Pion Production in Pion-Nucleon Collisions*

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The single production of pions occurring in pion-nucleon collisions at incident pion energies below $\sim$1.5 Bev has been considered in terms of the inelastic excitation of the nucleon to the isobaric state with isotopic spin and angular momentum $=\frac{3}{2}$, previously identified with the resonant state in the low-energy pion-nucleon scattering. An associated separate recoil pion accompanies the isobar, and conserves energy and momentum. The isobar subsequently decays into a nucleon and a pion. Expressions have been obtained for the branching ratios between the different reactions, $\pi^-+p\rightarrow\pi^-+\pi^++n$, and $\pi^-+\pi^0+p$, and for the separate energy spectra of the π^+, π^0, and π^- mesons. The momentum distributions of the pions and recoil nucleons have been calculated at incident pion energies of 0.93 Bev and 1.37 Bev, and are generally in reasonable agreement with the experimental data available at these energies.

I. INTRODUCTION

In previous publications[1,2] we have proposed and investigated an isobaric nucleon model[3] for pion-nucleon interactions from low energies to $\sim$1.4 Bev for both the elastic pion-nucleon scattering and the single pion production interactions. The model consists in assuming that the pion-nucleon interaction for pion kinetic energies $\lesssim$1–1.5 Bev proceeds predominantly via the real (i.e., nonvirtual) excitation of the nucleon to the isobaric level with isotopic spin (T) and angular momentum $(J)=\frac{3}{2}$ associated with the low-energy resonance ($\sim$190 Mev). This excitation can take place either directly (pion absorption followed by elastic re-emission) or indirectly (inelastic excitation of the level with an associated recoil pion). It therefore follows that the $T=\frac{1}{2}$ total cross section would necessarily be near zero below the effective threshold energy ($\sim$200 Mev) for formation of a $T=J=\frac{3}{2}$ isobar with an accompanying separate recoil pion which would allow the total T for the state to equal $\frac{1}{2}$.

A discussion of the behavior of the $T=\frac{1}{2}$ and $T=\frac{3}{2}$ state total cross sections from low energies to $\sim$1.4 Bev has been previously presented,[1,2] and it has been found that many of the general features of the cross sections can be satisfactorily explained by this model. The treatment of the isobar excitation process in pion-nucleon collisions was developed in analogy to our previously published isobaric nucleon model[3] for single pion production in nucleon-nucleon collisions with the recoil (unexcited) nucleon being replaced by the recoil pion.

The predicted pion momentum spectra for the π^+ and π^- lumped together from the reaction $\pi^-+p\rightarrow\pi^-+\pi^++n$ at 1.0 and 1.37 Bev have been previously presented[1,2] and were found to be in reasonable agreement with the experimental data at these energies.[4,5] This method of comparison is independent of the ratio of the $T=\frac{1}{2}$ and $T=\frac{3}{2}$ inelastic cross sections, and also the relative phase between the matrix elements, upon which the separate spectra of the π^+ and π^- mesons would depend.

The purpose of the present paper is to give a more complete account of the calculations reported in I, especially as concerns the momentum distributions of the outgoing particles at 1.0 and 1.37 Bev, for the cases of single pion production. We have also obtained the branching ratios for the various possible reactions and the corresponding expressions for the separate energy spectra of the π^+, π^0, and π^- mesons. These branching ratios and energy spectra have been compared to the experimental data.[4,5]

II. ENERGY SPECTRA OF PIONS AND NUCLEONS

In the present isobar model, the energy spectrum of all the pions from a reaction irrespective of charge is given by

$$\frac{d^2\sigma}{d\bar{T}_\pi d\bar{\Omega}_\pi}=B\int_{M_1}^{M_2}\sigma(m_I)FG_\pi dm_I+C\sigma(m_I)F\frac{dm_I}{d\bar{T}_\pi},\quad(1)$$

where $\sigma(m_I)$ is the total π^+-p scattering cross section for an isobar mass m_I (= total energy of the decay pion and nucleon in the isobar rest system), F is the two-body phase space factor for an isobar of mass m_I and the recoil pion, G_π is a factor giving the energy spectrum of the pions arising from the decay of the isobar of mass m_I, moving with a velocity $\bar{v}_I$ appropriate to the total energy $\bar{E}$ in the center-of-mass system of the original pion and nucleon. This center-of-mass system will be abbreviated as c.m.s. in the following, and quantities pertaining to the c.m. system will be barred. In Eq. (1), B and C are constants which merely serve to normalize the two terms. The integration over m_I extends over all values of m_I which are kinematically

* Work performed under the auspices of the U. S. Atomic Energy Commission.
1 S. J. Lindenbaum and R. M. Sternheimer, Phys. Rev. **106**, 1107 (1957). This paper will be referred to as I.
2 R. M. Sternheimer and S. J. Lindenbaum, Bull. Am. Phys. Soc. Ser. II, **2**, 176 (1957).
3 S. J. Lindenbaum and R. M. Sternheimer, Phys. Rev. **105**, 1874 (1957).
4 Walker, Hushfar, and Shephard, Phys. Rev. **104**, 526 (1956).
5 Eisberg, Fowler, Lea, Shephard, Shutt, Thorndike, and Whittemore, Phys. Rev. **97**, 797 (1955).

possible, between the following limits. The lower limit of m_I is $M_1 = m_p + m_\pi = 1.08$ Bev. The upper limit M_2 is either[6] $\bar{E} - m_\pi$ or M_b, whichever has the lower value. Here $M_b \equiv 1.58$ Bev, corresponding to an excitation energy of 500 Mev plus a pion rest mass. The second term of Eq. (1) (which is multiplied by C) gives the energy spectrum of the recoil pions. For each c.m.s. kinetic energy $\bar{T}_\pi$ of the recoil pion, there is an appropriate value of m_I, and the factor $dm_I/d\bar{T}_\pi$ represents the range dm_I per unit range $d\bar{T}_\pi$. In the present isobar model, different values of m_I are weighted by the factor $\sigma(m_I)F$, which accounts for the presence of this quantity in both terms of Eq. (1).

The momentum spectra of Walker *et al.*[4] and of Eisberg *et al.*[5] give the momentum distributions of the pions integrated over all angles. In this case, independently of the angular distribution of the isobar with respect to the incident direction, G_π is given by the following expression, provided one assumes that the isobar decays isotropically in its own rest system[7]:

$$G_\pi = 1/(\bar{T}_{\pi,\max} - \bar{T}_{\pi,\min}), \quad (\bar{T}_{\pi,\min} < \bar{T}_\pi < \bar{T}_{\pi,\max}), \quad (2)$$

$$G_\pi = 0, \quad (\bar{T}_\pi < \bar{T}_{\pi,\min} \text{ and } \bar{T}_\pi > \bar{T}_{\pi,\max}), \quad (2a)$$

where $\bar{T}_{\pi,\min}$ and $\bar{T}_{\pi,\max}$ are the minimum and maximum possible kinetic energies of the pions arising from the decay of the isobar (having mass m_I and velocity $\bar{v}_I$ in the c.m. system). Equations (2) and (2a) represent the well-known uniform distribution of energies of secondaries arising from a two-body decay. Upon inserting Eqs. (2), (2a) into Eq. (1), one obtains the resultant energy spectrum $d\sigma/d\bar{T}_\pi$ integrated over all angles.

It should be mentioned that in the more specific case, where one observes the pions at a particular angle (e.g., in counter experiments), the energy distribution will depend somewhat on the angular distribution $a(\bar{\theta}_{N^*})$ of the isobar N^* with respect to the incident direction. This situation is analogous to the corresponding one for nucleon-nucleon collisions.[3] The factor G_π in Eq. (1) then depends upon $a(\bar{\theta}_{N^*})$, and similarly C also becomes a function of the angle $\bar{\theta}_\pi$ of the recoil pion. In fact, since $\bar{\theta}_\pi = 180° - \bar{\theta}_{N^*}$, C will be simply proportional to $a(180° - \bar{\theta}_\pi)$.

The two terms of Eq. (1) will be called $I_{\pi,1}$ and $I_{\pi,2}$, respectively. $I_{\pi,1}$ gives the energy spectrum of the pions from the isobar decay, while $I_{\pi,2}$ is the energy spectrum of the recoil pions. $I_{\pi,1}$ and $I_{\pi,2}$ must be normalized to the same area, since there is a one-to-one correspondence between decay pions and recoil pions. Hence it follows that:

$$\int_0^{\bar{T}_{\pi,m}} I_{\pi,1} d\bar{T}_\pi = \int_0^{\bar{T}_{\pi,m}} I_{\pi,2} d\bar{T}_\pi, \quad (3)$$

where $\bar{T}_{\pi,m}$ is the maximum possible pion energy, above which $I_{\pi,1}$ and $I_{\pi,2}$ are zero. The corresponding pion momentum spectra will be denoted by $J_{\pi,1}$ and $J_{\pi,2}$. Thus

$$J_{\pi,i} = \bar{v}_\pi I_{\pi,i}, \quad (i = 1, 2), \quad (4)$$

where $\bar{v}_\pi$ is the pion velocity in the c.m. system.

