Reaction $p+p\rightarrow\pi^++d$ in the 425-Mev Energy Region*

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The 437-Mev external proton beam of the Carnegie Institute of Technology synchrocyclotron has been used to measure the angular distribution and total cross section of the reaction $p+p\rightarrow\pi^++d$. The proton beam impinged upon liquid hydrogen, and the resulting positive pions were detected in fast coincidence with their associated deuterons. The differential cross section was measured at seven pion center-of-mass angles in the range from 30° to 130°. Several corrections were applied to the observed counting rates, including a downward correction of about 7% to compensate for coincidence counts which arose from the reaction $p+p\rightarrow\pi^++p+n$. The corrected total cross section for the reaction $p+p\rightarrow\pi^++d$ is 1.23 ± 0.07 mb, with a center-of-mass angular distribution proportional to $(0.23\pm0.02)+\cos^2\theta$. In addition, a 53% polarized proton beam of energy about 415 Mev was used to measure azimuthal asymmetries in the $p+p\rightarrow\pi^++d$ reaction. The experimental technique was very similar to that used for the production cross-section measurements. The asymmetry ϵ, defined as the difference divided by the sum of the pion intensities on the right and left, was measured at pion center-of-mass angles of 90° and 50°. The magnitudes of the measured asymmetries were $\epsilon(90°)=0.20\pm0.03$ and $\epsilon(50°)=0.023\pm0.015$ in such a sense that the greatest meson intensity was on the left when the polarized beam originated in a scattering to the right. The results of the cross-section and asymmetry measurements are discussed in relation to phenomenological analyses of the threshold behavior of the $p+p\rightarrow\pi^++d$ reaction.

INTRODUCTION

THE interpretation of the data on nucleon-nucleon production of pions has been greatly facilitated by the phenomenological considerations of Watson and Brueckner,[1] Rosenfeld,[2] and Gell-Mann and Watson.[3] Their deductions are made from a "nuclear reaction" point of view, and assume the pseudoscalar nature of the pion field and conservation of isotopic spin. On such a basis, predictions regarding excitation functions and angular distributions can be made for incident nucleon energies not too far above threshold. In addition, some estimates can be made regarding the relative sizes of the various cross sections; for example, the strong interaction of the pion-nucleon system in the $I=\frac{3}{2}$, $J=\frac{3}{2}$ state can have a marked effect on certain cross sections.

The present experiment[4] is one of several[5] which have been performed at this laboratory in order to study pion production in p-p collisions in the 425-Mev region; the pertinent reactions are:

$$p+p\rightarrow\pi^++d, \quad (1)$$

$$p+p\rightarrow\pi^++p+n, \quad (2)$$

$$p+p\rightarrow\pi^0+p+p. \quad (3)$$

The charge independence assumption allows one to write these cross sections (in Rosenfeld's notation) as $\sigma_{10}(d)$, $\sigma_{10}(n+p)+\sigma_{11}$, and σ_{11}, respectively. The subscripts are the isotopic spin of the dinucleon system in the initial and final states, respectively. The $I=0$ final state may be either bound or unbound, as the notation indicates.

In addition to total cross section and angular distribution data, further experimental information could be gained by various kinds of polarization measurements; these include azimuthal asymmetry measurements in conjunction with a polarized incident proton beam, and a study of the polarization of the nucleons in the final state.

THEORY

In this section we restate some of the predictions of the phenomenological theory of pion production for the reaction $p+p\rightarrow\pi^++d$.[1–3,6–8]

The first point to be noted is that conservation of angular momentum and of parity dictate that only the following three partial wave combinations exist if the pions are to be produced in s or p states:

	$p+p$	π^+	d
(a)	$^3P_1$	s	$^3S_1$
(b)	$^1S_0$	p	$^3S_1$
(c)	$^1D_2$	p	$^3S_1$.

We limit our attention to these three processes in accordance with the expectation that pion partial waves with $l\geq2$ will not play a significant role in this energy region. We denote by δ_0 the ratio of the (complex) amplitude for process (b) to that for (c), and by δ_1, the ratio of the amplitude for process (a) to that for (c). Then the cross section for process (a), together

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[1] K. A. Brueckner and K. M. Watson, Phys. Rev. 83, 1 (1951).
[2] A. H. Rosenfeld, Phys. Rev. 96, 139 (1954).
[3] M. Gell-Mann and K. M. Watson, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Stanford, 1954), Vol. 4, p. 219.
[4] A preliminary account of this work has been given earlier: Fields, Fox, Kane, Stallwood, and Sutton, Phys. Rev. 95, 638 (1954). Some further details of the experimental arrangement are given in T. Fields, U. S. Atomic Energy Commission Report NYO-7103 (unpublished).
[5] See Fields, Fox, Kane, Stallwood, and Sutton, Phys. Rev. 109, 1713 (1958), following paper, and Stallwood, Sutton, Fields, Fox, and Kane, Phys. Rev. 109, 1716 (1958), this issue.
[6] R. E. Marshak and A. M. L. Messiah, Nuovo cimento 11, 337 (1954).
[7] L. Wolfenstein, Phys. Rev. 98, 766 (1955).
[8] F. Mandl and T. Regge, Phys. Rev. 99, 1478 (1955).

with δ_0 and δ_1, will specify the entire set of reaction amplitudes (apart from an over-all phase). Only reaction (c) can possess an intermediate pion-nucleon system in the $\frac{3}{2}$, $\frac{3}{2}$ state.

