during the complete thermal recovery of the copper specimen used in Run II should be about 1.1 cal/g.[24] This energy release when normalized to a 1 μohm cm resistivity change (using Cooper's initial resistivity change per d/cm^2)[2] is 5.4 (cal/g)/μohm cm. The only measurement of stored energy on deuteron irradiated samples was performed by Overhauser.[25] He found that between liquid nitrogen temperature and room temperature the stored energy release was 1.7 (cal/g)/μohm cm. No measurements on deuteron irradiated samples have been made below liquid nitrogen temperature. The only measurement of stored energy in copper below liquid nitrogen temperature was performed by Blewitt, Coltman, Holmes, and Noggle on neutron irradiated samples.[26] Their measurement indicates that the stored energy was less than 0.8 (cal/g)/μohm cm in the temperature range between 22°K and 60°K. It is clear that this value of stored energy is considerably smaller than the value predicted above. Since it is possible that the damage resulting from neutron irradiation is different from that produced by deuteron irradiation, a measurement of the energy released in Stage I recovery for the latter case should be performed. If the same result obtains for deuteron irradiation it appears that annihilation would not be the dominant process occurring in Stage I recovery.

[24] This value corresponds to an energy decrease of about 4.3 Mev from an incident deuteron energy of 10.7 Mev to 6.4 Mev.
[25] A. W. Overhauser, Phys. Rev. **94**, 1951 (1954).
[26] Coltman, Blewitt, and Noggle, Rev. Sci. Instr. **28**, 375 (1957); Blewitt, Coltman, Holmes, and Noggle, *Creep and Recovery* (American Society of Metals, Cleveland, 1957), p. 84.

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Ionization Rates for Electrons and Holes in Silicon

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The ionization rates for holes and electrons in silicon have been determined over the following ranges of field: for holes, $(2.5\text{–}6.0)\times10^5$ volts cm^{-1}; for electrons, $(2.0\text{–}5.0)\times10^5$ volts cm^{-1}. The ionization rate for electrons is higher than that for holes. The results suggest that the field dependence of the ionization rate for holes and, probably, for electrons also, can be expressed by $a\exp(-b/E)$, where E is the field. The constants a and b are different for electrons and holes.

INTRODUCTION

THE charge multiplication that results when carriers are injected into silicon *p-n* junctions was measured as a function of the reverse bias by McKay and McAfee[1,2] and explained as an avalanche process similar to that used by Townsend as a mechanism for breakdown in gases. Making the assumption that the ionization rates for holes and electrons were equal, McKay was able to deduce the field dependence of the ionization rate. More recently, by fitting empirical relations to the multiplication *versus* bias measurements, Miller[3] has been able to solve the analytical expressions for the multiplication for the case where the ionization rates for holes and electrons are different. Miller's results were confined to fields greater than 4×10^5 volts cm^{-1}.

By using more refined methods for measuring the multiplication as a function of bias, the ionization rates have now been determined over a very much wider range of fields for a given junction. In particular, it has proved possible to obtain the separate ionization rates for holes and electrons rigorously, that is, without having to use empirical relations of somewhat limited validity. The results reveal an interesting relation for the field dependence of the ionization rate.

EXPERIMENTAL

The technique used to determine the multiplication characteristics down to multiplications of 1.001 has been described elsewhere.[4] The charge multiplication, M, produced by carriers injected into a *p-n* junction was measured as a function of the reverse bias, V, for two different grown junctions. The high-resistivity side of junction A was n type while that of junction B was p type. The position of the light beam relative to the junction was adjusted so as to produce a maximum signal with zero or low reverse bias applied. Thus, the

[1] K. G. McKay and K. B. McAfee, Phys. Rev. **91**, 1079 (1953).
[2] K. G. McKay, Phys. Rev. **94**, 877 (1954).
[3] S. L. Miller, Phys. Rev. **105**, 1246 (1957).
[4] A. G. Chynoweth and K. G. McKay, Phys. Rev. **108**, 29 (1957).

initiation of multiplication was caused almost entirely by holes in the case of junction A (minority-carrier lifetime much greater on the n-side) and by electrons for junction B. Careful measurements of the junction capacity as a function of the reverse bias showed that each of the junctions followed the relation $C^nV=$constant, with $n=3.0$ for junction A and 2.9 for junction B. These values indicate that A follows very closely a parabolic field distribution while that of B is slightly distorted. The breakdown voltages of junctions A and B were 11.20 and 20.0 volts, respectively.