The expressions for the spectra $I_{\pi,1}$ and $I_{\pi,2}$ are similar to the expressions for the nucleon spectra $I_{N,1}$ and $I_{N,2}$, respectively, for the case of single N^* production in nucleon-nucleon collisions (see Figs. 7 and 8 of reference 3). The procedure of the calculation of $I_{\pi,1}$ and $I_{\pi,2}$ is completely analogous to the treatment of the corresponding spectra in reference 3 [Eqs. (18), (19), and (26)].

In order to obtain the isobar decay spectrum $I_{\pi,1}$, we have subdivided the range of m_I into intervals of 25 Mev. The various mass values will be called $m_{I,j}$ ($j = 1, 2, \cdots$). For each $m_{I,j}$, the following quantity $g_{j,1}$ was evaluated:

$$g_{j,1} \equiv \frac{\sigma(m_{I,j})F_j}{\bar{T}_{\pi,\max}^{(j)} - \bar{T}_{\pi,\min}^{(j)}}, \quad (5)$$

where $\bar{T}_{\pi,\min}^{(j)}$ and $\bar{T}_{\pi,\max}^{(j)}$ are the minimum and maximum possible energies of the pions from the isobar of mass $m_{I,j}$; F_j is the corresponding phase space factor pertaining to $m_{I,j}$. The partial spectrum due to the mass values near $m_{I,j}$ is given by a step function of magnitude $g_{j,1}$ extending from $\bar{T}_{\pi,\min}^{(j)}$ to $\bar{T}_{\pi,\max}^{(j)}$. The complete decay spectrum $I_{\pi,1}$ is obtained by drawing a smooth curve through the sum of the step functions $g_{j,1}$. Thus

$$I_{\pi,1}(\bar{T}_\pi) = (\textstyle\sum_j g_{j,1}^S)\Delta m, \quad (6)$$

where $\Delta m = 25$ Mev is the interval of $m_{I,j}$; $g_{j,1}^S \equiv g_{j,1}$ for $\bar{T}_{\pi,\min}^{(j)} < \bar{T}_\pi < \bar{T}_{\pi,\max}^{(j)}$, and $g_{j,1}^S \equiv 0$ for $\bar{T}_\pi < \bar{T}_{\pi,\min}^{(j)}$ and $\bar{T}_\pi > \bar{T}_{\pi,\max}^{(j)}$.

The spectrum of the recoil pions, $I_{\pi,2}$, is obtained from the following quantity $g_{j,2}$:

$$g_{j,2} \equiv \frac{\sigma(m_{I,j})F_j}{\bar{T}_\pi^{(j+)} - \bar{T}_\pi^{(j-)}}, \quad (7)$$

where

$$\bar{T}_\pi^{(j\pm)} \equiv \tfrac{1}{2}[\bar{T}_\pi^{(j)} + \bar{T}_\pi^{(j\pm1)}], \quad (8)$$

and $\bar{T}_\pi^{(j)}$ is the energy of the recoil pion pertaining to the isobar of mass $m_{I,j}$. We have: $\bar{T}_\pi^{(j)} = \bar{E} - \bar{E}_{N^*}^{(j)} - m_\pi$, where $\bar{E}_{N^*}^{(j)}$ is the total energy of the isobar in the c.m. system of the original π and proton.

The contribution of the values of m_I between $\frac{1}{2}(m_{I,j-1} + m_{I,j})$ and $\frac{1}{2}(m_{I,j} + m_{I,j+1})$ is represented by a step function of magnitude $g_{j,2}$, extending from $\bar{T}_\pi^{(j-)}$ to $\bar{T}_\pi^{(j+)}$. For neighboring values of $m_{I,j}$ ($m_{I,j}$ and $m_{I,j\pm1}$), the regions ($\bar{T}_\pi^{(j-)}$, $\bar{T}_\pi^{(j+)}$) are adjacent to each other. The complete recoil spectrum $I_{\pi,2}$ is obtained by drawing a smooth curve through the step functions $g_{j,2}$.

In Figs. 1 and 2, we show the momentum spectra for the reaction $\pi^- + p \to \pi^- + \pi^+ + n$ at incident pion ener-

[6] It is assumed that the units are such that $c = 1$.
[7] See reference 3, footnote 19.

gies $T_\pi = 0.93$ and 1.37 Bev.[4,5] In these figures, the π^+ and π^- mesons have been lumped together, so that the corresponding prediction of the isobar model is

$$d\sigma/d\bar{p}_\pi = J_{\pi,1} + J_{\pi,2}. \tag{9}$$

The dot-dashed curves in Figs. 1 and 2 show the separate momentum spectra $J_{\pi,1}$ and $J_{\pi,2}$. The dashed curve gives the prediction of the statistical theory of Fermi.[8,9] The previously published results of Fig. 2(a) of I were obtained using an incident pion energy $T_\pi = 1.0$ Bev. Actually the two parts of the experiment of Walker *et al.*[4] (see their Table I) were carried out at energies of 0.90 and 0.96 Bev, so that it is more realistic to perform the calculations for an average $T_\pi = 0.93$ Bev. The results shown in the present paper were calculated for this energy.

It is seen that the curves calculated from the isobar model have a double maximum, which is especially

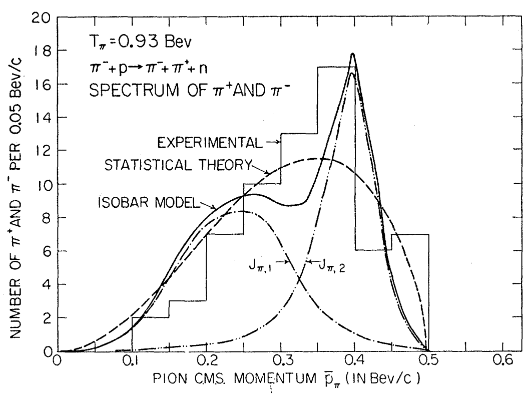

FIG. 1. Center-of-mass momentum spectrum of π^- and π^+ mesons combined from the reaction $\pi^- + p \rightarrow \pi^- + \pi^+ + n$ at an incident pion energy $T_\pi = 0.93$ Bev. The histogram represents the data of Walker *et al.* (reference 4). The solid curve was obtained from the isobar model. The dot-dashed curves give the contributions $J_{\pi,1}$ due to the pions from the isobar decay and $J_{\pi,2}$ due to the recoil pions. The dashed curve gives the result of the Fermi statistical theory. In all of the figures, the theoretical curves have been normalized to the number of observed cases.

marked at 1.37 Bev. The pronounced high-energy peak which occurs at a c.m.s. momentum $\bar{p}_\pi = 0.40$ Bev/c for $T_\pi = 0.93$ Bev and at 0.57 Bev/c for $T_\pi = 1.37$ Bev is due to the recoil pions, which are emitted with a momentum equal to the momentum of the isobar. Obviously, since the isobar is formed preferentially with a mass close to the resonance value of 1.22 Bev, the momentum of the recoil pions will generally have a value close to that which corresponds to an isobar of mass $m_I = 1.22$ Bev. On the other hand, the broad maximum of the isobar model curves at $\bar{p}_\pi \cong 0.26$ Bev/c is due to the isobar decay pions, which have a wide momentum distribution both because of the motion of the N^* and the Q value distribution in the rest system

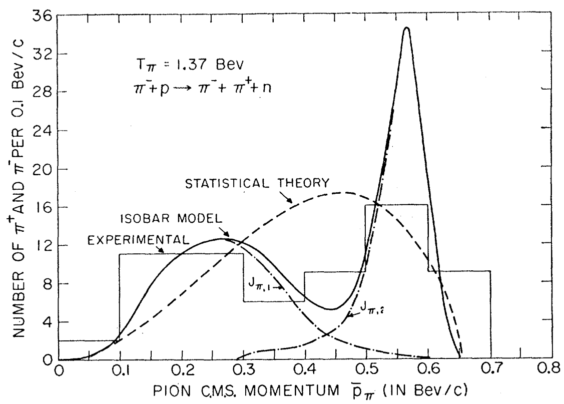

FIG. 2. Center-of-mass momentum spectrum of π^- and π^+ mesons combined from the reaction $\pi^- + p \rightarrow \pi^- + \pi^+ + n$ at an incident pion energy $T_\pi = 1.37$ Bev. The histogram represents the data of Eisberg *et al.* (reference 5). The solid curve was obtained from the isobar model. The dot-dashed curves give the terms $J_{\pi,1}$ and $J_{\pi,2}$. The dashed curve gives the result of the Fermi statistical theory.

of N^*, with a maximum at $\sim$140 Mev. The experimental pion spectrum at 0.93 Bev has a high-momentum peak in reasonable agreement with the prediction of the isobar model. The predicted slight depression between the low-energy and the high-energy peak is absent, and would be expected to be washed out by the crude momentum resolution resulting from the grouping of the experimental data. However, the general behavior of the data is consistent with the isobar model predictions. Nevertheless, one should note that the statistical uncertainties are quite large. The observed pion momentum spectrum at $T_\pi = 1.37$ Bev (Fig. 2) has a double maximum at the momenta expected from the isobar model. However, the statistical uncertainties are also quite large, so that one cannot necessarily conclude that this effect is real.