In the energy region near threshold, the cross section for process (a) will be characterized by an isotropic angular distribution and by a total cross section proportional to η, whereas the total cross sections for the p-wave processes will vary as η^3, with η the pion c.m. momentum (in units of μc). Furthermore, the angular distribution for the p wave processes will be given by $X+\cos^2\theta$, with

$$X=\left[\left|\frac{2-\sqrt{2}\delta_0}{1+\sqrt{2}\delta_0}\right|^2-1\right]^{-1}. \tag{4}$$

According to the above remarks, the total cross section may be written as

$$\sigma_T=\alpha\eta+\beta\eta^3, \tag{5}$$

where α and β are constants.

The angular distribution will be of the form $A+\cos^2\theta$, where

$$A=X+\frac{(X+\frac{1}{3})}{\eta^2}\frac{\alpha}{\beta}, \tag{6}$$

and where the second term is the contribution from s-wave pions.

The azimuthal asymmetry in the π-d reaction when the incident proton beam is polarized can be used to derive information about the relative contribution of meson s and p states, as pointed out by Marshak and Messiah.[6] The angular distribution of the asymmetry resulting from an s-p interference is of the form

$$\epsilon(\theta)=PQ\frac{A\sin\theta}{A+\cos^2\theta} \tag{7}$$

where θ is the pion c.m. angle, P is the beam polarization, $A+\cos^2\theta$ is the unpolarized angular distribution, and Q is a parameter of the theory. Q can be expressed in terms of α, β, δ_0, and δ_1 in the form

$$Q=\sqrt{2}\eta\eta_c\sin(\psi-\tau_1), \tag{8}$$

with

$$\eta_c=\left(\frac{\alpha}{\beta}\right)^{\frac{1}{2}}\frac{(1+|\delta_0|^2)^{\frac{1}{2}}}{|\delta_0+\sqrt{\frac{1}{2}}|}, \tag{9}$$

$$\psi=\arg(\delta_0+\tfrac{1}{2}), \tag{10}$$

$$\delta_1=|\delta_1|e^{i\tau_1}. \tag{11}$$

It can be seen that five pieces of experimental information are necessary in order to fix the above parameters at a given energy (apart from the usual ambiguities which may arise in attempting to derive phase shifts from experimental data). Four such pieces of information are α, β, A, and ϵ. The fifth must be obtained from a study of the polarization state of the emerging deuteron.[9]

GENERAL METHOD

Since the reaction to be studied contains only two particles in the final state, a fast-coincidence technique, detecting both particles, was used. Figure 1 shows a typical counter geometry. The deuteron laboratory angle is always less than 9.4°, because of the low velocity of the deuteron in the c.m. system.

In order to use such a method, it was necessary to examine the possibility that elastic scattering or one of the other pion production reactions could contribute to the observed counting rate. Elastic scattering can be effectively ruled out because, at angles where the counter geometry can accept p-p scatterings, the large angle proton, heading for the π counter, has insufficient energy to escape from the liquid hydrogen target. Neutral pion production will not contribute significantly to the counting rate since its cross section is only 10% of that for $p+p\rightarrow\pi^++d$ and since the two protons will not exhibit the unique angular correlation demanded by the counter geometry.

However, the third possibility, that of counting the pion and proton from the reaction $p+p\rightarrow\pi^++p+n$ (which we shall refer to as the πpn reaction) cannot be ruled out completely. In this case, if a high-energy pion is emitted, the center of mass of the n-p system will move in very nearly the same direction as would a deuteron, and there is thus the possibility that the proton will be counted in the deuteron counter. Or, from another point of view, one can say that a non-negligible fraction of the πpn pions which enter the π

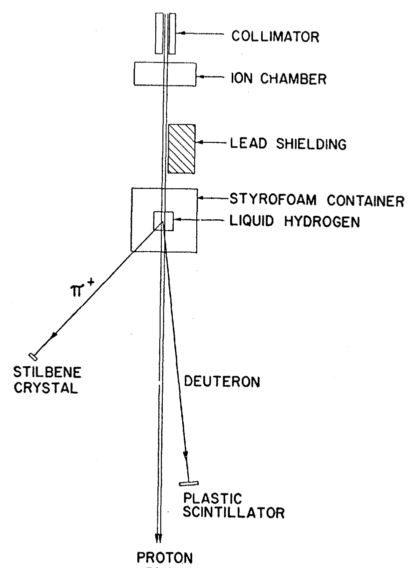

FIG. 1. Plan view of counter arrangement for π^+-d coincidence counting.

[9] R. Tripp, Phys. Rev. **102**, 862 (1956).

counter may be associated with protons which enter the deuteron counter. This possibility was investigated experimentally, and it was found that about 7% of the counting rate arose from the πpn reaction, making necessary a correction which will be discussed shortly.

APPARATUS AND BEAMS

A. Unpolarized Proton Beam

The normal external proton beam of the Carnegie synchrocyclotron spirals out of the machine (without the aid of a deflector) and passes into the experimental area through a steel collimator within the shield wall. In the experimental area is located a scattering table on which the hydrogen target and the counters are appropriately mounted. The general layout is shown in Fig. 2.

