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Paramagnetic Effect in Superconductors. VII. Shape of the Superconducting Domains*

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A recalculation of the density of the domains in the paramagnetic effect for large values of the ratio of length to diameter of the domains shows that, except for a thin layer near the surface, the density is always very small. Under these conditions the current through a single domain becomes too large to be neglected. From whatever point of view one considers the effect of this current, one will conclude that it will lower the value of the mean magnetic field between the domains below the value of the bulk critical field. Assuming that the same conditions persist in absence of an external longitudinal field, one can explain the increase of the critical resistance above one-half of the normal resistance and its dependence on electronic mean free path, sample diameter, and temperature.

An experimental study of the magnetic field in the center of a hollow indium wire and the longitudinal flux in a solid indium wire reveals that the paramagnetic effect gradually disappears at external fields below 0.5 amp/cm. This necessitates the assumption of disturbing influences which prevent the perfect alignment of the superconducting domains. It is believed that the disturbing influences, rather than the difference between the mean magnetic field and H_c, will lead to the corrections necessary to account for the observed limiting current I_g. A detailed treatment of the size of the domains and of their distribution should be made with the use of the thermodynamics of irreversible processes.

I. INTRODUCTION

LONDON'S theory[1] of the transition of a current-carrying wire was extended[2] in Part I of this series to include a superimposed longitudinal magnetic field H_{z0}. London's theory as well as its extension do not make any specific assumptions about the superconducting domains, but rather use a "smeared out" model, in which an anisotropic conductivity is linked to the mean magnetic inductance B and to the ratio l/a of the length to the diameter of the superconducting domains. It has been found experimentally[3] that the measurements of the longitudinal flux at large currents agree with the theoretical predictions if one chooses $l/a\approx 500$. Furthermore, it has been found[4-6] that the theory of the paramagnetic effect is still substantially correct for samples with a radius as small as $R=0.6$ mm. This implies that the diameter of the domains is still considerably smaller: $a<10^{-2}$ mm.

It was shown[7] in Part II that the theory predicts that the critical values of the resistance Ω_c as well as of the circular flux $\tilde{K}_{m\varphi}=\Phi_{\varphi\max}/\Phi_{\varphi n}$ are independent of the value of the longitudinal field H_{z0}. Both predictions are well confirmed by experimental observations.[8-10] This suggests that the arrangement of the superconducting domains is similar for pure current transitions and current transitions with superimposed longitudinal magnetic field.

In the following we will first show in diagrams the results which the present theory gives for large values of $C=l/a-1$. We will see that the theory leads to a solenoidal current layer near the surface which produces a strong longitudinal component of the magnetic field in the central part of the sample. This makes the central part almost normal-conducting, with a few superconducting domains almost aligned in the direction of the axis of the sample.

We will then discuss the implications of the assumption of long and thin superconducting domains and we will see that it is possible to explain at least qualitatively most of the differences between the experimental observations and the present theory.

Furthermore, we will describe some experiments in the region of very small values of H_{z0} which indicate that the paramagnetic effect is incomplete at very small values of H_{z0} contrary to the predictions of the present theory. This result shows the necessity to assume influences which disturb the ideal array of the superconducting domains.

Finally we will see that the present discussion leads to a series of questions, experimental as well as theoretical, which have to be answered before further quantitative progress can be made.

II. RESULTS OF THE PRESENT THEORY FOR LARGE VALUES OF C

The differential equations for the circular and longitudinal magnetic field [Part I, Eq. (20′)] have been solved in Parts I and II for various values of[11] $\varphi_0=H_{\varphi 0}/H_{z0}$ using a value of $C=10$. New experimental evidence (see reference 3) indicates that C is much