THEORY AND RESULTS

The ionization rates were computed from the multiplication curves first for the two cases: (i) where the parabolic field distribution was approximated by a constant field equal to the maximum of the actual field and extending for half the actual width of the junction, and (ii), where the actual field distribution was used. In both cases it was assumed that the ionization rates for holes and electrons were equal, the analysis being extended afterwards to the actual situation of different ionization rates.

(i) Square Field Approximation, $\alpha=\beta$

Let α and β be the ionization rates for holes and electrons, respectively. Then,[2]

$$1-\frac{1}{M}\approx\int_0^{W/2}\alpha(E_M)dx=\tfrac{1}{2}W\alpha(E_M), \tag{1}$$

where W is the actual width of the junction and $\alpha(E_M)$ is the ionization per unit path length at a field equal to E_M, the actual maximum field in the junction. For a linear-gradient junction,

$$E_M=1.5V/W=(1.5/W_1)V^{2/3}, \tag{2}$$

where V is the sum of the applied voltage and the built-in potential, the latter as well as the junction width being determined from the capacity measurements. W_1 is the junction width for a total potential difference across it of one volt. By using Eqs. (1) and (2) the field dependence of α can be obtained.

(ii) Parabolic Field, $\alpha=\beta$

For this case,[2]

$$\alpha(E_M)=\frac{2}{\pi}\left(\frac{1.5}{W_1{}^3}\right)^{\frac{1}{2}}\frac{d}{dE_M}\int_0^{E_M}\frac{(1-1/M)}{(E_M-E)^{\frac{1}{2}}}dE. \tag{3}$$

The integral was evaluated by machine after a sixth-order polynomial had been fitted to the experimental curve of $(1-1/M)$ against E_M.

(iii) Results of Above Methods of Analysis

For reasons discussed below, the results of the foregoing calculations are plotted in Fig. 1 in the form $\log_{10}\alpha$ *versus* $E_M{}^{-1}$. The solid lines labeled "holes" and "electrons" are the solutions of the parabolic field case for junctions A and B, respectively. The plot for junction A deviates less than 2% from a straight line for almost two decades. The plot for junction B is not so good; while it more or less follows a straight line at low fields it departs appreciably at the higher fields.

The plots obtained from the square field calculations show very similar shapes and slopes to the plots obtained from the more sophisticated treatment. The only important difference is that the latter result in rather higher values for the ionization rates. This, no doubt, arises from the arbitrary choice of an effective width for the constant field. Making the effective width somewhat less than half the actual width would bring the ionization rates more into agreement while having no effect on the value of E_M. Square field calculations made on several other junctions showed approximately straight plots in each case though there was some scatter when the various α *vs* E plots were superimposed. This scatter was probably due to nonideal field distributions which would result in errors in both α and E_M. Furthermore, the effective width used in the square field approximation varies with the actual width in a way that is not known with precision.

(iv) Calculation for the Case Where $\alpha\neq\beta$

For multiplication initiated by hole injection, we have[3]

$$1-\frac{1}{M}=\int_0^W\alpha(E)\exp\left\{-\int_0^x[\alpha(E)-\beta(E)]dx'\right\}dx. \tag{4}$$

A similar expression results for the case of electron injection if α and β are interchanged. The solution of these two simultaneous equations in α and β in the range of fields where the data overlap can be obtained for the case of a parabolic field by a method of successive approximations, as described in the appendix. The results of these calculations show that the ionization rates for holes as calculated for the case $\alpha=\beta$ and parabolic field distribution are very close to the true values. For example, though the correction to $\alpha_0(E_M)$ increased with the field, α_0 was reduced by only 7.5% at $E_M=4\times10^5$ volts cm^{-1}, an amount that is not very significant in the semilogarithmic plots. On the other hand, $\beta(E_M)$ is increased at the same field by about 20%. Thus the curve for electrons gets appreciably straightened by these calculations, as is shown in the figure. The small residual curvature could be accounted for, perhaps, by the departures of the field in junction B from a true parabolic distribution. Another possible source of error lies in the assumption concerning the relative lifetimes for minority carriers on either side of the junction. The lifetimes close to the space-charge region are not known and may be quite different from the lifetimes measured in the bulk crystal. However, based on bulk crystal measurements, the hole lifetime