The normal intensity of this beam was about 2×10^6 protons $cm^{-2}\ sec^{-1}$. The production cross sections were measured using a collimator opening (and thus beam size) of 0.50 in.×0.50 in. The energy content of this beam has been studied rather thoroughly,[10] yielding the result that, in addition to the main component of 437 ± 3 Mev protons, there is a lower energy "tail" of protons extending down to about 390 Mev with an intensity of about 5% of the total. Because of the relatively rapid change in production cross section with incident proton energy, a small correction has been made to the observed cross sections which compensates for this slight contamination of the incident beam.

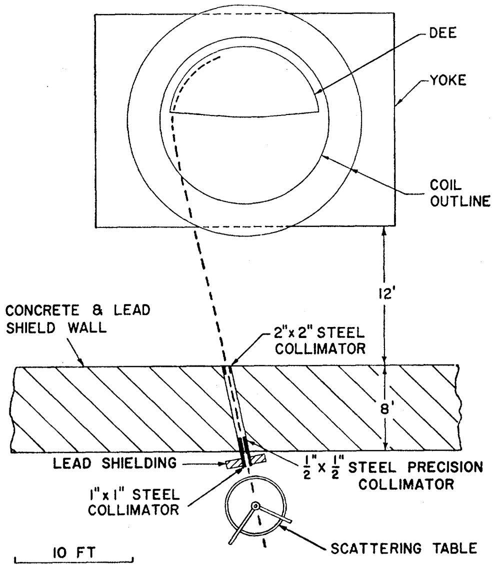

FIG. 2. Schematic diagram of cyclotron, shield wall, and external proton beam trajectory.

[10] Sutton, Fields, Fox, Kane, Mott, and Stallwood, Phys. Rev. **97**, 783 (1955).

B. Polarized Proton Beam

By properly positioning a target inside the cyclotron vacuum chamber, the internal cyclotron beam is scattered from this target, emerges from the cyclotron, and passes through the collimator used for the normal external beam. The internal beam is scattered through 10° from a carbon target, and the resulting beam has a measured polarization[11] of $(53\pm3)\%$.

The intensity of the scattered beam which was so obtained (Fig. 3) was about 2×10^5 protons $cm^{-2}\ sec^{-1}$, i.e., about 10% of the intensity of the normal external beam. This lower intensity made it desirable to use a larger collimator opening for this beam. The opening used was rectangular with a height of 2.0 in. and a width of 1.0 in.

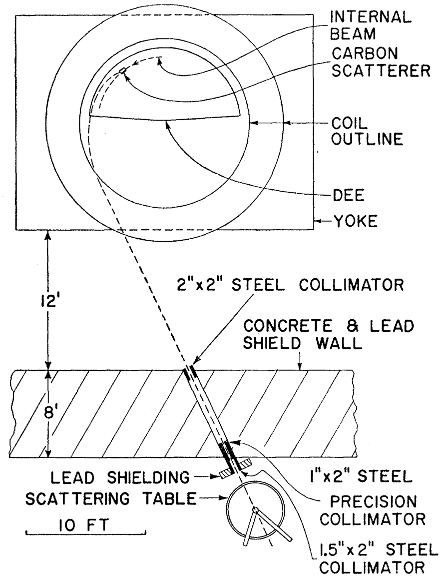

FIG. 3. Arrangement for obtaining a scattered proton beam.

A differential range technique was used to study the energy content of the beam. Figure 4 shows a differential range curve of a pure beam and of the scattered beam. From these curves one can conclude that the scattered beam had a mean energy of 415 Mev with an energy spread of about ±10 Mev.

C. Monitor

The proton beam, after it emerged from the collimator, passed through an argon-filled ionization chamber very similar to one used at Berkeley for the same purpose. The current from the ionization chamber charged a low leakage condenser yielding a voltage proportional to the integrated flux traversing the chamber. This voltage was measured with a dc amplifier of standard design. The leakage resistance of the system was high enough so that the amount of charge

[11] Kane, Stallwood, Sutton, Fields, and Fox (to be published).

leaking off the condenser during the usual charging times of one to five minutes was negligible.

In order to obtain the multiplication factor of the chamber (the number of ion pairs produced per proton traversing the chamber), it is necessary to know the energy loss of 437-Mev protons in argon and the average energy required to form an ion pair. The latter quantity we obtained from the Berkeley ionization chamber calibration[12] which was performed using a Faraday cup. The former we obtained from the tables of Aron *et al.*[13] This ionization chamber was used for all cross-section and asymmetry measurements. Thus the absolute cross sections contained in this paper depend on the calibration work at Berkeley, for which an estimated accuracy of 2% was quoted.[12,14]

D. Target

The liquid hydrogen which formed the target for the experiments herein described was held in a Styrofoam

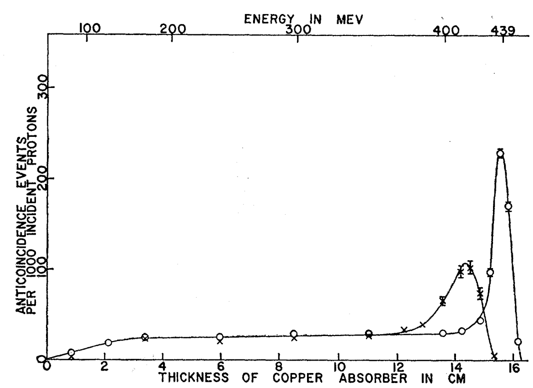

FIG. 4. Differential range curves. The copper absorber thickness and the energy scale have been corrected to allow for counter thickness.