* Supported by a Grant of the National Science Foundation.
[1] F. London, *Superfluids* (John Wiley and Sons, Inc., New York, 1950), Vol. 1, p. 120.
[2] Hans Meissner, Phys. Rev. **97**, 1627 (1955), referred to as "Part I."
[3] J. C. Thompson, Phys. Rev. **102**, 1004 (1956).
[4] Y. Shibuya and S. Tanuma, Sci. Repts. Research Insts., Tôhoku Univ. **A7**, 549 (1955).
[5] Y. Shibuya and S. Tanuma, Phys. Rev. **98**, 938 (1955).
[6] Hans Meissner, Phys. Rev. **103**, 39 (1956).
[7] Hans Meissner, Phys. Rev. **101**, 31 (1956), referred to as "Part II."
[8] Hans Meissner, Phys. Rev. **101**, 1660 (1956), referred to as "Part III."
[9] L. Rinderer, Helv. Phys. Acta **29**, 339 (1956).
[10] Hans Meissner, Phys. Rev. **109**, 668 (1958), referred to as "Part V."
[11] We are using the same notation as in Parts I and II.

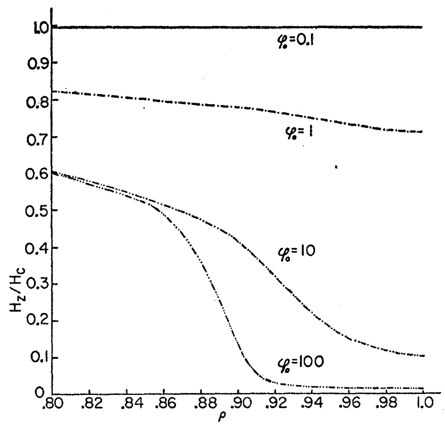

FIG. 1. Dependence of the longitudinal component of the magnetic field H_z/H_c on the radius $\rho=r/R$.

larger. Therefore we have recalculated all functions of interest using a value of $C=500$, and have plotted them in Figs. 1–4.

As in Part I, we are interested only in the case where the total magnetic field at the surface of the sample is equal to the critical field.

Figure 1 shows $H_z/H_c=\chi/(1+\varphi_0^2)^{\frac{1}{2}}$ plotted as function of the radius $\rho=r/R$. This figure shows only the outermost region of the sample $\rho>0.8$ in contrast to the Fig. 3(a) of Part I with which it should be compared. For $\rho<0.8$, the calculation shows that even for a very strong dominance of the current ($\varphi_0=100$) the longitudinal magnetic field is almost equal to the critical field for the central part of the sample.

Figure 2, which should be compared with Fig. 3(b) of Part I, shows a similar plot of φ/φ_0 *vs* ρ indicating the relatively small deviations from the curve for $C=\infty$ [Part I, Eq. (23)].

Figure 3 is similar to Fig. 1 of Part II and shows the radial variation of the mean induction. The latter is connected to the density of the normal regions by

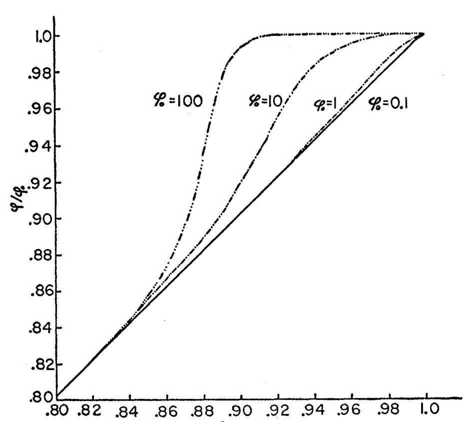

FIG. 2. Dependence of the circular component of the magnetic field $\varphi/\varphi_0=H_\varphi/H_{\varphi0}$ on the radius $\rho=r/R$.

Eq. (5) of Part I. One can see that for $\rho<0.8$ this density has a value of $\xi_{II}=(C-1)/C$ even for very large values of φ_0. (For $\varphi_0=\infty$ the density ξ_{II} would be zero at the center of the wire.)

Figure 4 shows, similarly to Fig. 2 of Part II, the radial dependence of the normalized longitudinal component of the current density. It can be seen that large deviations from one occur only in a zone $\rho>0.8$.