Figures 3 and 4 show the pion momentum spectra

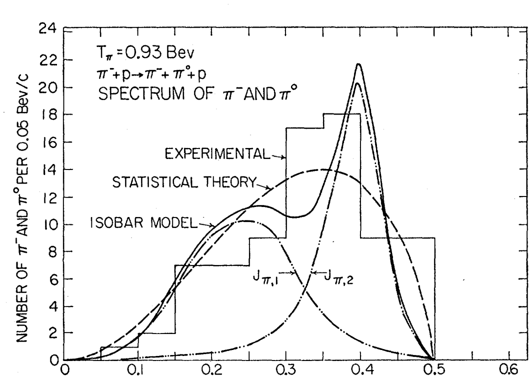

FIG. 3. Center-of-mass momentum spectrum of π^- and π^0 mesons combined from the reaction $\pi^- + p \rightarrow \pi^- + \pi^0 + p$ at $T_\pi = 0.93$ Bev. The histogram represents the data of Walker *et al.* (reference 4). The solid curve was obtained from the isobar model. The dot-dashed curves give the terms $J_{\pi,1}$ and $J_{\pi2,}$. The dashed curve gives the result of the Fermi statistical theory.

[8] E. Fermi, Progr. Theoret. Phys. (Japan) **5**, 570 (1950); Phys. Rev. **92**, 452 (1953); **93**, 1435 (1954).
[9] M. M. Block, Phys. Rev. **101**, 796 (1956).

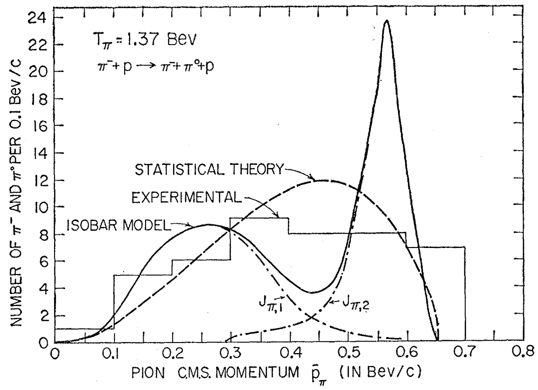

FIG. 4. Center-of-mass momentum spectrum of π^- and π^0 mesons combined from the reaction $\pi^- + p \to \pi^- + \pi^0 + p$ at $T_\pi = 1.37$ Bev. The histogram represents the data of Eisberg *et al.* (reference 5). The solid curve was obtained from the isobar model. The dot-dashed curves give the terms $J_{\pi,1}$ and $J_{\pi,2}$. The dashed curve gives the result of the Fermi statistical theory.

from the reaction $\pi^- + p \to \pi^- + \pi^0 + p$ at 0.93 and 1.37 Bev.[4,5] As in Figs. 1 and 2, we have combined both types of pions (π^- and π^0), so that the predicted spectrum shape is given by Eq. (9). The situation at 0.93 Bev, as shown by Fig. 3, is very similar to the corresponding results for the reaction $\pi^- + p \to \pi^- + \pi^+ + n$ (Fig. 1). The isobar model curve agrees on the whole better with the data than does the statistical theory. The experimental distribution has a strong maximum between 0.3 and 0.4 Bev/c, which is in approximate agreement with the calculated maximum due to the recoil pions. We note that the statistical theory does not predict a maximum at high momenta, such as is observed both for the $\pi^- + \pi^+ + n$ and $\pi^- + \pi^0 + p$ reactions.

As can be seen from Fig. 4, the curve obtained from the isobar model for the reaction $\pi^- + p \to \pi^- + \pi^0 + p$ is not in agreement with the data at 1.37 Bev. In particular, the experimental momentum distribution does not show a double maximum. However, it cannot be concluded at present that there is necessarily a real disagreement with the theory, in view of the limited statistics of the data for this reaction and the larger experimental uncertainties in the determination of the π^0 momenta as compared to the momenta of charged pions.

For the validity of the isobar model, it is necessary that the isobar and the recoil pion separate before the isobar decay takes place, or at least that the interaction between the isobar decay products and the recoil pion is relatively small. The same restriction was encountered previously in the treatment of the isobar model for nucleon-nucleon collisions.[3] However, the assumption of complete separation is probably better satisfied than for the nucleon-nucleon case at comparable energies, because of the large energy of the recoil pion. Thus the recoil pion tends to move a greater distance during the decay time ($\sim 10^{-23}$ sec) than the unexcited nucleon (or the second isobar N^*) in the nucleon-nucleon collision. Moreover, on account of its large velocity, the recoil pion will generally have too high an energy relative to the nucleon from the N^* decay to be strongly interacting with this nucleon via the $T = J = \frac{3}{2}$ resonance.

The spectrum of the recoil nucleons is obtained from the following equation, similar to the first term of (1):

$$\frac{d^2\sigma}{d\bar{T}_N d\bar{\Omega}_N} = B \int_{M_1}^{M_2} \sigma(m_I) F G_N dm_I, \tag{10}$$

where G_N is a factor giving the energy distribution of the nucleons arising from the decay of an isobar of mass m_I, with c.m.s. velocity $\bar{v}_I$. For the energy spectrum of nucleons integrated over all angles, $d\sigma/d\bar{T}_N$, G_N is given by the following equations, similar to (2) and (2a):

$$G_N = 1/(\bar{T}_{N,\max} - \bar{T}_{N,\min}), \quad (\bar{T}_{N,\min} < \bar{T}_N < \bar{T}_{N,\max}) \tag{11}$$

$$G_N = 0, \quad (\bar{T}_N < \bar{T}_{N,\min} \text{ and } \bar{T}_N > \bar{T}_{N,\max}), \tag{11a}$$

where $\bar{T}_{N,\min}$ and $\bar{T}_{N,\max}$ are the minimum and maximum possible kinetic energies of the nucleons from the isobar decay. One should note that the isobar model predicts the same spectrum $d\sigma/d\bar{T}_N$ for the recoil protons and neutrons.

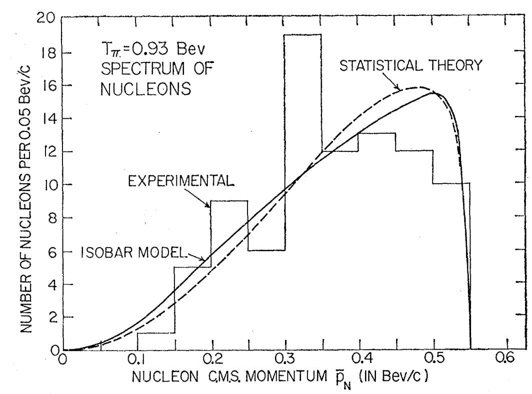

FIG. 5. Center-of-mass momentum spectrum of recoil nucleons from the reactions $\pi^- + p \to \pi^- + \pi^+ + n$ and $\pi^- + p \to \pi^- + \pi^0 + p$ at $T_\pi = 0.93$ Bev. The histogram represents the data of Walker *et al.* (reference 4). The solid curve was obtained from the isobar model. The dashed curve gives the result of the Fermi statistical theory.

Figures 5 and 6 show the momentum spectra of the recoil nucleons at 0.93 and 1.37 Bev. It is seen that the predictions of the isobar model are in reasonable agreement with the data. The curves obtained from the statistical theory appear to agree by coincidence with the results of the isobar model for the nucleon spectrum. It may be noted that the statistical theory spectra for both the nucleons and pions were obtained from the appropriate three-body phase space factor.[9]

Figure 7 shows the angular correlation between the two pions for the inelastic scattering cases at 0.93 Bev.[4] The theoretical curve obtained from the isobar model is in reasonable agreement with the data. The calculated curve is given by

$$\frac{dn}{d\cos\bar{\theta}_{\pi\pi}}=\int_{M_1}^{M_2}\sigma(m_I)FJ(\bar{\chi}_\pi)dm_I \bigg/ \int_{M_1}^{M_2}\sigma(m_I)Fdm_I, \tag{12}$$

where $\bar{\theta}_{\pi\pi}$ is the c.m.s. angle between the two pions, $J(\bar{\chi}_\pi)$ is the Jacobian for the decay of the isobar N^* into a pion making an angle $\bar{\chi}_\pi$ with the direction of N^*; $\bar{\chi}_\pi$ must be taken as $180°-\bar{\theta}_{\pi\pi}$. $J(\bar{\chi}_\pi)$ is given by[3]

$$J(\bar{\chi}_\pi)=\frac{1-\bar{v}_I^2}{(1+\bar{\rho}^2-2\bar{\rho}\cos\bar{\chi}_\pi-\bar{v}_I^2\sin^2\bar{\chi}_\pi)^{\frac{1}{2}}(1-\bar{\rho}\cos\bar{\chi}_\pi)}. \tag{13}$$

Here $\bar{v}_I$=velocity of isobar, and $\bar{\rho}\equiv\bar{v}_I/\bar{v}_\pi$, where $\bar{v}_\pi$ is the velocity of the pion in the c.m. system. As in the similar case for nucleon-nucleon collisions (see reference 3, Sec. VII), the integral in the numerator of Eq. (12) represents the weighted average of J over all mass values m_I, whereas the integral in the denominator serves merely to normalize the correlation function $dn/d\cos\bar{\theta}_{\pi\pi}$.