(polystyrene foam) container. The salient features of the container were walls about 2 inches thick, and a liquid-nitrogen-cooled baffle. The only essential way in which the Styrofoam container entered into the measured cross sections was in the path length of hydrogen (10 cm) which the proton beam traversed. This was measured with the target at room temperature and an allowance made for the estimated (3%) shrinkage of the Styrofoam and copper hydrogen cup. A check was provided by direct observation of the liquid hydrogen using a clear plastic lid instead of the Styrofoam one normally used. It is believed that the path length was known to an accuracy of 3%.

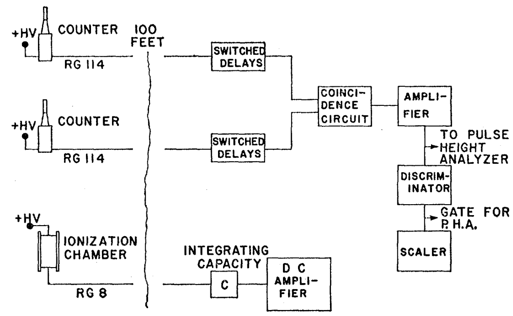

FIG. 5. Block diagram of electronic apparatus.

E. Counters

The mesons and deuterons were detected with scintillation counters of standard design. Each consisted of a stilbene crystal or a plastic scintillator viewed through an appropriate Lucite light pipe by an RCA 5819 photomultiplier tube. The tubes were well shielded against stray magnetic fields. Signals were developed at the last dynode or anode across 185-ohm resistors, with the resistor forming part of an *LC* network designed to minimize the loading effect of the tube and wiring capacity.

The counters were mounted on a scattering table on arms which pivoted about the center of the table. The edge of the table was graduated in units of 10 minutes of arc, with an estimated root-mean-square error in the markings of about 1 minute.

F. Electronics

Figure 5 shows a block diagram of the electronic equipment. The delay in each counter signal cable could be varied by means of switched delays in steps of 10^{-9} sec from 0 to 31×10^{-9} sec. Normally the counters were placed together in the direct beam (at reduced intensity) and their coincidence pulse height maximized by means of the switched delays. This procedure gave "zero delay" for the counters. Then, for π-d coincidence counting, a computed delay was added to the appropriate counter (the π counter in this case). This delay was computed from the known velocities of the pion and deuteron for various c.m. angles and from the counter distances.

The counter pulses, after emerging from the switched delays, were fed directly into a crystal diode coincidence circuit. The particular circuit we used accepted four input pulses, and could use one or two of these in anticoincidence. Here the counter pulses were clipped to a duration of about 2×10^{-9} second by shorted stubs, with the reflected pulse being absorbed at the counters. The resolving time of the circuit was thus about 4×10^{-9} second. This short resolving time was of great advantage in reducing the number of accidental coincidences; the proximity of the deuteron counter to the direct beam made its single counting rate very high.

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[12] Chamberlain, Segrè, and Wiegand, Phys. Rev. **83**, 923 (1951).

[13] Aron, Hoffman, and Williams, U. S. Atomic Energy Commission Report AECU-663, 1949 (unpublished), and W. Aron, University of California Radiation Laboratory Report UCRL-1325, 1951 (unpublished).

[14] Two experiments which provide an independent check of the ionization chamber calibration are those described by: Chamberlain, Pettengill, Segrè, and Wiegand, Phys. Rev. **93**, 1424 (1954), and Marshall, Marshall, and Nedzel, Phys. Rev. **92**, 834 (1954).

TABLE I. Experimental, corrected, and final corrected cross sections in the center-of-mass system.

Center-of-mass angle	d counter solid angle (10^{-3} sterad)	Experimental c.m. cross section (μb/sterad)	Corrected c.m. cross section (μb/sterad)	πpn correction (%)	Final corrected c.m. cross section (μb/sterad)
30°	6.7	166±10	183±11	−4.8±3	175±13
40°	4.4	135±7	146±8	−3.7±2	140±9
40°	6.9	143±9	153±10	−5.8±3	144±11
50°	6.7	104±8	114±9	−5.7±3	108±10
50°	3.3	110±8	122±9	−2.8±1.5	118±9
50°	7.5	114±8	125±9	−6.4±3.5	117±10
60°	4.4	84±3	87±4	−5.0±2	83±5
60°	6.9	88±4	92.5±4.5	−7.8±3.5	85±5
80°	4.4	48±4	50±4	−7.7±3	46±4
80°	6.9	53±4	55±4	−12±4.5	48±5
90°	3.3	35±4	36±4.5	−5.9±2	34±5
90°	7.5	39.5±5	40.5±5.5	−13±5	35±6
130°	4.4	101±6	109±7	...	...
130°	6.9	106±9	116±9	...	...

For the production cross-section measurements, it was particularly important to know the number of coincidence pulses, if any, which were of insufficient size to trip the discriminator; i.e., to know if the discriminator was set low enough. In order to investigate this question, a 25-channel pulse-height analyzer was used to obtain a pulse-height spectrum of the coincidence pulses, and the resulting spectrum showed the extent to which the discriminator setting was on a plateau. A typical fraction of coincidence pulses of insufficient height to trip the discriminator was 3%.

PROCEDURE FOR CROSS SECTION MEASUREMENTS

Figure 1 shows the experimental arrangement. The counter geometry was such that the π counter defined the solid angle; i.e., apart from complications discussed below, every pion (from π-d) which entered the π counter had its associated deuteron entering the d counter. These complications were two in number:

First, the pions and deuterons were multiply scattered within the liquid hydrogen and target walls. This rms multiple-scattering angle was quite small, generally about a degree, so that it was a sufficiently good approximation to say that as many pions were scattered into the π counter as out of it, provided that the deuteron counter was large enough to detect the deuterons associated with those pions scattered into the counter. Thus the d counter had to be as large as though the π counter were larger than its true size by an amount determined by this multiple scattering.

The second complication was similar to the first, except that the pions were changed in direction by π-μ decay rather than by multiple scattering. The situation was altered, moreover, by two facts: (1) the distribution in angle was more complicated; (2) only about 10% of the pions decayed before hitting the counter. For these reasons, the d counter size was determined without regard to π-μ decay and a correction (which turned out to be of negligible size) made to compensate for the π-d coincidence counts lost through this cause.

Previous to a run the counter angles were calculated from kinematics, and the counter sizes and distances were then computed from considerations of geometry, multiple scattering, and beam divergence. The exact counter sizes and distances used represented of course a compromise between angular resolution and intensity, and were also determined by the consideration of minimizing the required deuteron counter size in order to keep the accidental coincidence rate low.

Then, with the counters set up, the coincidence counts per unit integrated beam were observed. Letting $C(F)$ be the filled target rate, $C(F,A)$ be the filled target accidental rate, and $C(E)$ and $C(E,A)$ the corresponding quantities for the target empty, we can write the counts due to hydrogen as

$$C_{\mathrm{H}}=[C(F)-C(F,A)]-[C(E)-C(E,A)]. \quad (12)$$

This equation merely expresses the fact that the non-accidental coincidences from hydrogen equal those from the filled target minus those from the empty target, which is true as long as the presence of the liquid hydrogen does not affect the empty target events. The absorption and scattering by the liquid hydrogen, as well as its stopping power, are so small that this is a good assumption within the accuracy of this experiment.

Having determined C_{H}, the coincidence counts from hydrogen, for a given experimental arrangement, the c.m. differential cross section is computed from

$$d\sigma/d\Omega=JC_{\mathrm{H}}/(nN\Omega), \quad (13)$$

with C_{H}=coincidence counts from hydrogen per unit integrated beam, J=transformation factor between c.m. and lab differential cross section, n=number of

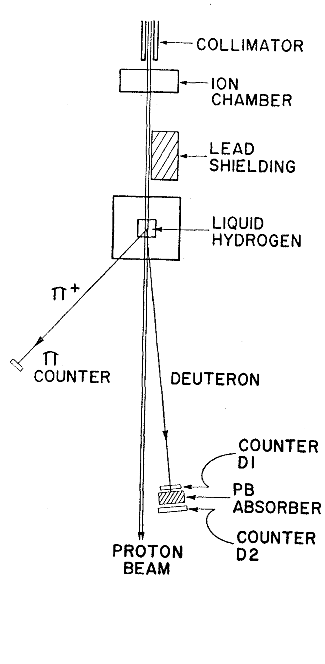

FIG. 6. The counter arrangement used for the "πpn experiment" as discussed in the text.

TABLE II. Final averaged cross sections in the center-of-mass system.

Center-of-mass angle	Averaged c.m. cross section (μb/sterad)
30°	175±13
40°	142± 7
50°	114± 6
60°	84± 3.5
80°	47± 3
90°	35± 4

target protons per cm^2, N = number of protons traversing the target per unit integrated beam, and Ω = solid angle subtended by the defining (π) counter.

RESULTS OF CROSS SECTION MEASUREMENTS

Table I lists the experimental cross sections, the "corrected cross sections," and the "final corrected cross sections."

The experimental cross sections are those obtained from Eq. (13). The corrected cross sections include six rather small corrections arising from the following sources:

1. π-μ decay of pions in flight to pion counter.
2. Slight low-energy contamination of incident proton beam.
3. Absorption of deuterons in hydrogen and target walls.
4. Absorption of pions in hydrogen and target walls.
5. Coincidence pulse height below the discrimination level.
6. The finite angular resolution of the pion counter.

Of these, only the last three were significantly angle-dependent. The net correction is seen to be of the order of 10% or less, and no single correction is greater than 4%. The errors in the corrected cross sections include the estimated uncertainties in the corrections.

The "final corrected cross sections" arise from downward correction of about 7% which has been made to allow for coincidence counts arising from the πpn reaction. The magnitude of this correction was determined from a separate experiment which involved measuring the absorption of deuterons from the π-d reaction. The experimental arrangement is shown in Fig. 6. It was very similar to the normal arrangement except that a third counter was placed behind the deuteron counter and used in anticoincidence with the other two. By placing an absorber thickness of about 0.7 times the deuteron range between it and the d counter, an anticoincidence count represented a π-d or πpn coincidence where the d or p stopped in the absorber. Deuterons could only stop in the absorber by nuclear absorption, and the number of such counts could be computed from measured deuteron absorption cross sections.[15] It was expected that essentially all πpn protons would stop in the absorber, inasmuch as both protons and deuterons had about the same velocity, yielding a proton range roughly equal to one-half the deuteron range. Thus the number of stopping protons was given by the number of anticoincidence counts minus the number of deuterons which underwent nuclear absorption.

The absorber used was lead since it is easier to analyze the absorption curves of deuterons in high-Z elements.[15] The expected absorption curve of a pure deuteron beam was then computed, taking into account the energy inhomogeneity in the deuterons due to the finite angular width of the π counter and to the slight contamination of the incident beam. The lead absorber thickness was chosen as great as possible without getting too near the "knee" of the absorption curve, and generally amounted to about 0.7 of the deuteron range.

The experimental data were treated in the following manner: by properly subtracting empty target and accidental coincidences, a fractional attenuation, equal to the anticoincidence events divided by the double coincidences, was computed. This was then compared with the pure deuteron attenuation computed as described in the preceding paragraph. The experimental attenuation was greater than that expected for a pure deuteron beam, and the difference between them represented directly the fraction of the doubles rate due to πpn events, and hence the relevant corrections were computed readily.

The "final corrected cross sections" exhibit little correlation with the deuteron counter solid angle, whereas the "corrected cross sections" seem to be larger for the larger deuteron solid angles, providing a qualitative check on the corrections for πpn events. Counting rate difficulties made the evaluation of the πpn correction for 130° c.m. infeasible, so no final corrected cross section is listed for that angle.

Table II gives the final averaged cross sections, and they are shown in Fig. 7. A least-squares fit of the form

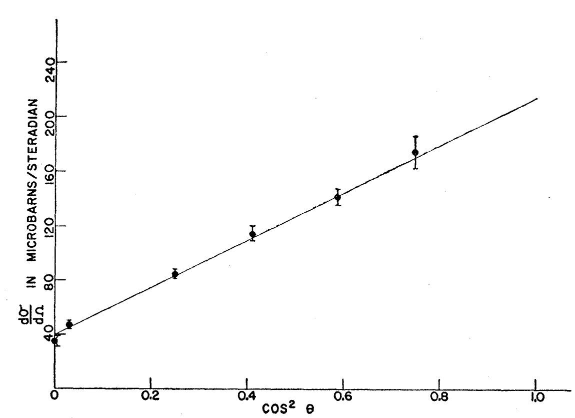

FIG. 7. The final c.m. differential cross sections *versus* the square of the cosine of the pion c.m. angle.

[15] Millburn, Birnbaum, Crandall, and Schecter, Phys. Rev. 95, 1268 (1954).

$d\sigma/d\Omega = B(A+\cos^2\theta)$ yields $B = 175\pm 12$ microbarns/sterad and $A = 0.23\pm 0.02$, corresponding to a total cross section of $\sigma_T = 1.23\pm 0.07$ millibarns. The error in A is obtained by compounding the statistical errors with the estimated errors in the corrections, whereas B and thus σ_T include an additional 4% representing a combination of the uncertainties in target thickness and ionization chamber calibration.

PROCEDURE FOR ASYMMETRY MEASUREMENT

For this measurement the polarized proton beam was used. The essential operation was to compare the π-d coincidence rate with the counters in two configurations which were in the plane of the first scattering and were reflected images about the beam direction. Many factors which could introduce errors into the production cross section measurement such as ionization chamber calibration, π-μ decay, etc., were of no concern in this case inasmuch as they affected the counting rate on both sides by the same factor. By frequent reversal about the beam direction of the scattering table arms which carried the counters, it was expected that even small drifts of the circuitry would produce negligible systematic errors.

At the beginning of each day's run the direction and polarization of the beam were measured. The experimental arrangement for the beam direction measurement is shown in Fig. 8. A beam "profile" was obtained by observing the counting rate in the profile counter telescope *versus* the angle of the telescope as measured on the scattering table. Figure 9 shows a typical beam profile. The beam intensity was greatly reduced for this measurement in order to avoid overloading the counter telescope which monitored the incident beam. The defining counter of the profile telescope was then used as the defining counter for the polarization measurement and as the defining (π) counter for the π-d asymmetry measurement. An x-ray film or Land film was occasionally used to be sure that the beam was traversing the center of the scattering table.

The polarization of the beam was measured by observing the asymmetry in elastic (or nearly elastic) scattering from carbon at 10° laboratory angle, with a standard experimental arrangement. This provided only a relative measurement of the beam polarization, and was used chiefly to be sure that the polarization was constant from day to day. In addition, differential range curves were usually taken to check the energy of the beam.

After these preliminary checks were completed, the counters were placed in an appropriate geometry for the particular c.m. angle (90° or 50°) being studied. Figure 10 shows the general features of the experimental arrangement. A two counter telescope was used to detect the mesons in order that triple coincidences,

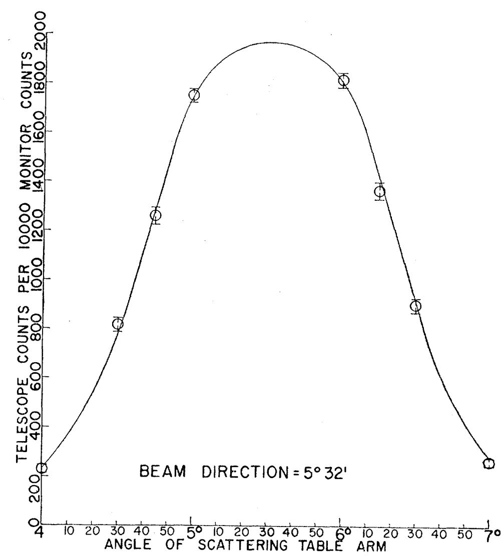

FIG. 9. A typical "beam profile" obtained with the apparatus of Fig. 8. The solid curve is drawn to be symmetrical about 5° 32′.