It follows from these diagrams that the present theory for $C=500$ and values of φ_0 up to $\varphi_0=100$ leads to a central, almost normal conducting core, filled with a longitudinal magnetic flux of an intensity almost equal to the critical field.

III. INFLUENCE OF THE SHAPE OF THE SUPERCONDUCTING DOMAINS

(a) The Current through the Domains and the Magnetic Field in between the Domains

Despite the existing differences between theory and experiment, we shall now assume that the theoretical

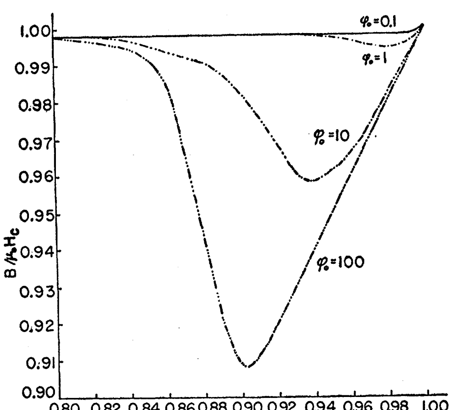

FIG. 3. Dependence of the mean magnetic induction B/μ_0H_c on the radius $\rho=r/R$. B/μ_0H_c is equal to the density of the normal conducting regions.

prediction about the central core is substantially correct. Furthermore, we shall tentatively assume that the diameter of the domains is about equal to the "range of order"[12] of the electrons, which is still considerably larger than the penetration depth.[13] For samples of low purity the range of order decreases and is of the size of the electronic mean free path [see reference 11, Eq. (16)].

The assumptions above require immediately a slight modification of the definitions of ξ_I and ξ_{II} in the present theory. In defining these densities in Part I, Eqs. (6) and (7), a two-dimensional approach was used. This was justified at that time because it was believed that only fairly high densities of the superconducting domains are of interest.

[12] A. B. Pippard, Proc. Roy. Soc. (London) **A216**, 547 (1953).
[13] D. Shoenberg, *Superconductivity* (Cambridge University Press, New York, 1952), p. 150.

At very low densities the problem has to be treated as three dimensional. We consider a "unit cell" which contains just one superconducting domain. (See Fig. 5). For clarity the shape of the domain drawn is a rectangular prism rather than an ellipsoid.) The direction of the domain will be in the direction of the mean magnetic induction **B**. The domain has a length l and a diameter a, while the dimensions of the unit cell are $l+d$ and $a+d$, respectively. An electric field applied in the η-direction (see Fig. 5) will be shorted out over the length l of the superconducting domain, so that the ratio $\xi_{\mathrm{I}E}$ of the mean electric field E_η to the local electric field e_η is, as in Part I, Eq. (7),

$$\xi_{\mathrm{I}E}=E_\eta/e_\eta=d/(l+d). \qquad (1)$$

Similarly one obtains for an electric field in the ζ

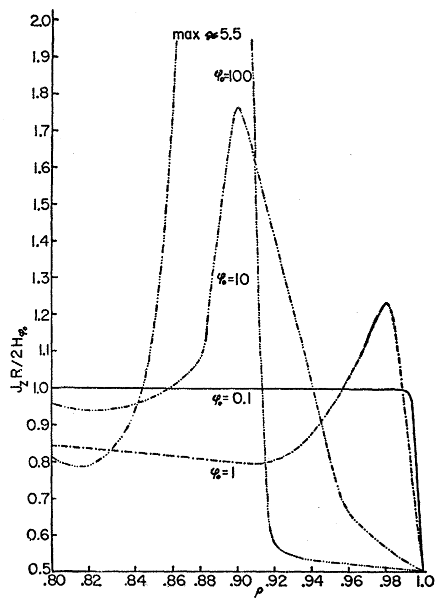

FIG. 4. Dependence of the longitudinal component of the current density $J_zR/2H_{\varphi 0}$ on the radius $\rho=r/R$.