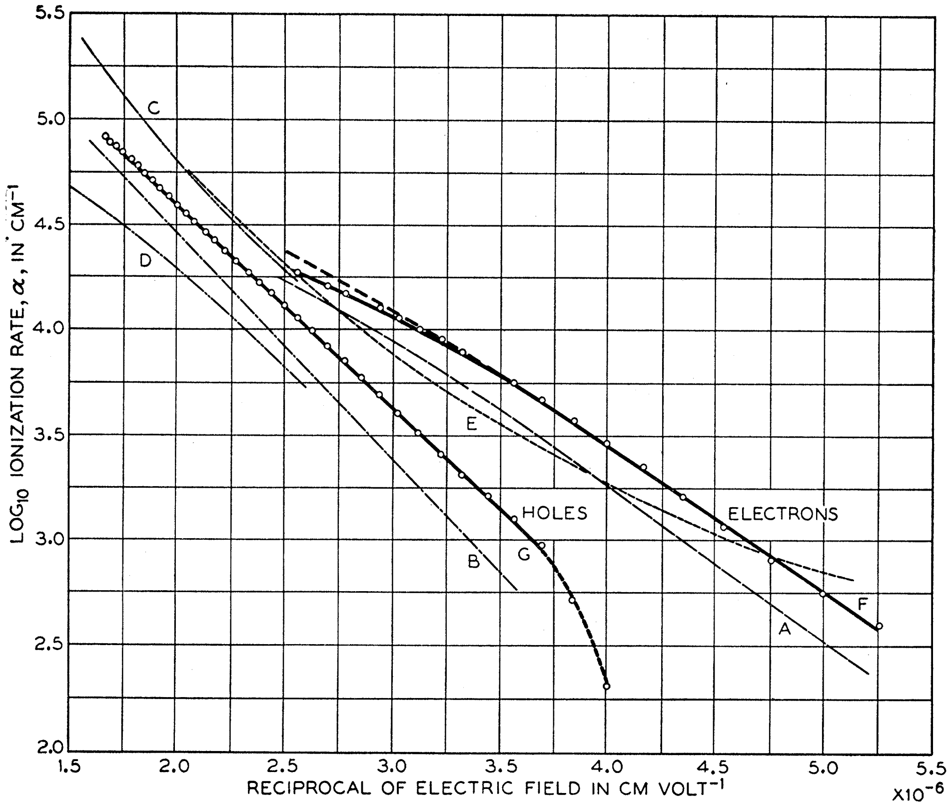

FIG. 1. The field dependence of the ionization rates for electrons and holes in silicon. Curves A and B are those obtained for electrons and holes, respectively, using the uniform field approximation and assuming that the ionization rates for electrons and holes are equal. Curves C and D represent the results obtained by Miller for electrons and holes respectively, while curve E represents McKay's averaged data. Curves F and G are those obtained for electrons and holes, respectively, for the case of a parabolic field distribution and assuming equal ionization rates for holes and electrons. No appreciable correction results to curve G when the ionization rates for holes and electrons are not assumed to be equal but the curve F (for electrons) deviates (as shown) to higher values of the ionization rate at the higher fields.

is very much greater than that for electrons for junction A but for junction B, the ratio of electron-to-hole lifetimes may not be so great. It can be concluded, therefore, that to within the limits of experimental error, plotting the logarithm of the ionization rate against the reciprocal of the electric field strength results in a straight line for holes and, probably, for electrons also.

DISCUSSION OF RESULTS

For comparison purposes the results obtained by Miller and McKay are also shown in Fig. 1. It is not possible to compare Miller's electron data with the present results but his hole curve appears to differ quite considerably. It is felt that this difference most likely results from the radically different methods used for analyzing the data.

McKay's curve is a composite from multiplication measurements on a number of junctions and reference to his original plot shows considerable scatter in the points. Furthermore, McKay's method of determining the multiplication was by measuring the *largest* charge pulses produced by carriers injected by α-particle bombardment. It is now apparent that the largest pulses would correspond always to electrons being injected at the p side of the junction because of their higher ionization rates. Thus, McKay's curve should correspond to the present one for electrons. Though it does appear to be closer to the electron rather than the hole curve, the agreement is not good. The discrepancies may be caused both by the scatter in McKay's data and by effects such as those discussed above in connection with the deviation of the electron curve from a straight line.

Ionization rates measured in gases can often be fitted by an expression of the form[5,6]

$$\alpha = a \exp(-b/E), \tag{5}$$

where a and b are constants. The results obtained for the better-behaved junction in particular (junction A) agree with Eq. (5) so closely that it seems there should be some theoretical justification for its use. However, Wolff's theory,[7] even in approximate form, does not lead to an E^{-1} dependence of an exponential factor, suggesting, therefore, that some fresh theoretical approach is necessary. Approximate theories[8] of the ionization rate in gases do lead to relation (5) though, as Rose has pointed out,[6] the theories fail to predict the observed linearity of $\ln\alpha$ against E^{-1} over so wide a range. Such a relation suggests a carrier energy distribution function approximating a Boltzmann distribution. The constant b may then be related to the ionization energy and the mean free path for the carriers between ionizing collisions.

[5] L. B. Loeb, *Basic Processes of Gaseous Electronics* (University of California Press, Berkeley, 1955), Chap. 8.
[6] D. J. Rose, Phys. Rev. **104**, 273 (1956).
[7] P. A. Wolff, Phys. Rev. **95**, 1415 (1954).
[8] For example, T. Kihara, Revs. Modern Phys. **24**, 45 (1952).

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APPENDIX. METHOD OF OBTAINING THE SOLUTIONS FOR α AND β IN THE CASE OF A PARABOLIC FIELD DISTRIBUTION, $\alpha \neq \beta$

The field distribution for junction A is

$$E(a)=E_M[1-(2a/W_p)^2],$$

where

$$-W_p/2 \leq a \leq +W_p/2,$$

W_p being the total width of the junction. Transferring the origin of the coordinate system to $a=-W_p/2$, i.e., $x=a+W_p/2$, we have

$$x=W_p[1\pm(E_M-E)^{\frac{1}{2}}]/2(E_M)^{\frac{1}{2}}. \quad (1)$$

Similarly, for junction β,

$$y=W_n[1\pm(E_M-E)^{\frac{1}{2}}]/2(E_M)^{\frac{1}{2}}. \quad (2)$$

The suffices n and p refer to the junctions where the initial injected carriers are electrons and holes, respectively. Now

$$\left(1-\frac{1}{M}\right)_p=\int_0^{W_p}\alpha(E)\times\exp\left\{-\int_0^x[\alpha(E')-\beta(E')]dx'\right\}dx=\int_0^{W_p}\alpha_0(E)dx, \quad (3)$$

where $\alpha_0(E)$ is the ionization rate calculated from the same multiplication data for $\alpha=\beta$. From Eqs. (1) and (2),

$$dx=f(E)dE \quad \text{and} \quad dy=g(E)dE.$$

Equating the integrands in Eq. (3), transforming the variable of integration, and identifying the upper limit of integration with E_M, we obtain

$$\ln[\alpha(E_M)/\alpha_0(E_M)]=\int_0^{E_M}\alpha(E)f(E)dE-\int_0^{E_M}\beta(E)f(E)dE. \quad (4)$$

But

$$W_pg(E)=W_nf(E). \quad (5)$$

Also,

$$\left(1-\frac{1}{M}\right)_p=2\int_0^{E_M}\alpha_0(E)f(E)dE,$$

$$\left(1-\frac{1}{M}\right)_n=2\int_0^{E_M}\beta_0(E)g(E)dE.$$

Using (5), we obtain

$$\left(1-\frac{1}{M}\right)_n=2\frac{W_n}{W_p}\int_0^{E_M}\beta_0(E)f(E)dE.$$

Thus, if α_0 is not too different from α and β_0 is not too different from β, we can write

$$\ln[\alpha_1(E_M)/\alpha_0(E_M)]^2=2\int_0^{E_M}\alpha_0(E)f(E)dE-2\frac{W_p}{W_n}\int_0^{E_M}\beta_0(E)g(E)dE, \quad (6)$$

where $\alpha_1(E_M)$ is a first approximation to $\alpha(E_M)$.

Now at a particular value of E_M we can fit the solution of the parabolic field case with $\alpha=\beta$ to that of the square field approximation by choosing an appropriate effective width, that is,

$$2\int_0^{E_M}\alpha_0(E)f(E)dE=\int_0^{W_p^*}\alpha_0(E_M)dx=\alpha_0(E_M)W_p^*,$$

where $W_p^*=k_pW_p$, k_p being a constant less than unity. Similarly,

$$2\int_0^{E_M}\beta_0(E)g(E)dE=\int_0^{W_n^*}\beta_0(E_M)dy=\beta_0(E_M)W_n^*,$$

where $W_n^*=k_nW_n$. Hence,

$$\ln[\alpha_1(E_M)/\alpha_0(E_M)]^2=\alpha_0(E_M)W_p^*-W_p\beta_0(E_M)W_n^*/W_n=W_p[\alpha_0(E_M)k_p-\beta_0(E_M)k_n].$$

Similarly, it can be shown that

$$\ln[\beta_1(E_M)/\beta_0(E_M)]^2=W_n[\beta_0(E_M)k_n-\alpha_0(E_M)k_p].$$

The right-hand sides of these two equations are known completely, thus enabling the approximate solutions $\alpha_1(E_M)$ and $\beta_1(E_M)$ to be obtained. Inserting these values into the right-hand sides yields the second approximations $\alpha_2(E_M)$ and $\beta_2(E_M)$. Over the range of fields where the data from the two junctions overlapped, the solutions converged very rapidly.