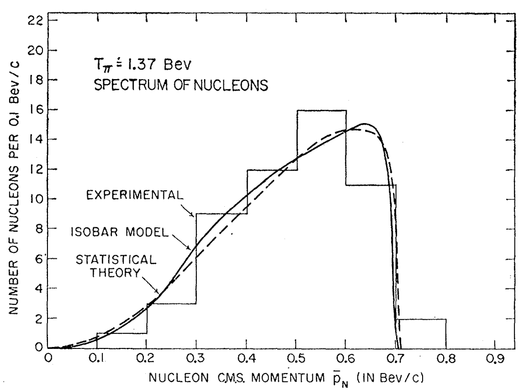

FIG. 6. Center-of-mass momentum spectrum of recoil nucleons from the reactions $\pi^-+p\rightarrow\pi^-+\pi^++n$ and $\pi^-+p\rightarrow\pi^-+\pi^0+p$ at $T_\pi=1.37$ Bev. The histogram represents the data of Eisberg *et al.* (reference 5). The solid curve was obtained from the isobar model. The dashed curve gives the result of the Fermi statistical theory.

In addition to the measurements of Walker *et al.*[4] at 0.93 Bev and of Eisberg *et al.*[5] at 1.37 Bev, Walker and Crussard[10] have made a study of π^--nucleon collisions in emulsion at 1.5 Bev. Although no detailed calculations were carried out for $T_\pi=1.5$ Bev, we note that the results of Walker and Crussard[10] seem to bear out qualitatively the predictions of the isobar model in several respects. In Fig. 14 of their paper, the combined momentum distribution of the pions observed in the single production cases shows a double peak with maxima at $\sim$0.3 and 0.6 Bev/c, in good agreement with the isobar model prediction. Figure 16 of reference 10 shows that the nucleons tend to come off at $\sim$180° to the fast pions. This is also expected from the isobar model, since the nucleons tend to follow the direction of motion of the isobar, whereas the fast pions are generally recoil pions which are emitted opposite to the isobar direction. Figure 18 of Walker and Crussard[10] shows that the two pions tend to be emitted at large angles with respect to each other with maximum probability at 180°. This feature was already apparent at 0.93 Bev (see Fig. 7). The pion from the isobar decay tends to follow the direction of the isobar and thus tends to go at $\sim$180° to the recoil pion, particularly at high incident energies for which the resulting velocity $\bar{v}_I$ of the isobar will be large. Indeed the distribution $dn/d\cos\bar{\theta}_{\pi\pi}$ is considerably more peaked for backward angles at 1.5 Bev[10] than at 0.93 Bev[4] (i.e., the ratio of the number of backward to forward events is 78/15 =5.20 at 1.5 Bev as compared to 57/34=1.68 at 0.93 Bev). In Fig. 23 of their paper, Walker and Crussard[10] have plotted the momentum distribution of the pions from the reaction $\pi^-+n\rightarrow 2\pi^-+p$. This distribution has a pronounced double peak, and provides additional support for the present isobar model involving the production of a fast recoil pion.

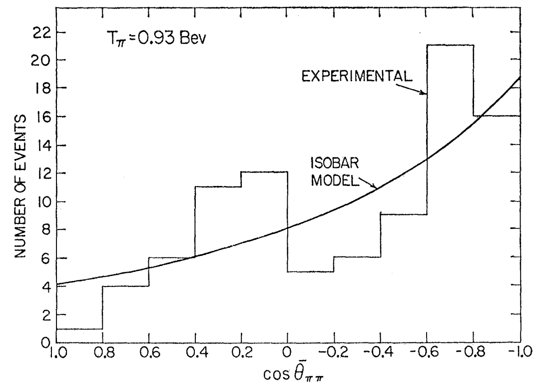

FIG. 7. Center-of-mass distribution of the angle $\bar{\theta}_{\pi\pi}$ between the two pions for the single production cases at $T_\pi=0.93$ Bev. The histogram represents the data of Walker *et al.* (reference 4). The curve was obtained from the isobar model and has been normalized to the number of observed cases.

If one combines all of the pion spectra observed both at 1.37 Bev by Eisberg *et al.*[5] and at 1.50 Bev by Walker and Crussard,[10] one obtains the histogram shown in Fig. 8 (solid lines). This histogram represents 222 pions (108 at 1.37 Bev and 114 at 1.50 Bev), so that the statistics are relatively good. For comparison, we have shown in Fig. 8 the histograms which would be expected from the isobar model (dashed lines) and from the statistical theory (dot-dashed lines). The momentum interval was taken as 0.2 Bev/c, in order to improve the statistics and to reduce the effects due to the difference between the two incident energies, $T_\pi=1.37$ and 1.50 Bev. Actually the differences in the spectra are expected to be small, except near the momentum cutoff, which is at $\bar{p}_\pi=0.65$ Bev/c for $T_\pi=1.37$ Bev and 0.70 Bev/c for $T_\pi=1.50$ Bev. The histograms for the isobar model

[10] W. D. Walker and J. Crussard, Phys. Rev. 98, 1416 (1955).

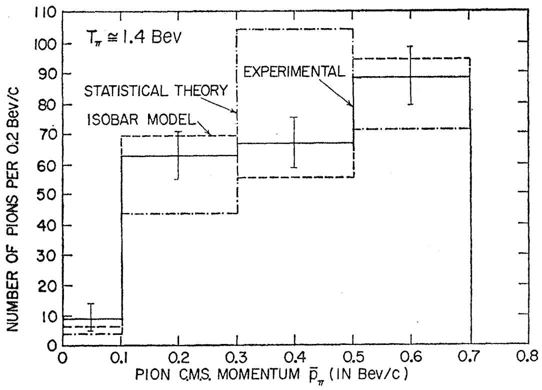

FIG. 8. Center-of-mass momentum spectrum of all pions combined at 1.37 and 1.50 Bev. The solid lines show the histogram obtained by combining the data of Eisberg *et al.*[5] at 1.37 Bev and those of Walker and Crussard[10] at 1.50 Bev. The dashed lines give the prediction of the isobar model. The dot-dashed lines show the result of the Fermi statistical theory.

and for the statistical theory were obtained by taking the area of the corresponding theoretical curves in each momentum interval and by using a rectangle having the same area. Of course, the complete area of each curve was normalized to the total number of pions. The appropriate curve for the isobar model was that previously calculated for $T_\pi=1.37$ Bev (see Figs. 2 and 4). The histogram for the statistical theory is based on the relativistic three-body phase space factor[9] calculated for an average energy $T_\pi=1.44$ Bev. As discussed above, the difference between $T_\pi=1.37$ and 1.44 Bev is not regarded as significant, especially in view of the wide momentum interval used.

For the histogram of the experimental data, the vertical bars represent one standard deviation. It is seen that the isobar model predictions are in good agreement with the data, whereas the spectrum of the statistical theory is not. Thus the isobar model histogram is within one standard deviation of the data, except for the interval 0.3–0.5 Bev/c, where the difference is 1.4 standard deviations. By contrast, the statistical theory spectrum lies outside the statistical errors throughout the momentum range; the differences are 2.4, 4.6, and 1.8 standard deviations for the regions 0.1–0.3, 0.3–0.5, and 0.5–0.7 Bev/c, respectively. Figure 8 shows that the statistical theory is in disagreement with the general momentum dependence of the spectrum, whereas the isobar model agrees quite well. The isobar model predicts a relatively flat spectrum up to 0.5 Bev/c, and a pronounced high-energy peak between 0.5 and 0.7 Bev/c. This prediction is confirmed by the data which show a significantly larger number of pions in the 0.5–0.7 Bev/c interval than in any of the lower-momentum groups. On the other hand, the statistical theory gives a maximum in the middle region (0.3–0.5 Bev/c), which is in pronounced disagreement with the data (4.6 standard deviations). Actually, the minimum of the isobar model curve ($J_{\pi,1}+J_{\pi,2}$) occurs in this middle region (at $\bar{p}_\pi=0.44$ Bev/c), but this feature does not appear prominently in the histogram, since the 0.3–0.5 Bev/c region also includes a part of the maximum due to the decay pions ($J_{\pi,1}$).

It can thus be concluded that the isobar model predictions are in good agreement with the combined pion spectrum for the 1.37–1.50 Bev region, whereas the statistical theory is in pronounced disagreement with the data. In particular, the statistical theory would predict a maximum at intermediate momenta (0.3–0.5 Bev/c), whereas the data show a maximum at high momenta (0.5–0.7 Bev/c), which is satisfactorily accounted for by the isobar model.

In connection with the assumption of isotropic decay of the N^* in its own rest system, which was made in Eq. (2), we note that the spectrum of the recoil pions is obviously independent of this assumption. Thus, irrespective of the angular distribution of the isobar decay products, we expect no effect on the shape of the high-energy peak due to the recoil pions. On the other hand, the decay pion spectrum would be affected by a departure of the angular distribution from isotropic. In particular, if the isobar decays preferentially forward and backward in the rest system, this would broaden the distribution $I_{\pi,1}$, with relatively more high- and low-energy pions, and fewer pions of intermediate energies.

Crew, Hill, and Lavatelli[11] have applied the isobar model to the production of pions in pion-nucleon collisions at incident energies of 0.93, 1.37, and 1.50 Bev. These authors have used the Monte Carlo method to obtain the momentum spectra of all pions combined from both reactions. Their results are in general similar to those presented above, considering the wide momentum interval of the groupings and the statistical uncertainties present in their Monte Carlo method of evaluation.

III. BRANCHING RATIOS FOR VARIOUS REACTIONS AND SEPARATE ENERGY SPECTRA OF π^+, π^0, AND π^- MESONS

From the assumption that the pion-nucleon interactions proceed via formation of a nucleon isobar with $T=J=\frac{3}{2}$, and using conservation of isotopic spin, one can obtain the branching ratios for the various possible charge states of the pions and the corresponding pion energy spectra.

We shall consider first the case of incident π^- mesons on protons. As is well known, the total $\pi^- - p$ cross section is given by

$$\sigma_{\pi^- - p}=\tfrac{2}{3}\sigma_{\frac{1}{2}}+\tfrac{1}{3}\sigma_{\frac{3}{2}}, \qquad (14)$$

where $\sigma_{\frac{1}{2}}$ and $\sigma_{\frac{3}{2}}$ are the total cross sections for isotopic spin $T=\frac{1}{2}$ and $\frac{3}{2}$, respectively. The same relation holds for the elastic part and the inelastic part of the cross section separately. For the state $T=\frac{1}{2}$, the wave function of the final state consisting of the isobar and

[11] Crew, Hill, and Lavatelli, Phys. Rev. **106**, 1051 (1957).

the recoil pion is given by

$$\Psi_{\pi^- - p}(T=\tfrac{1}{2}) = (\tfrac{1}{2})^{\frac{1}{2}}\psi_{-\frac{3}{2}}\eta_1 - (\tfrac{1}{3})^{\frac{1}{2}}\psi_{-\frac{1}{2}}\eta_0 + (\tfrac{1}{6})^{\frac{1}{2}}\psi_{\frac{1}{2}}\eta_{-1}, \quad (15)$$

where ψ_{t_z} is the wave function of the isobar with z-component of isotopic spin t_z, and η_{t_z} is the wave function of the recoil pion (η_1 corresponds to π^+, etc.). Similarly, for the state $T=\frac{3}{2}$, Ψ is given by

$$\Psi_{\pi^- - p}(T=\tfrac{3}{2}) = -(\tfrac{2}{5})^{\frac{1}{2}}\psi_{-\frac{3}{2}}\eta_1 - (\tfrac{1}{15})^{\frac{1}{2}}\psi_{-\frac{1}{2}}\eta_0 + (\tfrac{8}{15})^{\frac{1}{2}}\psi_{\frac{1}{2}}\eta_{-1}. \quad (16)$$

In the decay of the isobar, $\psi_{\frac{1}{2}}$, $\psi_{-\frac{1}{2}}$, and $\psi_{-\frac{3}{2}}$ are equivalent to the following:

$$\psi_{\frac{1}{2}} = (\tfrac{2}{3})^{\frac{1}{2}}\mu_0\chi_{\frac{1}{2}} + (\tfrac{1}{3})^{\frac{1}{2}}\mu_1\chi_{-\frac{1}{2}}, \quad (17)$$

$$\psi_{-\frac{1}{2}} = (\tfrac{1}{3})^{\frac{1}{2}}\mu_{-1}\chi_{\frac{1}{2}} + (\tfrac{2}{3})^{\frac{1}{2}}\mu_0\chi_{-\frac{1}{2}}, \quad (18)$$

$$\psi_{-\frac{3}{2}} = \mu_{-1}\chi_{-\frac{1}{2}}, \quad (19)$$

where μ_{t_z} and χ_{t_z} are the wave functions of the decay pion and the nucleon, respectively ($\chi_{\frac{1}{2}}$=proton, $\chi_{-\frac{1}{2}}$=neutron). We define ρ as the ratio:

$$\rho \equiv \sigma_{\frac{3}{2},\,\mathrm{inel}}/(2\sigma_{\frac{1}{2},\,\mathrm{inel}}), \quad (20)$$

where $\sigma_{\frac{1}{2},\,\mathrm{inel}}$ and $\sigma_{\frac{3}{2},\,\mathrm{inel}}$ are the inelastic (pion production) parts of $\sigma_{\frac{1}{2}}$ and $\sigma_{\frac{3}{2}}$, respectively. The normalized energy spectra for the decay poins and the recoil pions are denoted by $I_{\pi,1}$ and $I_{\pi,2}$, respectively.

The following reactions can take place for single pion production:

$$\pi^- + p \to \pi^- + \pi^+ + n, \quad (\mathrm{I})$$

$$\pi^- + p \to \pi^- + \pi^0 + p, \quad (\mathrm{II})$$

$$\pi^- + p \to 2\pi^0 + n. \quad (\mathrm{III})$$

The pion spectra are obtained from the expression for $|\Psi_{\pi^- - p}{}^{(\mathrm{final})}|^2$,

$$|\Psi_{\pi^- - p}{}^{(\mathrm{final})}|^2 = |\Psi_{\pi^- - p}(T=\tfrac{1}{2}) - \rho^{\frac{1}{2}}\exp(i\varphi) \times \Psi_{\pi^- - p}(T=\tfrac{3}{2})|^2, \quad (21)$$

where φ is the phase difference between the matrix elements for pion production in the $T=\frac{1}{2}$ and $T=\frac{3}{2}$ state. One thus obtains the following pion spectra for the three reactions:

$$I_{\pi^- - p}{}^{(\mathrm{I})}(\pi^-) = (\tfrac{1}{2} + \tfrac{2}{5}\rho + a)I_{\pi,1} + (\tfrac{1}{18} + \tfrac{8}{45}\rho - \tfrac{2}{9}a)I_{\pi,2}, \quad (22)$$

$$I_{\pi^- - p}{}^{(\mathrm{I})}(\pi^+) = (\tfrac{1}{18} + \tfrac{8}{45}\rho - \tfrac{2}{9}a)I_{\pi,1} + (\tfrac{1}{2} + \tfrac{2}{5}\rho + a)I_{\pi,2}, \quad (23)$$

$$I_{\pi^- - p}{}^{(\mathrm{II})}(\pi^-) = (\tfrac{1}{9} + \tfrac{1}{45}\rho - \tfrac{1}{9}a)I_{\pi,1} + (\tfrac{1}{9} + \tfrac{16}{45}\rho - \tfrac{4}{9}a)I_{\pi,2}, \quad (24)$$

$$I_{\pi^- - p}{}^{(\mathrm{II})}(\pi^0) = (\tfrac{1}{9} + \tfrac{16}{45}\rho - \tfrac{4}{9}a)I_{\pi,1} + (\tfrac{1}{9} + \tfrac{1}{45}\rho - \tfrac{1}{9}a)I_{\pi,2}, \quad (25)$$

$$I_{\pi^- - p}{}^{(\mathrm{III})}(\pi^0) = [\tfrac{2}{9}(1-a) + \tfrac{2}{45}\rho](I_{\pi,1} + I_{\pi,2}), \quad (26)$$

where the notation $I_{\pi^- - p}{}^{(\mathrm{I})}(\pi^-)$ means the π^- spectrum from reaction (I), etc. In Eqs. (22)–(26), a is defined as:

$$a \equiv 2(\rho/5)^{\frac{1}{2}}\cos\varphi. \quad (27)$$

The terms containing a represent the interference terms between the production in the $T=\frac{1}{2}$ and $T=\frac{3}{2}$ states. The normalization of Eqs. (22)–(26) is such that the sum of all the I's is:

$$\sum_{ij} I_{\pi^- - p}{}^{(i)}(\pi^j) = (1+\rho)(I_{\pi,1} + I_{\pi,2}). \quad (28)$$

It may be noted that the sums

$$[I_{\pi^- - p}{}^{(\mathrm{I})}(\pi^-) + I_{\pi^- - p}{}^{(\mathrm{I})}(\pi^+)]$$

and

$$[I_{\pi^- - p}{}^{(\mathrm{II})}(\pi^-) + I_{\pi^- - p}{}^{(\mathrm{II})}(\pi^0)]$$

are both proportional to $I_{\pi,1} + I_{\pi,2}$, as is also obvious on general grounds, since in each reaction, the numbers of recoil pions and isobar decay pions irrespective of charge are equal. This feature has been used above in Figs. 1–4 [see Eq. (9)] to obtain momentum spectra which are independent of ρ and φ.

From the coefficients of Eqs. (22)–(26) one can determine the relative probabilities $P_{\pi^- - p}{}^{(i)}$ of the three reactions, resulting from the previously stated assumptions of the isobar model. One obtains

$$P_{\pi^- - p}{}^{(\mathrm{I})} = (1+\rho)^{-1}[\tfrac{5}{9} + \tfrac{26}{45}\rho + \tfrac{7}{9}a], \quad (29)$$

$$P_{\pi^- - p}{}^{(\mathrm{II})} = (1+\rho)^{-1}[\tfrac{2}{9} + \tfrac{17}{45}\rho - \tfrac{5}{9}a], \quad (30)$$

$$P_{\pi^- - p}{}^{(\mathrm{III})} = (1+\rho)^{-1}[\tfrac{2}{9} + \tfrac{2}{45}\rho - \tfrac{2}{9}a]. \quad (31)$$

For the case of incident π^+ on protons, only the $T=\frac{3}{2}$ state is involved, and the total wave function Ψ for the final state is given by

$$\Psi_{\pi^+ - p} = -(\tfrac{2}{5})^{\frac{1}{2}}\psi_{\frac{1}{2}}\eta_1 + (\tfrac{3}{5})^{\frac{1}{2}}\psi_{\frac{3}{2}}\eta_0. \quad (32)$$

The possible reactions are

$$\pi^+ + p \to \pi^+ + \pi^0 + p, \quad (\mathrm{IV})$$

$$\pi^+ + p \to 2\pi^+ + n. \quad (\mathrm{V})$$

From Eqs. (17)–(19) and (32), one obtains the following energy spectra for the π^+ and π^0 mesons from these two reactions:

$$I_{\pi^+ - p}{}^{(\mathrm{IV})}(\pi^+) = (\tfrac{3}{5})I_{\pi,1} + (\tfrac{4}{15})I_{\pi,2}, \quad (33)$$

$$I_{\pi^+ - p}{}^{(\mathrm{IV})}(\pi^0) = (\tfrac{4}{15})I_{\pi,1} + (\tfrac{3}{5})I_{\pi,2}, \quad (34)$$

$$I_{\pi^+ - p}{}^{(\mathrm{V})}(\pi^+) = (\tfrac{2}{15})(I_{\pi,1} + I_{\pi,2}). \quad (35)$$

The relative probabilities of the two reactions are

$$P_{\pi^+ - p}{}^{(\mathrm{IV})} = \tfrac{13}{15}; \quad P_{\pi^+ - p}{}^{(\mathrm{V})} = \tfrac{2}{15}. \quad (36)$$

Thus the isobar model predicts that reaction (IV) is far more frequent than (V) (by a factor of 6.5). An experimental check of this prediction would obviously be of considerable value as a test of the present isobar model.

In connection with Eqs. (15), (16), and (32), we note that it is not necessary to symmetrize Ψ with respect to the wave functions ψ_{t_z} and η_{t_z}, since the isobar and the recoil pion are different particles in this treatment, and the isobar decay is assumed to occur after separation of the two particles to a distance beyond the range of the interaction of the recoil pion with the isobar decay products.

It is of interest to compare the prediction of the isobar model for the ratio $P_{\pi^--p}^{(\mathrm{II})}/P_{\pi^--p}^{(\mathrm{I})}\equiv R$ with the experimentally observed ratio R_{exp} of the reactions (I) and (II). R is given by

$$R=(10+17\rho-25a)/(25+26\rho+35a). \qquad (37)$$

We note that the theoretical value of R can vary over a wide range. The maximum possible value of R, R_{max}, is 2.51, and is obtained for $\rho=1.093$ and $\varphi=180°$. The minimum possible R, R_{min}, is 0.044, corresponding to $\rho=0.561$ and $\varphi=0°$.

In order to compare the calculated R with the experimentally observed ration R_{exp}, one must make some assumptions about ρ [Eq. (20)]. The comparison will first be made at 0.93 Bev. At this energy, the total cross sections $\sigma_{\frac{1}{2}}$ and $\sigma_{\frac{3}{2}}$ are[12]: $\sigma_{\frac{1}{2}}=61$ mb, $\sigma_{\frac{3}{2}}=23$ mb. We shall first assume that the ratio of the inelastic parts of $\sigma_{\frac{3}{2}}$ and $\sigma_{\frac{1}{2}}$ is given simply by the ratio of the total cross sections $\sigma_{\frac{3}{2}}$ and $\sigma_{\frac{1}{2}}$:

$$\sigma_{\frac{3}{2},\,\mathrm{inel}}/\sigma_{\frac{1}{2},\,\mathrm{inel}}=\sigma_{\frac{3}{2}}/\sigma_{\frac{1}{2}}. \qquad (38)$$

Equation (38) gives $\rho=0.189$. The value of Walker *et al.*[4] from the cloud chamber data at 960 Mev is:

$$R_{\mathrm{exp}}=(6.9\pm2)/(9.5\pm2)=0.73\pm0.26.$$

Upon inserting $R=0.73$ and $\rho=0.189$ in Eq. (37) and solving for a, and then φ [Eq. (27)], one finds $\varphi=116°$. The extreme values of R_{exp}: 0.47 and 0.99 correspond to $\varphi=93°$ and 135°, respectively. Thus by assuming a value of ρ and using R_{exp}, one can obtain a crude estimate of the phase angle φ.

It will now be shown that the value of φ thus obtained is not very sensitive to the assumptions made about ρ. At 0.93 Bev, of the total $\sigma_{\pi^--p}=48$ mb, the elastic cross section σ_{el} accounts for 20 mb, while the inelastic cross section σ_{inel} is 28 mb. If one makes the extreme assumption that all of $\sigma_{\frac{3}{2}}$ is elastic in order to obtain a lower limit for ρ, one finds $\sigma_{\frac{1}{2},\,\mathrm{inel}}=42$ mb, $\rho=0$, and $R=0.40$, which is somewhat below the range of R_{exp}. On the other hand, to obtain an upper limit for ρ, one can consider the case where all of $\sigma_{\frac{3}{2}}$ is inelastic. Actually this extreme case is not physically possible, since there is always an appreciable diffraction scattering associated with the inelastic processes. Hence the upper limit so obtained is actually an overestimate. With this assumption, one finds $\sigma_{\frac{3}{2},\,\mathrm{inel}}=23$ mb, $\sigma_{\frac{1}{2},\,\mathrm{inel}}=31$ mb, so that $\rho=0.37$. Upon using $\rho=0.37$ in Eq. (37), and with $R_{\mathrm{exp}}=0.73$, one obtains $\varphi=109°$, which is of the same order as the value $\varphi=116°$ found with the estimate of $\rho(=0.189)$ of Eq. (38).

In order to make a similar comparison at $T_\pi=1.37$ Bev, we note that[12] $\sigma_{\frac{1}{2}}=25$ mb, $\sigma_{\frac{3}{2}}=41$ mb. With the assumption of Eq. (38), one finds $\rho=0.82$. In order to obtain a lower limit for ρ, we assume that all of the elastic scattering ($\sigma_{\mathrm{el}}\cong7$ mb)[5] is due to the $T=\frac{3}{2}$ state. This gives $\rho=0.43$. Finally, upon assuming that all of $\sigma_{\frac{3}{2}}$ is inelastic, one finds $\rho=1.34$ for an upper limit. Experimentally (see Table III of reference 5), there are 16 cases of reactions in which $p+\pi^-+\pi^0$ are produced, and 6 events which could be either $p+\pi^-+\pi^0$ or $p+\pi^-+2\pi^0$. There are 8 events which are certainly $n+\pi^++\pi^-$ and 23 cases of either $n+\pi^++\pi^-$ or $n+\pi^++\pi^-+\pi^0$. It is likely that most of the doubtful cases are actually single production events. If one includes all of the doubtful cases, one obtains $R_{\mathrm{exp}}=22/31=0.71\pm0.20$, whereas if only half of these cases represent single production, R_{exp} would be $19/19=1\pm0.3$. The errors given above represent estimates of the statistical uncertainties of the data. If one assumes $R_{\mathrm{exp}}=1$, Eq. (37) gives $\varphi=117.4°$ for $\rho=0.82$. This result for φ is not sensitive to the assumed value of ρ. Thus for $R=1$, one finds $\varphi=122.5°$ and $\varphi=115.8°$ for the extreme values of $\rho=0.43$ and 1.34, respectively.

Walker and Crussard[10] have made an investigation of π^--n interactions in emulsion. By using charge symmetry, the results of Eq. (36) obviously can be transformed to apply to π^--n interactions. Thus the isobar model predicts a value of 6.5 for the ratio of $\pi^-+\pi^0+n$ to $2\pi^-+p$ events. Walker and Crussard[10] have found 10 cases of the latter reaction and 107 cases which are either elastic scatterings or $n+\pi^-+\pi^0$ or $n+\pi^-+2\pi^0$. The possibility that of these 107 cases, ~65 were $n+\pi^-+\pi^0$ events (which is the predicted number) seems to be compatible with their results.

The π^- spectrum of Eq. (22) can be written as follows:

$$I_{\pi^--p}^{(\mathrm{I})}(\pi^-)=c_1I_{\pi,1}+c_2I_{\pi,2}=c_2(\xi I_{\pi,1}+I_{\pi,2}), \qquad (39)$$

where c_1, c_2, and ξ are defined by

$$c_1\equiv\tfrac{1}{2}+\tfrac{2}{5}\rho+a, \qquad (40)$$

$$c_2\equiv(1/18)+(8/45)\rho-(2/9)a, \qquad (41)$$

$$\xi\equiv c_1/c_2=(45+36\rho+90a)/(5+16\rho-20a). \qquad (42)$$

Similarly, the π^+ spectrum of Eq. (23) becomes

$$I_{\pi^--p}^{(\mathrm{I})}(\pi^+)=c_2(I_{\pi,1}+\xi I_{\pi,2}). \qquad (43)$$

In general, for values of φ which are consistent with our knowledge of R_{exp}, the resulting values of ξ are appreciably larger than 1. Thus the isobar model predicts that in the reaction $\pi^-+p\rightarrow\pi^-+\pi^++n$, the π^+ should be predominantly fast recoil pions, while the π^- should be emitted mostly from the isobar decay, with correspondingly low c.m. system energies.

At 0.93 Bev, using the values $\rho=0.189$ and $\varphi=116°$ obtained above, one finds $\xi=3.20$. The resulting π^+ and π^- spectra are shown in Figs. 9 and 10. It is seen that the π^+ spectrum predicted by the isobar model is in better agreement with the data than the statistical

[12] Cool, Piccioni, and Clark, Phys. Rev. **103**, 1082 (1956).

theory spectrum (dashed curve). Owing to the large value of ξ, the calculated π^+ spectrum has only one maximum, namely that due to the recoil pions. This maximum is in general agreement with the peak of the experimental distribution between 0.35 and 0.40 Bev/c.

For the π^- spectrum at 0.93 Bev (Fig. 10), the main maximum of the isobar model curve is due to the decay pions (at $\bar{p}_\pi \cong 0.25$ Bev/c), whereas the smaller subsidiary maximum at $\bar{p}_\pi \cong 0.40$ Bev/c is due to the recoil pions. In agreement with the predictions of the isobar model, the experimental distribution is very broad and practically flat from 0.2 to 0.4 Bev/c without a prominent narrow peak at high momenta, such as is observed for π^+.

At 1.37 Bev, the separate π^+ and π^- spectra have been calculated using the values $\rho=0.82$ and $\varphi=117.4°$ determined above. This gives $\xi=1.60$. For the π^+ spectrum shown in Fig. 11, the predictions of the isobar model are in disagreement with the data. The expected

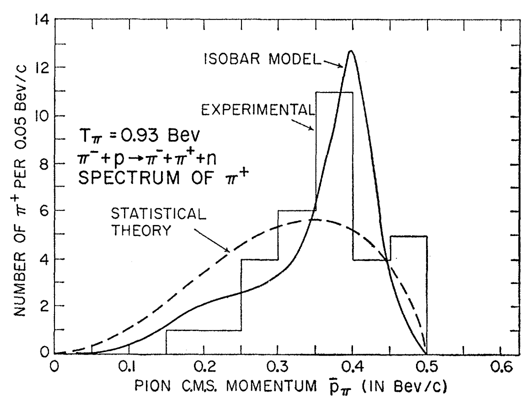

FIG. 9. Center-of-mass momentum spectrum of π^+ mesons from the reaction $\pi^-+p\to\pi^-+\pi^++n$ at $T_\pi=0.93$ Bev. The histogram represents the data of Walker *et al.* (reference 4). The solid curve was obtained from the isobar model. The dashed curve gives the result of the Fermi statistical theory.

prominent high-energy maximum due to the recoil pions (at $\bar{p}_\pi=0.57$ Bev/c) is not observed. Instead there is a maximum at low momenta (0.1 to 0.2 Bev/c) which is not obtained from the calculations. The calculated spectrum is very insensitive to the assumptions made about ρ, so that the uncertainty in ρ cannot be responsible for the disagreement. As discussed above, if one uses $R=1$, one finds that, using the lower limit for $\rho=0.43$, φ is 122.5°, which gives $\xi=1.77$. Similarly, for the upper limit $\rho=1.34$, φ is 115.8°, giving $\xi=1.49$. These results for ξ do not differ appreciably from the value $\xi=1.60$ used in obtaining Fig. 11. The value $R=1$ used in these considerations is probably near the upper limit of the range of experimental values R_{exp}. If one would use a lower value of R, the resulting ξ would increase, leading to a greater disagreement for the π^+ spectrum. Thus for $R=0.71$ and using $\rho=0.82$, Eq. (37) gives $\varphi=102.8°$, which would lead to $\xi=2.69$.

The very limited statistics may be responsible for

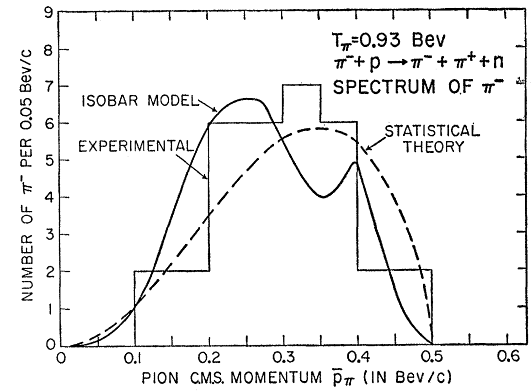

FIG. 10. Center-of-mass momentum spectrum of π^- mesons from the reaction $\pi^-+p\to\pi^-+\pi^++n$ at $T_\pi=0.93$ Bev. The histogram represents the data of Walker *et al.* (reference 4). The solid curve was obtained from the isobar model. The dashed curve gives the result of the Fermi statistical theory.

part of the disagreement for the π^+ spectrum at 1.37 Bev. It is also possible that the maximum of the experimental spectrum at low momenta (0.1 to 0.2 Bev/c) is due in part to the inclusion of some double-pion production events $n+\pi^++\pi^-+\pi^0$. For these cases, the average pion momentum is expected to be considerably lower than for the $n+\pi^++\pi^-$ events.

The π^- spectrum at 1.37 Bev is shown in Fig. 12. It is seen that the isobar model predictions agree in general features with the experimental data. In particular, the curve of the isobar model has two maxima at approximately the two peaks of the experimental distribution (~0.25 and 0.55 Bev/c).

IV. SUMMARY AND CONCLUSIONS

We have presented a model of pion production in pion-nucleon collisions, based on the idea that the incident pion excites the nucleon to the isobaric state with $T=J=\frac{3}{2}$, which subsequently decays into a nucleon and a pion. The conservation of isotopic spin

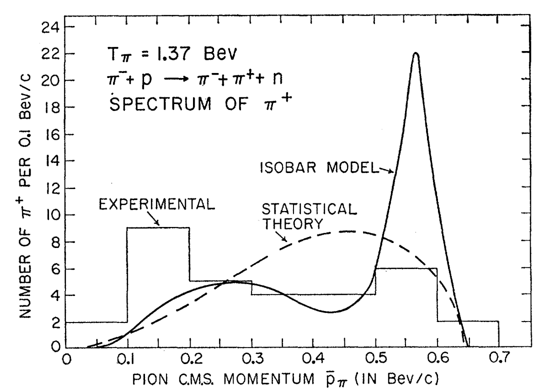

FIG. 11. Center-of-mass momentum spectrum of π^+ mesons from the reaction $\pi^-+p\to\pi^-+\pi^++n$ at $T_\pi=1.37$ Bev. The histogram represents the data of Eisberg *et al.* (reference 5). The solid curve was obtained from the isobar model. The dashed curve gives the result of the Fermi statistical theory.

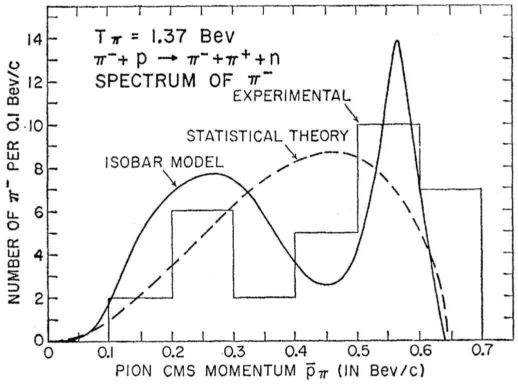

FIG. 12. Center-of-mass momentum spectrum of π^- mesons from the reaction $\pi^- + p \rightarrow \pi^- + \pi^+ + n$ at $T_\pi = 1.37$ Bev. The histogram represents the data of Eisberg *et al.* (reference 5). The solid curve was obtained from the isobar model. The dashed curve gives the result of the Fermi statistical theory.

is assumed both in the production and the decay of the isobar. This model is completely analogous to the isobar model for single pion production in nucleon-nucleon collisions, which has been previously discussed.[3] In particular, the single production by pions is similar to the formation of $N^* + N$ in the nucleon-nucleon case, with the unexcited nucleon (N) being replaced by the recoil pion.