direction

$$\xi_{\mathrm{II}E}=E_\zeta/e_\zeta=/d(a+d). \qquad (2)$$

The ratio of the mean induction B to μ_0 times the local field h is no longer equal to $\xi_{\mathrm{II}E}$ but is given by

$$\xi_{\mathrm{II}B}=B/\mu_0h=[(a+d^2)-a^2]/(a+d)^2$$
$$=1-(1-\xi_{\mathrm{II}E})^2. \qquad (3)$$

It can be readily checked that most of the calculations presented in part I remain unchanged, since $\xi_{\mathrm{I}E}$ and $\xi_{\mathrm{II}E}$ enter. Only the calculation of the magnetic flux requires the use of $\xi_{\mathrm{II}B}$ rather than $\xi_{\mathrm{II}E}$, leading to somewhat larger values of the flux.

We shall now try to estimate the current through one domain assuming that the domain is parallel to the z axis and that the mean current density J_z is about equal to the current density in the normal conducting

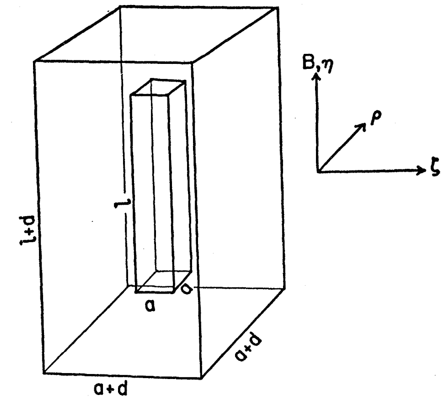

FIG. 5. "Unit cell" of cross section $(a+d)^2$ and length $l+d$ containing one superconducting domain of cross section a^2 and ength l. For clarity the superconducting domain is drawn as a rectangular prism rather than an ellipsoid.

state. With these simplifying assumptions, practically all of the current entering the unit cell at the top will eventually go through the superconducting domain in an axial direction. The current will therefore be

$$I_d=(a+d)^2J_z, \qquad (4)$$

with

$$J_z=2H_{\varphi 0}/R, \qquad (5)$$

according to our assumptions above. This current will produce a circular magnetic field at the surface of the domain with a value of $H_\rho=I_d/\pi a$. Assuming $H_{\varphi 0}\gg H_{z0}$ leads to $H_{\varphi 0}=H_c$ since the total field at the surface of the sample shall always be critical. With this we obtain

$$H_\rho/H_c=2(a+d)^2/\pi aR. \qquad (6)$$

Although this derivation is admittedly very rough, it allows one to see what quantities enter into the problem and in what direction changes are to be expected.

The first effect is that the mean value of the magnetic field $|\mathbf{H}|=H_\eta$ is not equal to H_c but is given by

$$H_c^2=H_\eta^2+H_\rho^2,$$

which leads to

$$H_\eta=H_c[1-4(a+d)^4/\pi^2a^2R^2]^{\frac{1}{2}}. \qquad (7)$$

There can be a further deviation from this value if the domain is so thin that its critical field differs from the bulk critical field H_c. This deviation will be a decrease if the current through the domain is dominating, and an increase if the magnetic field around it is dominating.

In all events, the value of the mean magnetic field which enters into the calculation [Part I, Eq. (14)] will be smaller than the bulk critical field if the sample is subject to a sizable current, that is, if $H_{\varphi 0}/H_{z0}>1$.

(b) Critical Resistance

The present theory (see Part II) gives for the value of the critical resistance $\Omega_c/\Omega_n=\frac{1}{2}$, independent of the value of a superimposed field H_{z0}. While, as mentioned earlier, the experiments confirm the independence of H_{z0}, they all give values of Ω_c/Ω_n somewhat larger than $\frac{1}{2}$ (see Rinderer[9] and Scott[14] and Parts V and VI[15]).

