high enough scattering cross section, which is likely because the two atoms per unit cell are identical. Experiments on compounds have not been performed yet.

That space-charge instabilities caused by the negative dc conductivity can be avoided, and how, will be shown with the detailed theory.

---

* Now at Philips Laboratories, Hamburg, Germany.
[1] W. Shockley and W. P. Mason, J. Appl. Phys. **25**, 677 (1954).
[2] Dresselhaus, Kip, and Kittel, Phys. Rev. **98**, 368 (1955).
[3] W. Shockley, Bell. System Tech. J. **30**, 990 (1951).
[4] E. J. Ryder, Phys. Rev. **90**, 766 (1953).

Penetration of Magnetic Fields through Superconducting Films

A. L. SCHAWLOW
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received January 17, 1958)

BARDEEN, Cooper, and Schrieffer[1] have recently formulated a molecular theory of superconductivity. Their analysis leads to a nonlocal relation between current and field not very different from that proposed originally by Pippard[2] in analogy with the anomalous skin effect theory. The consequences of such a relation are quite different from that of the London theory when the magnetic field varies rapidly with distance, and can be tested experimentally.

Several years ago, experiments were conducted on penetration of magnetic fields through thin cylindrical films, in an attempt to measure the penetration depth parameter (λ) and to determine the law of penetration. The films, of tin or lead, were deposited on tubes of glass, brass, or plastic, and were ordinarily about 6 cm long and 4 mm in diameter. The tubes were rotated about their axes by a motor during deposition, and film thicknesses were measured by multiple beam interferometry. In the experiment, an alternating field ($\sim$500 cps) was provided by a solenoid outside and coaxial with the film, and field penetration was detected by a small pickup coil inside the tube. With the aid of careful shielding (which is made possible by the favorable geometry) and good amplification, alternating fields of the order of 10^{-5} oersted were detectable. The experiment was complicated by sudden breakthroughs and nonlinear penetration which could be localized to particular spots on the films. To ensure linearity it was, therefore, necessary to work with external ac fields of not more than 1% of the critical field, although superimposed dc fields had very little effect.

Even with small applied ac fields, the observed penetrations were too large for the London theory with any reasonable value of λ. Therefore measurements were suspended before much quantitative data had been accumulated. However, this is a situation in which the magnetic field varies by a factor of the order of 10^4 in a distance of a few hundred angstroms. Thus a nonlocal theory should give considerably different results, and the experiments might give a critical test of this theory.

According to the London theory,

$$H_0/H_i=(r/2\lambda)\sinh(d/\lambda), \tag{1}$$

where r is the cylinder radius, H_i is the magnetic field inside the cylinder, H_0 is the field outside, and d is the film thickness.

On the basis of the nonlocal theory, using the solution obtained by Peter,[3] one finds

$$\frac{H_0}{H_i}=\frac{3rd^2}{8\lambda^2\xi_0}\left\{\ln\left(\frac{\xi}{d}\right)+0.423+\frac{2d}{3\xi}-\frac{d^2}{12\xi^2}\left[\ln\left(\frac{\xi}{d}\right)+2.006\right]\right\},$$

where ξ is the range of coherence, and ξ_0 is the range in a large, unstrained sample.

Experimentally, we have data on two films. (1) Tin film on brass, $r=2.77$ mm, $d=935$ A, (a) at $T=3.23$°K, $H_0/H_i=5.9\times10^3$; (b) at 2.40°K, $H_0/H_i=1.41\times10^4$. (2) Tin film on glass, $r=2.00$ mm, $d=660$ A; at 2.25°K, $H_0/H_i=6.7\times10^3$. To fit these values with the London equation we would need the following values of λ_0: (1a), $\lambda_0=1010$ A; (1b), $\lambda_0=955$ A; (2), $\lambda_0=980$ A, where $\lambda_0=\lambda[1-(T/T_c)^4]^{\frac{1}{2}}$ is the penetration depth at absolute zero. These values are impossibly large, and do not agree with any of the experimental measurements.

On the other hand, if we use the nonlocal theory with the experimental value of $\lambda_0=5.1\times10^{-6}$ cm, and the theoretical[1] $\xi_0=2500$ A, and solve for ξ, we obtain: (1a), $\xi=1000$ A; (1b), $\xi=1260$ A; (2), $\xi=1450$ A. These values of ξ are quite reasonable, and are comparable with the normal electron free path in similar films.

These measurements of field penetration through thin films give strong support to the nonlocal theory, but are not yet complete enough to investigate the details of the relation between current and field.

I wish to thank E. M. Kelly for evaporating the films, G. E. Devlin for his assistance in the experiment, and H. W. Lewis, P. W. Anderson, M. Tinkham, G. Wannier, V. Van Lint, and M. Peter for helpful discussions.

---

[1] Bardeen, Cooper, and Schrieffer, Phys. Rev. **108**, 1175 (1957).

[2] A. B. Pippard, Proc. Roy. Soc. (London) **A216**, 547 (1953).

[3] M. Peter, Phys. Rev. **109**, 1857 (1958), following Letter.

Penetration of Electromagnetic Fields through Superconducting Films

Martin Peter
Bell Telephone Laboratories, Murray Hill, New Jersey

(Received January 17, 1958)

In their new theory, Bardeen *et al.*[1] arrive at a nonlocal expression for the connection between fields and currents in superconductors which has the form of a system of integro-differential equations. The equation proposed by Pippard[2] is of this type and has been shown to be in fair agreement with the theory of Bardeen *et al.*:

$$-\mathbf{j}(\mathbf{r})=\frac{3c}{\xi_0(4\pi)^2\lambda_L{}^2}\int\frac{\mathbf{r}'(\mathbf{r}'\cdot\mathbf{A})e^{-r'/\xi}}{r'^4}dr', \tag{1}$$

where $\mathbf{A}\equiv\mathbf{A}(\mathbf{r}+\mathbf{r}')$. An approximate solution is given below for the case of superconducting films, assuming that the electromagnetic field is known on one side of the film, and that the integral is to be taken over the film only. The calculations apply to the experiments of Schawlow[3] and also to experiments on transmission of plane waves through plane films,[4] if x_1 is replaced by $L/2\pi$.[5] (We set $x_1=$ inner radius of cylindrical shield, $L=$ vacuum wavelength, $d=$ film thickness, $\lambda_L=$ London penetration depth.) Terms of the order d/x_1 have been neglected in the calculations. Thus we obtain Eq. (9) of Pippard's paper,[2] but with finite limits of integration: