Neutral Pions from Proton-Proton Collisions*

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The production of neutral pions in p-p collisions has been measured from 346 to 437 Mev. The method was to detect single photons at laboratory angles of 30°, 60°, and 90° from the decay of neutral pions in the reaction $p+p\rightarrow\pi^0+p+p$. Total cross sections which were obtained in this experiment and previous results for proton energies near threshold, have an energy dependence consistent with the phenomenological theory. Additional data were obtained on neutral pion production in proton bombardment of several complex nuclei from Li to Pb. A cross section for bound neutrons was obtained by a comparison of results from H_2O and D_2O targets.

INTRODUCTION

THE most striking feature of the cross section for neutral pion production in proton-proton collisions is the steep excitation function. The earliest measurements of the reaction were carried out by Mather and Martinelli[1] at an incident proton energy of 340 Mev. They found a cross section of 0.010 ± 0.003 millibarn. Moyer and Squire[2] have obtained additional data in the range 315 to 330 Mev. For energies well above threshold the cross section is greatly increased as found by Marshall *et al.*[3] at 430 Mev, Tyapkin *et al.*[4] at 480 Mev and 670 Mev, and Soroko[5] and Prokoshkin and Tyapkin[6] in the range 400 to 660 Mev.

In order to study the main features of the above reaction in more detail, we have measured the production of neutral pions by proton bombardment of hydrogen at energies from 346 to 437 Mev.[7] These results fill a gap in the existing data between 340 and 430 Mev.

Considerable success in the interpretation of data on the production of neutral and charged pions near threshold has been achieved by Gell-Mann and Watson[8] and by Rosenfeld.[9] This treatment is on the hypothesis of charge independence, the conservation of angular momentum and parity, and the pseudoscalar nature of the pion. Table I, which is taken from Rosenfeld's paper, lists the allowable nucleon transitions, and the corresponding angular distributions and energy spectra of the pions. The cross section for neutral pion production in proton-proton collisions is denoted by σ_{11} in the notation of Rosenfeld, where the first index represents the isotopic spin of the initial state, and the second that of the final state of the two nucleons. The theory is intended to apply only near threshold, so attention is limited to angular momentum states S or P of the final two-proton system and angular momentum s or p of the pion. The spin and angular momentum of the two-nucleon system are indicated in the conventional manner.

The experimental results clearly point to a contribution from processes which leave the nucleons in a P state (class Pp with $\sigma_{11}\propto\eta^8$ or Ps with $\sigma_{11}\propto\eta^6$ or both) since the steep excitation function cannot be explained by the class $Ss(\sigma_{11}\propto\eta^2)$ alone. An analysis of all the data shows that the measurements can be well described by an excitation function of the form:

$$\sigma_{11}=A\eta^2+B\eta^6+C\eta^8.$$

A, B, and C are three empirical parameters which give the contributions to meson production of the three processes predicted for this reaction near threshold.

A subtraction, $\frac{1}{2}\sigma(D_2O-H_2O)$, was used to determine the cross section for neutral pion production from neutrons bound in D_2. Similar measurements have

TABLE I. Analysis of the reaction: $\sigma_{11}(p+p\rightarrow\pi^0+p+p)$; η is the maximum c.m. momentum available to the pion, measured in units of $m_{\pi^0}c$ where m_{π^0} is the rest mass of the neutral pion.

Class	Classification of states: Nucleons Initial	Final	Meson	J_{final}	Angular distribution	Excitation function
Ss	$^3P_0$	$^1S_0$	s	0	isotropic	η^2
Sp	none	$^1S_0$	p	1	···	forbidden
Ps	$^1S_0$	$^3P_0$	s	0	isotropic	η^6
	none	$^3P_1$	s	1	···	···
	$^1D_2$	$^3P_2$	s	2	isotropic	η^6
Pp	$^3P_1$	$^3P_0$	p	1		
	$^3P_{0,1,2}$ or $^3F_2$	$^3P_1$	p	0,1,2	$c+\cos^2\theta$	η^8
	$^3P_{1,2}$ or $^3F_{2,3}$	$^3P_2$	p	1,2,3		

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[1] J. W. Mather and E. A. Martinelli, Phys. Rev. **92**, 780 (1953).
[2] B. J. Moyer and R. K. Squire, Phys. Rev. **107**, 283 (1957).
[3] Marshall, Marshall, Nedzel, and Warshaw, Phys. Rev. **88**, 632 (1952).
[4] Tyapkin, Kozodaev, and Prokoshkin, Doklady Akad. Nauk S.S.S.R. **100**, 689 (1955).
[5] L. M. Soroko, Zhur. Eksptl. i Teoret. Fiz. **30**, 296 (1956) [translation: Soviet Phys. JETP **3**, 184 (1956)].
[6] I. D. Prokoshkin and A. A. Tyapkin, *Proceedings of the CERN Symposium on High-Energy Accelerators and Pion Physics, Geneva, 1956* (European Organization of Nuclear Research, Geneva, 1956), Vol. II, p. 385.
[7] Stallwood, Fields, Fox, and Kane, Bull. Am. Phys. Soc. Ser. II, **1**, 71 (1956).
[8] M. Gell-Mann and K. M. Watson, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Stanford, 1954), Vol. 4, p. 219.
[9] A. H. Rosenfeld, Phys. Rev. **96**, 139 (1954).

been made elsewhere at other energies.[3,4,9] The relative neutral pion yields for several nuclei from lithium to lead under bombardment by 437-Mev protons are presented. The data are in reasonable agreement with the assumption that only surface nucleons contribute to the pion production.

All data reported here have been obtained by the detection of single photons originating in the decay of neutral pions. Various possible coincidence measurements were ruled out because of the resultant low intensity. The detection of single photons, however, makes the interpretation of the data less straightforward. The observed angular distribution of events has to be related to the total neutral pion cross section through the sensitivity for photons of the detecting counters and the γ-ray spectra. In this experiment the γ-ray spectra were calculated according to the predictions by the phenomenological theory of (a) the angular distribution and (b) the energy spectrum of the neutral pions. The energy spectra for the various classes of neutral pion production, as given by Gell-Mann and Watson,[8] are listed in Table II.

TABLE II. Energy spectra in the center-of-mass system normalized to one meson; $t=E_\pi/E_{tot}$; E_π=kinetic energy of the meson in the c.m. system; E_{tot}=total kinetic energy in the c.m. system after the meson is produced.

Class	$d\sigma_{11}/dt$
Ss	$2t/\pi(1-t)$
Ps	$16t(1-t)^{\frac{3}{2}}/\pi$
Pp	$128t^{\frac{3}{2}}(1-t)^{\frac{3}{2}}/3\pi$

APPARATUS

The proton beam was allowed to strike liquid hydrogen. The apparatus for measuring the high-energy γ rays arising from the reaction is shown in Fig. 1. The reader is referred to a previous report[10] for details concerning the energy and spread in energy of the protons, the ionization chamber used as monitor, and the general layout of the cyclotron.

The energy of the external beam was varied from 437 to 346 Mev by the use of carbon degraders placed in the path of the beam at the entrance to the channel in the shield wall. The resulting energies were obtained by a differential range measurement and use of the range-energy tables of Aron *et al.*[11] These measurements also provided assurance that only slight additional spread in energy was introduced by the degraders.

The liquid hydrogen was contained in a special thin-walled Styrofoam and copper container. The protons traversed about 10 cm of liquid hydrogen. The ratio of the surface density of hydrogen to that of the container was 4.5. The effect from hydrogen was at least as large as that from the empty target plus background.

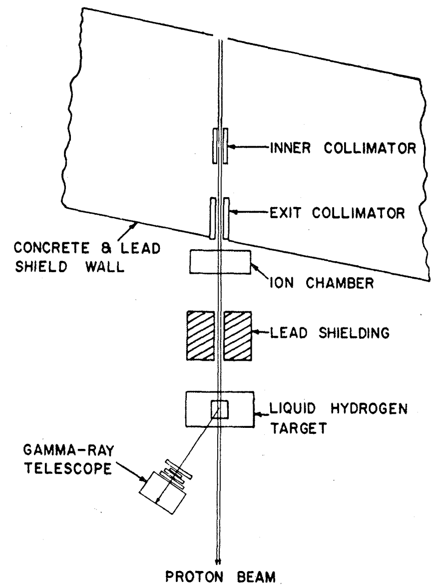

FIG. 1. Experimental arrangement for photon detection from hydrogen.

The γ-ray detector consisted of three plastic scintillators and a water Čerenkov counter numbered 1, 2, 3, 4 in order of increasing distance from the target. The defining element of the telescope was a lead converter 0.52 cm thick and 3.18 cm in diameter placed immediately behind the first counter. It subtended a solid angle of 2.96×10^{-3} sterad at the target. Some photons, originating from neutral mesons in the target, after traversing anticoincidence counter 1 gave rise to shower electrons in the converter which were detected by a coincidence among counters 2, 3 and the water Čerenkov counter, 4. The purpose of the anticoincidence counter was to reject charged particles incident upon the telescope from all regions of the target irradiated by the proton beam. The geometry of the other counters provided efficient counting of the shower electrons. Counters 1 (2.25 in. diam), 2 (1.70 in. diam), and 3 (2.00 in. diam) were plastic scintillators, 0.40 in. thickness, viewed through short light pipes by RCA 6199 photomultipliers. The Čerenkov counter consisted of a water radiator contained in a thin-walled Lucite cell 3 in. in diameter and $2\frac{3}{4}$ in. long which was viewed directly by an RCA 5819. All counters were well shielded from magnetic fields.

The refractive index of water is such that a charged particle requires a kinetic energy greater than 0.48 of its rest mass for Čerenkov radiation to be emitted. Thus under the conditions of the experiment the counter was insensitive to all particles except electrons and charged mesons. This helped considerably in reducing the background. The response of the water

[10] Sutton, Fields, Fox, Kane, Mott, and Stallwood, Phys. Rev. **97**, 783 (1955).

[11] Aron, Hoffman, and Williams, U. S. Atomic Energy Commission Report, AECU-663, 1949 (unpublished) and W. Aron, University of California Radiation Laboratory Report UCRL-1325 (unpublished).

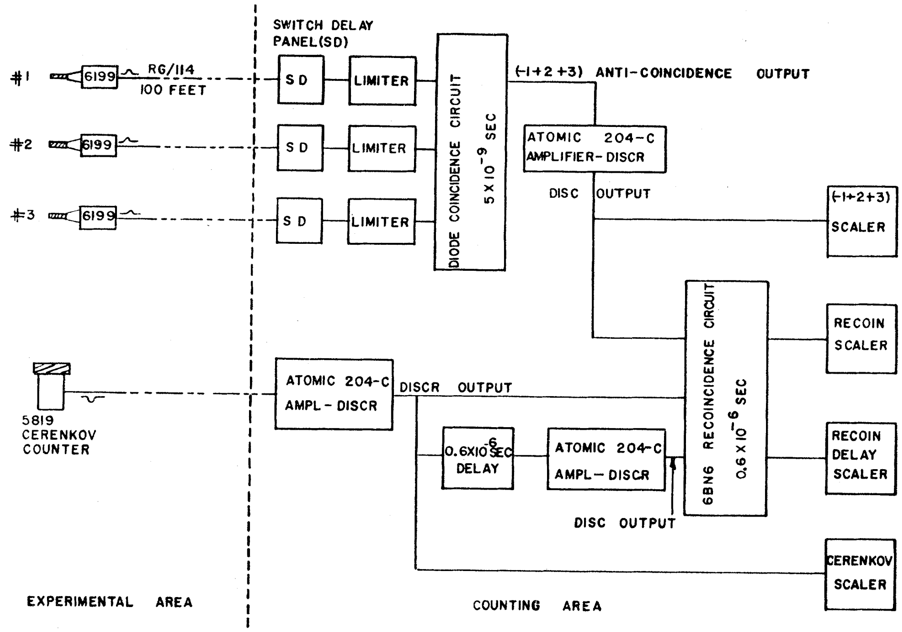

FIG. 2. Block diagram of electronics.

Čerenkov counter has been explored with pions ($\beta=0.87$) and found to be satisfactorily uniform over its face. In addition, a plus-height analysis of the Čerenkov pulses from conversion electrons from decay γ rays of neutral pions produced in a carbon target bombarded by the 437-Mev proton beam showed that that a negligible number of Čerenkov pulses were so small as to be rejected by the discriminator settings. It was concluded the Čerenkov counter detected high-energy electrons with very nearly 100% efficiency. The equality of the counting rates $(2+3-1)$ and $(2+3-1+\text{Čerenkov})$ gave further support for this conclusion.

Signals from each of the three plastic scintillators were properly delayed, limited, and clipped before being fed into a crystal diode coincidence circuit (resolving time 5×10^{-9} sec) with counter 1 in anticoincidence. The output of this coincidence-anticoincidence circuit (ac output) was further placed in coincidence with amplified, shaped Čerenkov signals. This final coincidence output is called a "recoincidence." A block diagram of the electronics is shown in Fig. 2.

It was found that the main source of accidentals consisted of a pulse in the Čerenkov counter coinciding with an unrelated pulse from the ac output. These were monitored by observing recoincidences when the Čerenkov signals were delayed by 0.6×10^{-6} sec. The ac, recoincidence, delayed recoincidence, and Čerenkov counting rates were all recorded.

With this arrangement it was possible to obtain satisfactory plateaus, for example those shown in Fig. 3. They were obtained with carbon as the target in the 437-Mev proton beam. Plateau corrections to account for pulses too small to trip either discriminator were felt to be unnecessary. Measurement of the γ-ray yield from carbon was a quick and convenient method of adjusting the electronics. The good agreement of the carbon effects, taken periodically during all runs and on different days, provided assurance of stability and reproducibility of conditions.

EXPERIMENTAL PROCEDURE

The experiment was performed by allowing protons to bombard various targets. The proton beam was automatically shut off when the ionization chamber monitor reached a preset voltage corresponding to a certain integrated beam flux.

In order to prevent overloading of the anticoincidence counter by the large flux of charged particles from the target, the beam was operated at an intensity of 2×10^6 protons cm^{-2} sec^{-1}. At this level, the number of events rejected by random pulses from the anticoincidence counter was measured to be much less than 1%. The background from the general radiation outside the cyclotron shielding made it desirable to use as large a collimator opening as could be tolerated by the targets. In studying the yield from hydrogen the collimator size was $1\frac{1}{2}$ in. square.

The recoincidence counting rates for the following combinations of targets and converter were recorded:

(Pb,T) — the rate with converter and full target,
(Pb,0) — the rate with converter and empty target,
(0,T) — the rate without converter and with full target,
(0,0) — the rate without converter and with empty target.

$R=[(\text{Pb},\text{T})-(\text{Pb},0)]-[(0,\text{T})-(0,0)]$ is defined as the recoincidence rate (with accidentals subtracted), and was taken to be the number of events per unit integrated beam produced in liquid hydrogen and detected through the presence of the lead converter.

The differential cross sections for detecting the conversion electrons from the γ rays were calculated as $R/(Nn\Omega)$, where R is the recoincidence rate just defined, N is the incident proton flux in protons cm^{-2} sec^{-1}, n is the surface density of the target in nuclei per cm^2, and Ω is the solid angle of the defining element of the proton detector.

EFFICIENCY OF THE COUNTER TELESCOPE FOR PHOTONS

The γ-ray energies from neutral pions produced by 437-Mev protons on hydrogen vary from 15 Mev to 300 Mev. Over this range the γ-ray detection efficiency of the counter telescope varied from zero to about 0.45 for a lead converter of 0.52 cm thickness. A calculation based on the Monte Carlo results of Wilson[12] was used to estimate the sensitivity as a function of energy, $\epsilon(w)$, for the detector. Wilson's calculation provides the expected number of electrons leaving the lead converter for photon-initiated showers. From this one can then estimate the probability that the electrons successfully traverse the rest of the counter telescope. The efficiency calculated in this way is shown in Fig. 4.

To assign an absolute efficiency, the telescope was calibrated using the decay photons, at 30° in the laboratory, from neutral pions produced in the charge

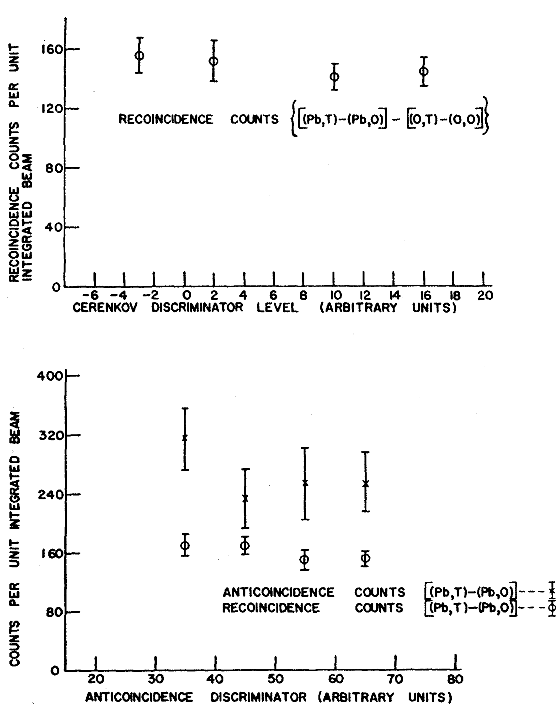

FIG. 3. Plateau characteristics. $E_p=437$ Mev; carbon target $=0.7$ in. thick; $\theta=30°_{\text{lab}}$.

[12] R. R. Wilson, Phys. Rev. **86**, 261 (1952).

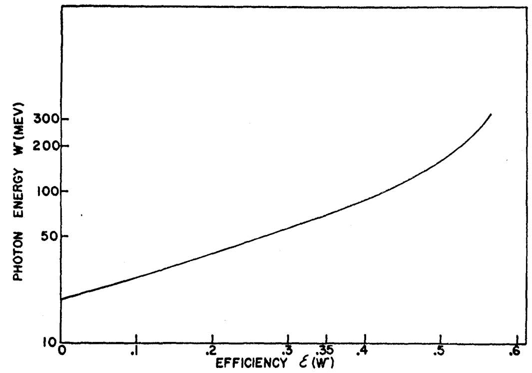

FIG. 4. Efficiency of gamma-ray telescope. Calculated for a lead converter thickness of 0.52 cm.

exchange scattering of 170-Mev negative pions by hydrogen. The angular distribution of this reaction has been measured by Ashkin *et al.*[13] enabling one to calculate the energy spectrum of photons at any angle in the laboratory. By folding the energy sensitivity $\epsilon(w)$ with the distribution of γ rays $J(\theta,w)$, the expected number of events can be determined. The ratio of measured to expected counting rate was (1.07 ± 0.17). This factor was used along with the calculated efficiency in Fig. 4 in calculating results.

In order to obtain a confirmation of the efficiency, we made a measurement of the yield from carbon at 340 Mev and 47°. Previous experiments on neutral meson production from carbon made by Crandall and Moyer[14] with a pair spectrometer provide an energy spectrum of the decay photons from the neutral pions. The ratio of the counting rate observed to the counting rate expected[14] was 0.96 ± 0.21, showing agreement within the rather large statistical error.

Various tests of the performance of the γ-ray detector yielded satisfactory results: the area, shape and thickness of the lead converter were changed and the separation of the components of the γ-ray telescope was altered.

CORRECTIONS

The data have been corrected for the following effects:

(1) The strong energy sensitivity of the cross section which made it necessary to consider the beam contamination and slowing down of the beam in the targets.

(2) The attenuation of photons, originally heading for the converter, in the target material, target container, air, and anticoincidence counter.

(3) The cold helium gas in the empty hydrogen target, which had to be introduced in order to prevent condensation of moisture and air.

(4) The alternate mode of decay of the neutral pion into an electron pair and a γ photon.

[13] Ashkin, Blaser, Feiner, and Stern, Phys. Rev. **101**, 1149 (1956).

[14] W. E. Crandall and B. J. Moyer, Phys. Rev. **92**, 749 (1953)

(5) The finite solid angle of the liquid hydrogen target.

Possible effects of negative pions which could stop or undergo charge exchange scattering in the target and of neutrons in the beam where estimated to be negligible compared with statistical errors.

RESULTS AND ANALYSIS

A. Hydrogen

In analyzing the hydrogen data it is necessary to consider how the expected angular distribution of recoincidence events is related to the possible neutral pion energy spectra and angular distributions.

Let us assume that the predictions of the phenomenological model are valid and that meson production in class Pp predominates at a proton energy of 437 Mev. As previously indicated, the meson cross section in the center-of-mass system is expected to be of the form[15]

$$d\sigma_{11}/d\Omega' dt' = \sum a_l P_l(\cos\theta') 128 t'^{\frac{3}{2}}(1-t')^{\frac{3}{2}}/3\pi$$

(primed quantities being measured in the c.m. system), where $t' = (E_{\pi'}/E'_{\mathrm{tot}})$ = ratio of kinetic energy of meson to total kinetic energy available in the c.m. system. The case $a_0 = 1/(4\pi)$ corresponds to an isotropic angular distribution of the neutral pions in the c.m. system.

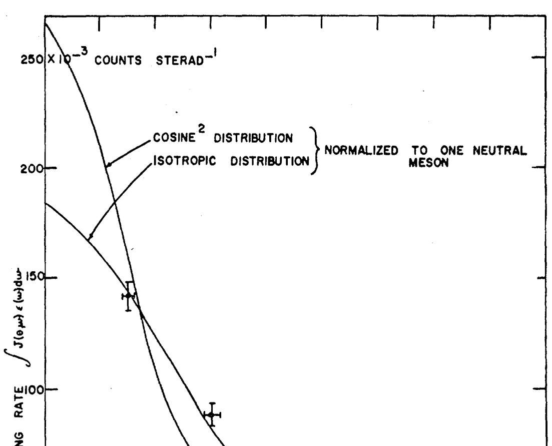

FIG. 5. Counting rate as a function of laboratory angle for the reaction $\sigma(p+p \to \pi^0+p+p)$. $E_p = 437$ Mev. The ordinates of the points are calculated as $R/(Nn\Omega\sigma)$ where the symbols are explained above under "Experimental Procedure." To obtain the fit shown in the figure σ was taken to be 147 microbarns.

[15] The recoil energy of the two-nucleon system has been neglected.

The case $a_0 = 1/(4\pi)$, $a_2 = 1/(2\pi)$ corresponds to a (cosine)^2 distribution in the c.m. system.

$J'(\theta' w')$, the intensity of γ rays with energy w', angle θ, and in the interval $dw' d\Omega'$, in the c.m. system of the collision may be expressed as

$$J'(\theta' w') = 2\sum a_l P_l(\cos\theta') 128 P_l(\cos\lambda) \times t'^{\frac{3}{2}}(1-t')^{\frac{3}{2}} dt'/3\pi\mu\gamma'\beta',$$

where λ = c.m. angle between the direction of the neutral pion and the decay photon, $\mu\gamma'\beta'$ = momentum of the neutral meson in units $m_{\pi^0}c$. Transforming to the laboratory system, we have

$$J(\theta,w) = \gamma_0(1+\beta_0 \cos\theta') J(\theta' w'),$$

where

$$w = \gamma_0(1+\beta_0\cos\theta') w', \quad \beta_0 = v/c, \quad \gamma_0 = (1-\beta_0^2)^{-\frac{1}{2}},$$

and v_0 is the velocity of the c.m. system. $J(\theta,w)$ has been calculated with an IBM-650 computer for several energy and angular distributions. The expected intensity at any angle θ is given by $I(\theta) = \int J(\theta,w) \times \epsilon(w) dw$. $I(\theta)$ has been plotted in Fig. 5 for the cases of isotropic and (cosine)^2 distributions of the neutral pions, the energy distribution given above, and for the efficiency for photon detection in Fig. 4. The experimental points are also shown in Fig. 5. The data are seen to fit the isotropic distribution slightly better than the (cosine).^{2}

TABLE III. Cross sections from hydrogen.

Incident proton energy Mev	Corrected $R/(Nn\Omega)$ at 30° μb sterad^{-1}	Total cross section μb
437	20.9±0.8	147±30
408	11.1±2.3	81±22
383	4.4±2.2	33±17
367	3.4±1.4	25±12
346	2.1±1.4	16±11

Letting $A(\theta) = 1/\int J(\theta,w)\epsilon(w)dw$, the total cross section for neutral pion production was calculated from

$$\sigma(pp,\pi^0) = A(\theta) R/(Nn\Omega).$$

The total cross section, $\sigma(pp,\pi^0)$, was finally calculated to be 147±30 microbarns independent of the assumed angular distribution of the pions. In view of the insensitivity of the results to these two distributions, no estimate could be made of their relative contributions to the total cross section.

The errors listed in the final cross section arise from the following sources: (1) a 5% statistical error in the number of counts, (2) a 5% systematic error arising from uncertainties in the surface density of hydrogen, and the monitor calibration, (3) an 18% uncertainty in the absolute efficiency for the detection of photons, and (4) a 2% error due to uncertainties in corrections.

The differential cross sections from hydrogen at $\theta = 30°$, as a function of incident proton energy, are contained in Table III. The total cross sections were computed by using in $A(\theta)$ the mean of the two meson angular distributions. Consideration has been given to the possibility of a change in the telescope efficiency with proton bombarding energy through changes in the relative contributions from different classes of meson production. The effects were much smaller than the statistical errors. The cross sections, which were measured at 346, 367, 383, 408, and 437 Mev, are plotted in Fig. 6 along with the available data from other laboratories.

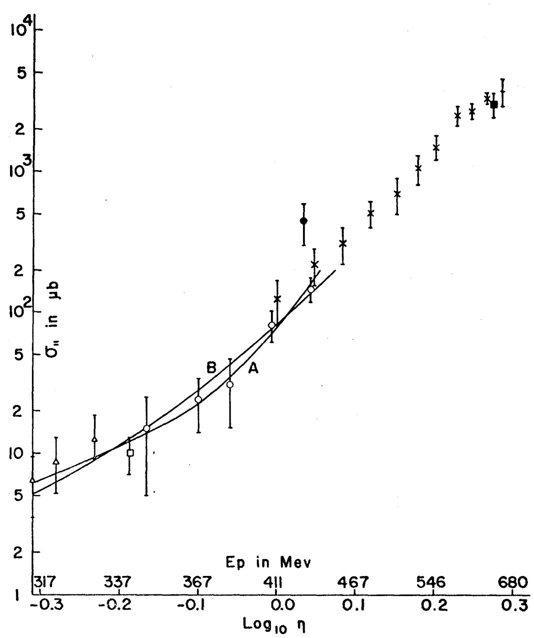

FIG. 6. Total cross section (σ_{11}) for the production of neutral pions in p-p collisions. E_p is the kinetic energy of the bombarding protons and η is the maximum momentum of the mesons in the system in units $m_{\pi^0}c$. The sources of the data are as follows: △ reference 2; □ reference 1; o this work; × reference 6; ● reference 3; + reference 4; ■ Balashov, Zhukov, Pontecorvo, and Selivanov as quoted in reference 5. Curve A is the function $\sigma_{11}=27\eta^2-25\eta^6+75\eta^8$ which gives the best least squares fit to the low-energy points △, □, and o. Curve B is the function $\sigma_{11}=19\eta^2+62\eta^6$ which gives the best fit to the low energy points △, □, and o using only terms in η^2 and η^6.

B. Carbon

The following ratio of the yields from hydrogen and carbon was used as a check during all the runs:

$$(d\sigma/d\Omega_{pp})/(d\sigma/d\Omega_{pC})_{\theta=30°}=0.032\pm0.001 \text{ per nucleus.}$$

On the assumption that the angular distribution and the efficiency for detection of π^0 decay photons were similar in hydrogen and carbon, we obtained for the ratio of the total cross sections in hydrogen and carbon

$$\sigma_{\text{tot}}(pp,\pi^0)/\sigma_{\text{tot}}(pC,\pi^0)=0.039\pm0.004 \text{ per nucleus,}$$

and for the total cross section in carbon (3.7 ± 0.7) millibarns per nucleus. (The above two ratios differ slightly because of differences in some of the corrections.)

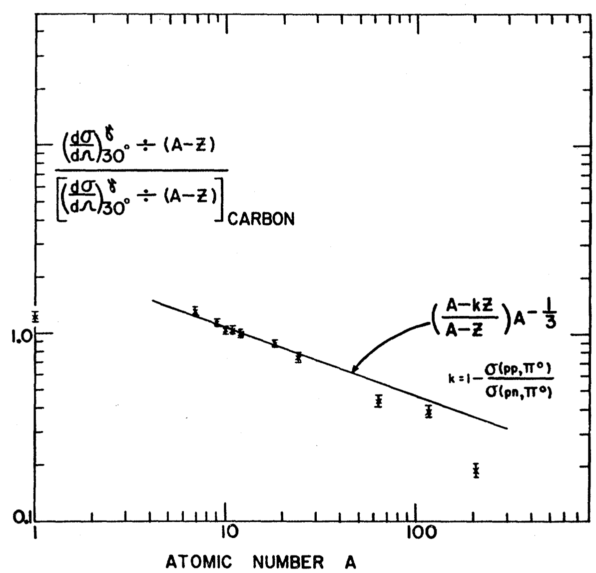

FIG. 7. Neutral meson production in proton bombardment of nuclei at 437 Mev. The ordinates are the proton yields at a laboratory angle of 30° relative to carbon per neutron.

C. Deuterium

The measured difference in counting rate (D_2O-H_2O) at 30° yields $R/(Nn\Omega)$ per neutron $=(152\pm6)\times10^{-30}$ cm^2/sterad. Assuming an angular distribution and efficiency for detection of γ rays from the decay of neutral pions accompanying the reaction $p+n\rightarrow\pi^0+$nucleons to be the same as in the reaction $p+p\rightarrow\pi^0+p+p$, we calculated $[\sigma(p\text{D},\pi^0)-\sigma(pp,\pi^0)]=(1.00\pm0.21)$ millibarn as the value for the neutral pion yield from bound neutrons. Furthermore, we could obtain

$$\sigma(p\text{D},\pi^0)=\sigma(pp,\pi^0)+[\sigma(p\text{D},\pi^0)-\sigma(pp,\pi^0)]$$
$$=1.15\pm0.22 \text{ millibarns.}$$

D. Neutral Meson Yield from Complex Nuclei

The γ-ray counting rates at $\theta=30°$ for 437-Mev protons interacting with the targets D, Li, Be, B^{10}, B^{11}, C, O, Mg, Cu, Sn, and Pb were measured. For targets other than D and O the yields were measured directly from the targets which consisted of the elements in various solid or powdered forms. The yield for oxygen was obtained from a subtraction of the results from H_2O and H_2. The yield for deuterium was obtained from a D_2O-H_2O subtraction as described above. The results have been expressed in terms of the yield per neutron relative to carbon and are shown in Fig. 7. The solid curve on the graph represents the dependence of neutral pion production per neutron on atomic number assuming that only the surface nucleons contribute.

DISCUSSION

In reviewing the data obtained from this experiment, it is of interest to consider all the results on neutral meson production in proton-proton collisions and their relation to the phenomenological description of neutral pion production by nucleons.

The total cross section for the reaction $p+p\rightarrow\pi^0+p+p$, expressed as a function of the maximum meson momentum in the c.m. system η, is expected to be a sum of contributions from *Ss*-, *Ps*-, and *Pp*-class meson production, which would vary as η^2, η^6, and η^8, respectively. Within this scheme the present data and the results of the Berkeley measurements near threshold[1,2] can be represented best by a function of the form

$$\sigma_{11}=(27\pm10)\eta^2-(25\pm90)\eta^6+(75\pm70)\eta^8.$$

The least-squares fit is practically as close with a function involving only η^2 and η^8. It is somewhat worse with one involving only η^2 and η^6,

$$\sigma_{11}=(19\pm6)\eta^2+(62\pm15)\eta^6.$$

The two expressions given for σ_{11} are plotted in Fig. 6. The first one, involving three powers of η, suffers from the drawback of having at the upper end of the energy range a slope which is incompatible with other measurements[6] (see Fig. 6). The second expression for σ_{11} above, involving only terms in η^2 and η^6, has the drawback of apparently conflicting with measurements of the angular distribution of the pions.[6] These data yielded nonisotropic angular distributions at energies down to 445 Mev, the lowest investigated. On the basis of the phenomenological theory, this should mean that σ_{11} includes a term in η^8.

Thus, while we can conclude that the coefficient of η^2 is now determined to be about 25 within about 50%, little is known, unfortunately, about the coefficients of η^6 and η^8. We have not extended the analysis to include the data of Prokoshkin[6] in a higher but overlapping energy range (see Fig. 6). Although the agreement between their data and ours is fairly good it is possible that there is a difference in the absolute values due to the use of different monitoring techniques.

Neither theory nor experiment permits quantitative conclusions to be drawn concerning the interaction of high-energy neucleons with complex nuclei. In discussing observations dealing with meson production even in deuterium, one must consider the possibility that the yield from one nucleon may be influenced by the other. At 437 Mev the wavelength ($\lambda\!\!\!^-$) of the protons is 2×10^{-14} cm, which is small compared with the distance between the nucleons.[16] Thus, one might expect the cross section to be very nearly the sum of the individual cross sections. However, Glauber[17] has discussed the nonadditivity of free-particle cross sections in high-energy measurements and has shown, to the correct order of magnitude, that the difference can be thought of as an eclipsing of the nucleons in the deuteron whenever the incident wavelength is much smaller than the range of interaction of the nucleons and the incident particle ($\lambda\!\!\!^-/mc=1.4\times10^{-13}$ cm).

With this in mind, a comparison can be made between the free neutron and proton cross sections and the deuteron cross section. A subtraction measurement of the γ rays at 30° from D_2O and H_2O was interpreted above to give a proton-deuteron cross section, $\sigma(p\text{D},\pi^0)=(1.15\pm0.22)$ mb, at 437 Mev. If we assume that $\sigma(p\text{D},\pi^0)$ is the sum of the free particle cross sections, then, in the notation of Rosenfeld,

$$\sigma(p+[p+n]\rightarrow\pi^0+\text{nucleons})=\sigma(pp,\pi^0)+\sigma(pn,\pi^0+\text{nucleons})=\sigma_{11}+\tfrac{1}{2}\sigma_{10}(\text{D})+\tfrac{1}{2}\sigma_{10}(\text{unbound})+\tfrac{1}{2}\sigma_{01}.$$

The values of these independent cross sections at energies near 437 Mev, corresponding to $\eta=1.07$, are known: $\sigma_{11}=(0.147\pm0.030)$ mb, $\sigma_{10}(\text{D})=(1.23\pm0.07)$ mb[18] $\sigma_{10}(\text{unbound})=(1.06\pm0.24)$ mb[19] and σ_{01} may be neglected.[20] Thus, we obtain $\sigma(p+[p+n]\rightarrow\pi^0+\text{nucleons})=1.23\pm0.12$ mb corrected to 437 Mev. In view of the experimental and theoretical uncertainties, the agreement with the above experimental results is better than might be expected.

For heavier elements, the relative cross sections per neutron as functions of atomic number are shown in Fig. 7. They are satisfied by an $A^{-\frac{1}{3}}$ law down to light elements. This agrees with the assumption that only the surface neutrons contribute, and differs markedly from the result obtained at Berkeley[21] (at much lower average energy) where the production of neutral pions was proportional to the total number of neutrons at low atomic number and to the number of surface neutrons for elements with $A>25$. The higher energy available in this experiment may be responsible for the difference. Similar observations have been made by Tyapkin *et al.*[4] using incident protons of energy 670 Mev.

[16] r_{0t} (triplet n-p effective range) $=1.7\times10^{-13}$ cm.

[17] R. J. Glauber, Phys. Rev. **100**, 242 (1955).

[18] Fields, Fox, Kane, Stallwood, and Sutton, Phys. Rev. **96**, 812 (1954).

[19] See Fields, Fox, Kane, Stallwood, and Sutton, Phys. Rev. **109**, 1713 (1958), preceding paper.

[20] Evidence that σ_{01} is small comes from experiments on the reactions $\sigma(pp,\pi^+d)$[16], $\sigma(pp,\pi^+pn)$[17], and $\sigma(np,\pi^0)$ which was studied by Rosenfeld, Solmitz, and Hildebrand [Bull. Am. Phys. Soc. Ser. II, **1**, 72 (1956)], and R. A. Schluter [Phys. Rev. **96**, 734 (1954)].

[21] R. W. Hales and B. J. Moyer, Phys. Rev. **89**, 1047 (1953).