It is easy to show that a decrease of the mean magnetic field leads to an increase of the value of the critical resistance. We consider the case that the sample is subject to a current only. The equation $\mathbf{J}=\text{curl}\mathbf{H}$ leads, with the assumption $\mathbf{H}\equiv H_\eta$ and independent of r, to $J=H_\eta/r$. Following the derivation in reference 1, page 120, one obtains

$$\Omega_c/\Omega_n=\tfrac{1}{2}H_c/H_\eta, \tag{8}$$

which is larger than $\frac{1}{2}$ if $H_\eta<H_c$. This calculation cannot be strictly correct, since at the surface of the sample $r=R$ we have

$$B=\mu_0\xi H_\eta=\mu_0 H_c \tag{9}$$

implying $\xi>1$, which is not possible. It follows that the mean magnetic field cannot be unequal to H_c and independent of r at the same time. As rough as this estimate is, it still seems to give a plausible explanation for the fact that the values of Ω_c/Ω_n found experimentally are all somewhat larger than $\frac{1}{2}$.

The hysteresis which is observed in transitions with uninterrupted current (see references 9 and 14 and Part V) is now easily understood. In approaching the normal conducting state the sample stays superconducting until the critical field is exceeded at the surface of the sample. It then goes over into a state where the mean magnetic field H_η is smaller than the bulk critical field. In approaching the superconducting state, however, the sample can stay in the mixed state to values of $H_{\varphi 0}$ smaller than H_c, conceivably as small as H_η. This state is, of course, metastable, and a small fluctuation, especially the reversal of the current, can throw it into the stable, completely superconducting state.

(c) Diameter of the Superconducting Domains

The experimentally observed dependence of Ω_c/Ω_n on the radius R of the sample and on the electronic mean free path (see references 9 and 14 and Parts V and VI) can now be easily explained. From Eq. (7) we can see that H_η decreases with decreasing radius which, according to Eq. (8), leads to an increase in Ω_c/Ω_n, in qualitative agreement with the experiments.

The dependence of Ω_c/Ω_n on electronic mean free path follows from the assumption that the diameter a of the domains is connected with the range of order of the superconducting electrons. According to Pippard [reference 12, Eq. (16)] the range of order decreases with decreasing electronic mean free path. Assuming that the distance d between the domains does not decrease too much at the same time, it follows from Eq. (7) that H_η decreases with decreasing electronic mean free path, and from Eq. (8) that Ω_c/Ω_n increases with decreasing electronic mean free path, in qualitative agreement with the experimental observations [see Part V, Fig. 11(b) and Part VI, Fig. 7(b)].

(d) Deviations Near the Critical Temperature

The present theory states that all functions, after proper normalization, should be independent of the absolute value of H_c, that is, independent whether the experiment is performed close to T_c or at some distance from it. Contrary to this, a number of anomalies have been observed in the neighborhood of the critical temperature.

In Part II, Fig. 6 it was observed that the assumption of a "mixed" core surrounded by a normal conducting sheath fails close to the critical temperature, and it was remarked that the transition region seems to be extended to values $H>H_c$,[16] i.e., that the sheath is not completely normal conducting.

The current transition of samples of lower purity is also spread out near T_c as observed by Rinderer (see reference 9) and in Part V. Rinderer was especially careful with the attachment of the potential taps, thus omitting "tails," and could prove (see reference 9, Figs. 5 and 7) that the first rise occurs even near T_c at the usual value of the critical field and that the sample is always more superconducting than it would be in a corresponding state at a lower temperature (see reference 9, Fig. 5).

A similar interpretation can also be given for the reduction of the circular flux at low currents, that is, near T_c, found in Part III (see Figs. 5 and 6). The current is, in these cases, more evenly distributed over the radius. This can be explained by assuming that more superconducting domains are at large radii, that is in fields $H>H_c$.