with their lower associated accidental coincidence rate, could be used as π-d coincidences. As Fig. 10 shows, lead shielding was placed on both sides of the incident beam in order to shield the deuteron counter for both counter configurations. Furthermore, copper absorber, of thickness about 0.7 of the deuteron range, was placed in front of the deuteron counter in order to prevent the πpn reaction from contributing to the counting rate. The counters were reversed about the beam direction at least 4 times in every run (a run consisted of from 10 to 18 hours of π-d counting time).

As with the production cross-section measurements, accidentals constituted a non-negligible portion of the coincidence counting rate and so were also measured to the required statistical accuracy. It should be noted, however, that of the various possible types of accidentals among the three counters, all were negligible

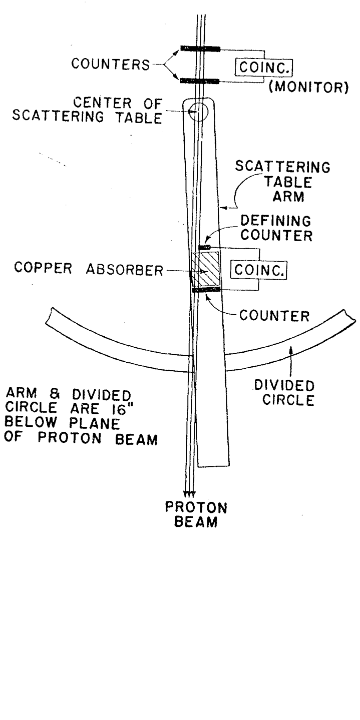

FIG. 8. Counter arrangement used to measure the direction of the polarized beam.

except those consisting of a true coincidence between the two meson counters and a random count in the deuteron counter.

The azimuthal asymmetry, ϵ, we took to be

$$\epsilon=(R-L)/(R+L), \tag{14}$$

where R=intensity on right, and L=intensity on left. We arbitrarily chose R and L to refer to the meson of the π-d reaction. The various experimental coincidence counting rates (per unit integrated beam) we represented by $C(F)$, the filled target rate; $C(F,A)$, the filled target accidental rate; $C(E)$, the empty target rate; and $C(E,A)$, the empty target accidental rate. Then the net hydrogen counting rate on the right was given by

$$R=[C_R(F)-C_R(F,A)]-[C_R(E)-C_R(E,A)], \tag{15}$$

and that on the left by the same equation with R replaced by L. Thus a measurement of those eight counting rates sufficed to determine ϵ. On each run, the available counting time was distributed among them in such proportions as to minimize the statistical error on ϵ.

TABLE III. Azimuthal asymmetry $\epsilon=(R-L)/(R+L)$.

Center-of-mass angle	ϵ (in %)
90°	-21.5 ± 9.5
90°	-18.0 ± 5.5
90°	-15.5 ± 7.0
90°	-23.8 ± 4.5
50°	-2.5 ± 2.9
50°	-1.2 ± 2.1
50°	-3.3 ± 2.0

RESULTS OF ASYMMETRY MEASUREMENTS

Three runs were made at 50° c.m. and four at 90° c.m. Table III lists the results of the seven runs.

The errors for $\epsilon(50°)$ shown in Table III are the statistical standard deviations obtained from the observed distribution in the number of counts per unit integrated beam. The errors obtained in this manner agreed closely in all cases with those obtained from the square root of the number of counts, indicating that drifts in the circuitry produced negligible errors, and also that counter angles, positions, etc. were quite reproducible. At 90° c.m. ϵ is much larger than at 50° c.m., and the laboratory cross section is quite constant *versus* laboratory angle. It is therefore expected that systematic errors should be quite small at 90°.

The final averaged asymmetries are

$$\epsilon(90°)=-0.20\pm0.03 \quad\text{and}\quad \epsilon(50°)=-0.023\pm0.015.$$

As can be seen from the definition of ϵ given in the previous section, the negative sign means that the pions were predominantly produced on the left, with the polarized beam originating in a scattering to the right.

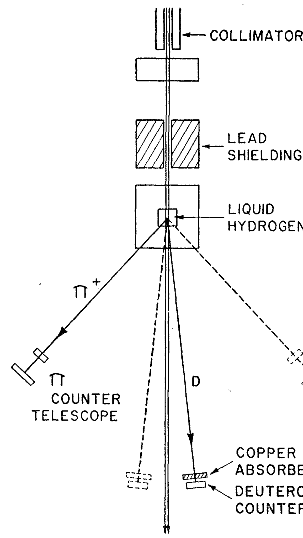

FIG. 10. Counter arrangement used to measure asymmetries in the π-d reaction. The dotted lines show the reflected positions of the counters.

A further check for systematic errors was provided by a π-d asymmetry measurement using the normal unpolarized external beam degraded to 415 Mev. The experiment was performed at 90° c.m. and yielded $\epsilon=-0.01\pm0.04$.

DISCUSSION

In Figs. 11[16–19] and 12 are shown σ_T *vs* η and A *vs* $1/\eta^2$, respectively. All of the presently available data on the $p+p\rightarrow\pi^{+}+d$ reaction are shown. It is seen that the agreement with the phenomenological threshold theory [Eqs. (5) and (6)] is good, in the case of σ_T for $\eta<1.3$ (approximately) and in the case of A, for $\eta<0.8$ (approximately). Since the criterion for the validity of the threshold approximation is that the c.m. wavelength of the pion be somewhat larger than the range of interaction, a rough estimate of the upper limit on η for the theory to be valid would be given by λ_π (de Broglie)$=\lambda_\pi$(Compton), which occurs at $\eta=1$. This is in reasonable accord with the above experimental upper limits.

It is not surprising that the angular distribution parameter A deviates from the threshold behavior at lower energies than does the total cross section, inasmuch as A depends on both the amplitude and phase of δ_0, whereas the total cross section depends only on the amplitude of δ_0. Further information on the variation of the phases of the reaction amplitudes with η can be obtained by noting that the asymmetry measurement yields a value of Q which, through Eqs. (8)–(11),

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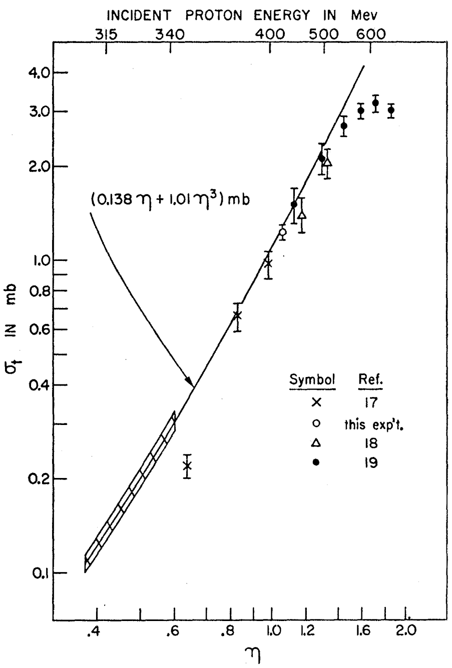

FIG. 11. Total cross section for $p+p\rightarrow\pi^{+}+d$ *versus* η, the c.m. pion momentum divided by $m_{\pi}+c$. The solid line is a least-squares fit given by Crawford and Stevenson[16] to their data which are represented by the shaded region. We have not shown the earlier Berkeley data, as they are generally in good agreement with that of Crawford and Stevenson and of somewhat less accuracy.

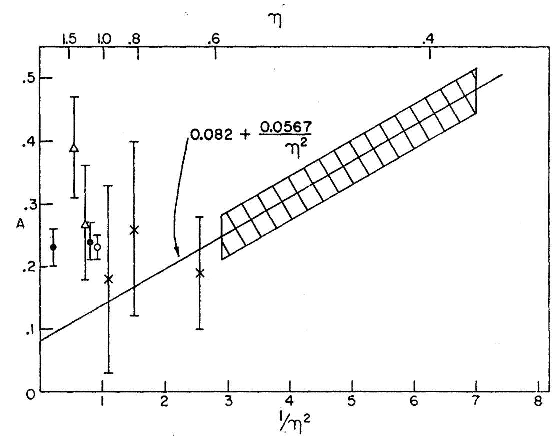

FIG. 12. Angular distribution parameter A *versus* η. The legend and caption of Fig. 11 are applicable to this figure also.

determines $\sin(\psi-\tau_1)$. The experimental results yield a value[16] of $\sin(\psi-\tau_1)$ of 0.70 ± 0.14 at $\eta=0.41$ and 0.63 ± 0.14 at $\eta=0.97$.

As previously remarked, a fifth independent experiment, a measurement of the polarization state of the emerging deuterons is necessary to fix the values of the reaction amplitudes at a given energy. Such an experiment was performed by Tripp at $\eta=0.58$ and through a theorem relating the amplitudes in the pion production reaction to those for elastic p-p scattering,[3] the trigonometric ambiguities were resolved. The fifth parameter so measured was essentially ψ [Eq. (10)]. It should be noted that the set of amplitudes[9] deduced by Tripp indicates that reaction C is the dominant of the two p wave production reactions, contributing about 90% of the p-wave intensity. In order to determine the detailed nature of the deviations from threshold behavior for $\eta\approx1$, it will be necessary to perform the deuteron polarization experiment in this energy region. It seems likely, however, that no features of the reaction are essentially different (at $\eta\approx1$) from the threshold behavior.

The possibility that pion d waves are of sufficient amplitude to affect the picture appreciably at $\eta\approx1$ would seem unlikely by analogy with pion-nucleon scattering. The present experiment indicates in two ways that d-state effects are small. The angular distribution of $\epsilon(\theta)$, given in Eq. (7) for s and p waves only, predicts $|\epsilon(50°)|$ to be 0.055 ± 0.010 if the experimental value of $|\epsilon(90°)|$ is used, whereas the experimental value of $\epsilon(50°)$ is 0.023 ± 0.018. Since the quoted asymmetry errors are statistical only, the fair agreement between these values, together with the apparent absence of a $\cos^4\theta$ term[8] in $d\sigma/d\Omega$, indicates that d-state effects can be neglected in the energy region $\eta\approx1$.

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