It has been previously shown[1] that the isobar model for pion-nucleon interactions enables one to explain why the $T = \frac{1}{2}$ cross section $\sigma_{\frac{1}{2}}$ is essentially zero below $T_\pi \sim 200$ Mev and rises rapidly above this energy. According to the present model, the pion-nucleon interaction can take place in the $T = \frac{1}{2}$ state only if it is possible to form a $T = \frac{3}{2}$ isobar and a separate recoil pion. This reaction has a threshold at ~ 200 Mev. Reasonable agreement with the observed energy dependence of $\sigma_{\frac{1}{2}}$ near threshold can be obtained by assuming that $\sigma_{\frac{1}{2}}$ is proportional to $\int_{M_1}^{M_2} \sigma_{\frac{3}{2}} F dm_I$, where $\sigma_{\frac{3}{2}}$ is the $\pi^+ - p$ scattering cross section and F is the two-body phase space factor.

In the isobar model, one expects that the elastic scattering in the $T = \frac{1}{2}$ state ($\sigma_{\frac{1}{2}, \mathrm{el}}$) is in the nature of a diffraction scattering accompanying the inelastic cross section, and is therefore a direct consequence of the inelastic processes. The smallness of $\sigma_{\frac{1}{2}, \mathrm{el}}$ below the threshold energy and the rapid rise of $\sigma_{\frac{1}{2}, \mathrm{el}}$ above 200 Mev are both consistent with this prediction. We note that at 0.93 Bev, where $\sigma_{\frac{1}{2}}$ has its maximum, $\sigma_{\frac{1}{2}, \mathrm{el}}$ has reached a value of ~ 25 mb, whereas at 1.37 Bev, beyond the peak of $\sigma_{\frac{1}{2}}$, $\sigma_{\frac{1}{2}, \mathrm{el}}$ has decreased to ~ 5 mb. It thus appears that the energy dependence of $\sigma_{\frac{1}{2}, \mathrm{el}}$ is qualitatively similar to the behavior of the total $T = \frac{1}{2}$ cross section $\sigma_{\frac{1}{2}}$.

The isobar model has been applied to obtain the momentum spectra of the pions and recoil nucleons at incident pion energies $T_\pi = 0.93$ and 1.37 Bev, for which experimental data are available. One of the characteristic predictions of the present model is the double maximum of the combined momentum distribution for both pions from the reaction considered ($\pi^- + \pi^+ + n$ or $\pi^- + \pi^0 + p$). The high-energy peak arises from the recoil pions, whereas the broad maximum at low energies is due to the pions from the isobar decay. The double peak is especially pronounced at 1.37 Bev, and has been observed for the reaction $\pi^- + p \rightarrow \pi^- + \pi^+ + n$ at this energy.[5] Moreover, a double maximum was found by Walker and Crussard[10] at 1.5 Bev for the momentum distributions of the pions both from $\pi^- - p$ and $\pi^- - n$ interactions. The predicted momentum spectra of the recoil nucleons are in reasonable agreement with the data at 0.93 and 1.37 Bev. The distribution of the c.m.s. angle $\bar{\theta}_{\pi\pi}$ between the two pions was calculated at 0.93 Bev, and was found to agree reasonably well with the histogram obtained by Walker *et al.*[4]

The predictions of the statistical theory of Fermi have also been obtained. At 0.93 Bev, the statistical theory spectrum for the pions generally disagrees with the data. The statistical theory fails to explain the presence of the peak at high momenta in the experimental distributions for π^+ or for both pions combined, while the isobar model does explain this feature. The same is true for the combined spectrum from the reaction $\pi^- + p \rightarrow \pi^- + \pi^+ + n$ at 1.37 Bev. For the $\pi^- + \pi^0 + p$ events at 1.37 Bev, the statistical theory agrees somewhat better with experiment than the isobar model. However, there are considerable uncertainties in the data, so that no definite conclusions can be drawn from this result. For the nucleon spectra, the statistical theory predicts approximately the same shape as the isobar model, and gives reasonable agreement with the data.

The branching ratios for the various reactions involving single pion production have been obtained, together with the corresponding expressions for the separate energy spectra of the π^+, π^0, and π^- mesons. These quantities involve the ratio ρ of the inelastic cross sections $\sigma_{\frac{3}{2}, \mathrm{inel}}$ and $2\sigma_{\frac{1}{2}, \mathrm{inel}}$, and the phase difference φ between the matrix elements for pion production in the $T = \frac{1}{2}$ and $T = \frac{3}{2}$ states. Although the total cross sections $\sigma_{\frac{1}{2}}$ and $\sigma_{\frac{3}{2}}$, and the fraction of $\sigma_{\pi^- - p}$ which is elastic, are known throughout the energy range used, the value of ρ can be determined at present only within certain limits. However, it has been shown that from the experimental ratio R_{exp} of the $\pi^- + \pi^0 + p$ reaction to the $\pi^- + \pi^+ + n$ reaction, one can determine the phase difference φ between the matrix elements for pion production. The value of φ so obtained ($\sim 120°$) is quite insensitive to the uncertainty in ρ.

The isobar model predicts that in the reaction $\pi^- + p \rightarrow \pi^- + \pi^+ + n$, the π^+ should be predominantly fast, being generally the recoil pion, while the π^- should be slower on the average, since it is generally the pion emitted in the isobar decay. We have therefore obtained the separate π^+ and π^- spectra at 0.93 and 1.37 Bev, in order to compare them with the experimental distributions.[4,5] At 0.93 Bev, the agreement of the calculations with the data is reasonably good. The observed π^- spectrum has a broad momentum distribution, while the π^+ spectrum has a maximum at high momenta (0.35–0.40 Bev/c), as is expected from the present model. However, at 1.37 Bev, the predicted π^+ spectrum is in diagreement with the data, and there is an indication that the fraction of the recoil pions which are π^+ is smaller than would be obtained from the isobar model.

It should be mentioned that the particular isobar theory proposed here can only be applied directly to single pion production in pion-nucleon collisions. At energies above $\sim$1.5 Bev, where double production becomes important, one would have to consider modifications of the present treatment of the isobar model. One such possibility is that two or more recoil pions are coupled to the $T=J=\frac{3}{2}$ isobar at these higher energies. It is also possible that a different isobar having $T=\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$, or $\frac{7}{2}$ with the same or different J values is involved in cases where double pion production takes place. Even at the higher energies ($\gtrsim$1.5 Bev), where double production plays an important role, the single pion production may still proceed primarily via the $T=J=\frac{3}{2}$ state and would then be described by the present treatment. However, a different single production isobaric level could become important.

The predictions of the isobar model are markedly different from the results of the statistical theory. Therefore, when better experimental data become available, it should be relatively simple to discriminate between the two models. At 0.93 Bev, although the statistics are limited and the momentum intervals of the data are rather wide, the over-all pattern of the experimental results is in much better agreement with the isobar model than with the statistical theory. At 1.37–1.5 Bev, some of the predictions, notably the combined pion spectra, seem to agree reasonably well with the data. However, the calculated separate π^+ spectrum seems to be in disagreement, but the statistics are quite limited for this spectrum. The very limited statistics and other errors in the available data at 1.37–1.5 Bev make it difficult to draw any definite conclusions about the general validity of the isobar model in this energy range.

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Interactions of 38- and 61-Mev Positive Pions in Deuterium*†

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Measurements have been made of the reactions (1) $\pi^++d\to\pi^++d$, (2) $\pi^++d\to\pi^++n+p$, and (3) $\pi^++d\to p+p$ at incident pion lab energies of 38 and 61 Mev using a liquid deuterium target and scintillation counters. A separation of the three processes is obtained by determining the mass and energy of reaction products using a pulse-height analysis technique on the signals from the scintillation counters. Results from process (3) are compared with other processes involving two nucleons and a meson and results from processes (1) and (2) are compared with impulse approximation calculations.

I. INTRODUCTION

WHEN positive pions are incident on deuterons the following five reactions may proceed:

$$\pi^++d\to\pi^++d \quad \text{(elastic scattering)}, \tag{1}$$
$$\to\pi^++n+p \quad \text{(inelastic scattering)}, \tag{2}$$
$$\to p+p \quad \text{(nonradiative absorption)}, \tag{3}$$
$$\to\pi^0+p+p \quad \text{(charge exchange scattering)}, \tag{4}$$
$$\to\gamma+p+p \quad \text{(radiative absorption)}. \tag{5}$$

Much of the previous work on the interactions of pions in deuterium has been concerned with the measurement of the total cross section for all five processes.[1–3] The nonradiative absorption process was the first to be individually measured,[4] because of its significance in determining the spin of the pion. More recently attempts have been made to measure the cross sections for individual processes in counter experiments at 119 Mev,[5,6] 94 and 76 Mev,[7] and 45 Mev.[8] The most complete work has been done at 85 Mev,[9] where cross sections for each

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