It seems that these anomalies can be explained with the aid of Ginsburg and Landau's new phenomenological theory.[17] This theory gives first order transitions if the diameter of a sample (or domain) is larger than a certain critical value[18] $a_k=\sqrt{3}\delta$, second-order transitions if it is smaller than a_k (δ is the penetration depth). The quantity δ increases with decreasing electronic mean free path (see reference 12) and increases sharply at temperatures close to T_c (see reference 13, p. 143). As we have remarked above, the diameter of the domains decreases with decreasing electronic mean free path. It is then possible, especially in the neighborhood of T_c, that in samples of lower purity the diameter of the domains becomes smaller than a_k. They will then undergo second order, rather than first order, transitions which will allow them to form more freely. Moreover their critical fields will be higher than the bulk critical

[14] R. B. Scott, J. Research Natl. Bur. Standards **41**, 581 (1948).
[15] H. Meissner and R. Zdanis, Phys. Rev. **109**, 681 (1958), referred to as "Part VI."

[16] Owing to a misprint the quoted equation unfortunately reads $H<H_c$.
[17] V. L. Ginsburg and L. D. Landau, J. Exptl. Theoret. Phys. U.S.S.R. **20**, 1064 (1950).
[18] V. P. Silin, J. Exptl. Theoret. Phys. U.S.S.R. **21**, 1330 (1951).

field, since their size is comparable to the penetration depth. (It is assumed that the current through the domains is not too large, which will be true in the outermost regions of the sample where the domains are tilted against the direction of the current.) Therefore domains can form in the outermost regions of the sample which, due to the high fields, should stay normal conducting. This is precisely what is needed to explain the anomalies discussed above.

(e) Transitions in a Longitudinal Magnetic Field

The present theory gives infinitely sharp transitions if these are forced by a longitudinal external magnetic field and observed with a negligibly small measuring current: $H_{\varphi 0} \ll H_{z0}$. Experiments of this type have been performed by Sizoo *et al.*,[19] by de Haas and Voogd[20] and by McDonald and Mendelssohn.[21] The first two groups observed very large hysteresis and stepwise transitions with some samples, and slight hysteresis and smooth transitions with other samples, while the last group found, for proper geometry, no hysteresis and smooth transitions. McDonald and Mendelssohn explained the difference of their results by their improved geometry. It is of course well known that improper attachment of the potential taps can lead to "tails" and that "shadows" from bulbous ends can lead to hysteresis. Nevertheless, it seems that there is some real difference. The strongly stepwise transitions with very large hysteresis were observed only for the thinnest samples if they consisted of not more than a few crystallites (the tin and indium samples of de Haas and Voogd). It seems as if here only a few domains are formed. This assumption is in agreement with recent measurements on very thin tin whiskers by O. Lutes.[22] In all other cases many superconducting domains are formed, a few of which can persist to external fields larger than the critical field of bulk superconductors, leading to an extension of the transition curve toward higher fields. A small increase in the measuring current actually makes the transition curves sharper (as long as $H_{\varphi 0} \ll H_{z0}$) since the superconducting domains then carry a sizable current which reduces the value of the mean magnetic field H_η.

If hysteresis is found in these cases at all, it is very small, of the type discussed with the current transitions, and would probably vanish at sufficiently low measuring currents.

IV. PARAMAGNETIC EFFECT AT LOW VALUES OF H_{z0}

The present theory predicts that for sufficiently large values of $C = l/a - 1$ the paramagnetic effect should practically always be set up leading to a longitudinal field of almost critical strength at the center of the sample. No direct check of this prediction existed so far aside from the fact that the critical resistance did not change when a longitudinal field is superimposed.

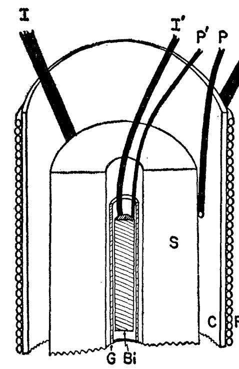

FIG. 6. Schematic diagram of the indium sample XV. The sample S is provided with current (I) and potential leads (P). The current returns through the copper tube C, which carries a field coil F. A bismuth wire Bi with current (I') and potential leads (P') is mounted in a glass capillary G and inserted in the center hole of the sample.

After it was established that indium gives reliable results also in extruded, rather than in the form of single crystals (see Part VI), it became feasible to make hollow wires of indium by an extrusion process.

The sample No. XV was extruded from 99.97% pure indium of the Indium Corporation of America. It had an o.d. of 1.94 mm and an i.d. of (nominal) 0.5 mm and was 50 mm long. Potential taps were attached with In-Sn solder at a distance of about 5 mm from each end. The sample had an icepoint resistance of 9.914×10^{-4} ohm; the residual resistance ratio was $r_0 = 2.2 \times 10^{-4}$. The sample was mounted in a concentric copper tube of 10 mm o.d., 60 mm length which served as current return and holder for the field coil of 52 mm length. A bismuth wire of about 0.2 mm diameter and 15 mm length was provided with current and potential leads and mounted in a thin glass capillary which was inserted into the 0.5 mm hole of the sample (see Fig. 6).

The earth's magnetic field was compensated by a pair of Helmholtz coils to less than 3×10^{-3} amp/cm. The cryostat and automatic temperature control were the same as described in Parts III and V.

The magnetic field in the center hole was measured with the bismuth wire as function of the sample current for various values of the longitudinal field H_{z0} at two temperatures below the critical temperature of indium. Freezing-in of the flux was prevented by always removing the magnetic field at very high currents, restoring it at zero current and always measuring with rising current.

Figure 7 shows the resistance of the sample and the magnetic field in the center hole for various values of the longitudinal field H_{z0} plotted as function of the circular field $H_{\varphi 0}$. The sharp resistance transition (note the tremendous spread of the $H_{\varphi 0}$ axis) shows the good quality of the sample. Nevertheless the magnetic field in the center H_{zi} gradually disappears at low values of H_{z0}.

Figure 8 shows the dependence of the maximum value of H_{zi} on H_{z0} more clearly, indicating that the

[19] Sizoo, de Haas, and Onnes, Comm. Leiden 180c (1926).
[20] W. T. de Haas and J. Voogd, Comm. Leiden 191d (1928).
[21] D. K. C. McDonald and K. Mendelssohn, Proc. Roy. Soc. (London) **A200**, 66 (1949).
[22] O. Lutes, Phys. Rev. **105**, 1451 (1957).

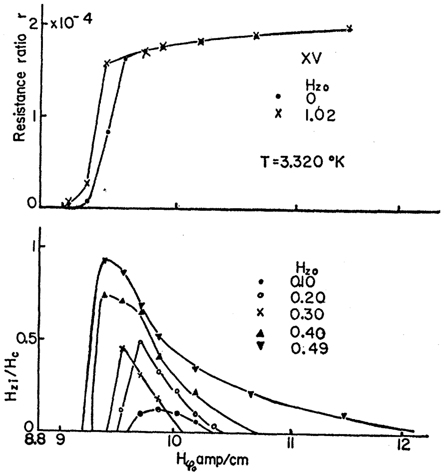

FIG. 7. Resistance (upper part) and longitudinal field (lower part) H_{zi} in the center hole as a function of the circular magnetic field $H_{\varphi 0}$ produced by the current at the surface of the indium sample XV for various values of the superimposed longitudinal field H_{z0} at a temperature of $T=3.320$°K.

theoretical prediction of almost critical values of the longitudinal magnetic field in the center is not fulfilled for values of $H_{z0}<0.5$ amp/cm.

One will, of course, query immediately whether this deviation is only caused by the existence of the inner boundary, which certainly is not included in a correct way in the present theory. A search through the literature shows that nobody has measured below $H_{z0}=0.5$ amp/cm and that the curves of Shibuya and Tanuma (see reference 4, Fig. 16) drop off conspicuously around $H_{z0}=0.5$ amp/cm. Therefore it has been found worth while to make an immediate, even if rough, check of the paramagnetic effect at values of $H_{z0}<0.5$ amp/cm.

The indium sample XVII was an extruded (solid) wire, about 50 mm long, 1.94 mm in diameter. The search coil of 10 000 turns No. 40 wire was wound directly upon the sample and covered a length of 40 mm. Despite the use of a special winding machine, the sample was somewhat damaged during the winding process, resulting in a high residual resistance ratio $r_0=23\times10^{-4}$. The sample was placed in the center of a copper tube of 19 mm o.d. which served as a return for

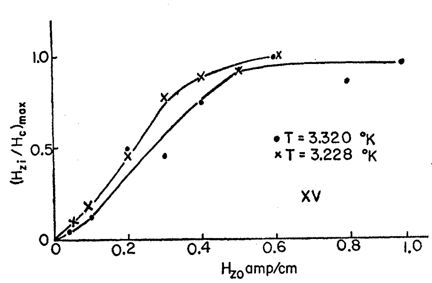

FIG. 8. Dependence of the maximum value of the longitudinal field H_{zi} in the center hole of the indium sample XV on the value of the superimposed longitudinal magnetic field.

the current and as holder for the field coil of 200 mm length. Part of the nuisance flux was compensated by a coil of larger diameter than the sample, wound with 1000 turns of No. 40 wire and placed in the same field coil at some distance from the sample. All other arrangements were the same as described above.

The longitudinal flux was measured by observing the deflection of a ballistic galvanometer connected to the search coil while the magnetic field was reversed. Figure 9 shows a plot of the longitudinal flux *vs* sample current for different temperatures and a value of the longitudinal field of $H_{z0}=0.20$ amp/cm. Close to the critical temperature of In ($T_c=3.412$°K), the flux behaves normally, the maximum values $\tilde{K}_m$ increasing with current. The increase is considerably smaller than that found by Thompson (ese reference 3) for his very pure single crystal samples. At large currents, however, the maximum is smaller instead of larger.

Figure 10, where $\tilde{K}_m$ is plotted as function of H_{z0} for fixed temperature, shows this behavior better. Fixed

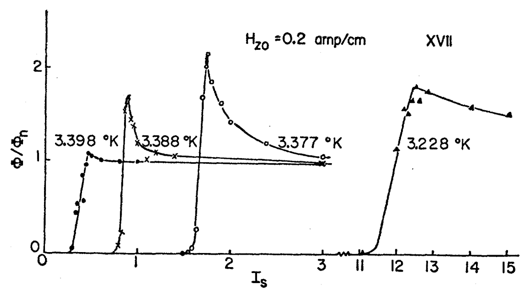

FIG. 9. Dependence of the longitudinal flux on sample current for a value of the superimposed longitudinal field $H_{z0}=0.2$ amp/cm at various temperatures for indium sample XVII, 1.94-mm o.d.

temperature means fixed H_c and, at low values of H_{z0}, fixed $H_{\varphi 0}$. Since $\tilde{K}_m$ increases in the regular region of sufficiently large H_{z0} with $\gamma=(H_{\varphi 0}/H_{z0})[1-(I_g/I)]$ (see Part I), one would expect $\tilde{K}_m$ to increase for fixed temperature with decreasing H_{z0}. On the contrary, Fig. 10 shows that below $H_{z0}=0.5$ amp/cm $\tilde{K}_m$ drops, reaching a value of $\tilde{K}_m=1$ at $H_{z0}=0$.

It is very probable that this drop occurs at still lower values of H_{z0} for samples of better quality, but it certainly will always be there.

Table I gives a complete list of all measurements on the solid indium sample. Using only the measurements at the two lowest values of γ at fields of $H_{z0}=0.2$ and 0.5 amp/cm, one obtains a value of $I_g=0.31\pm0.02$ and a value of $\gamma^*=1.0\pm0.3$. At larger values of γ the points deviate in a manner such that $\tilde{K}_m$ is no longer a function of γ only. Nevertheless it can be said that at lower values of $H_{\varphi 0}$ the drop in $\tilde{K}_m$ occurs at lower values of H_{z0}.

V. DISCUSSION

The following conclusions can be drawn from this investigation:

(1) One can qualitatively account for a number of differences between experiments and the present theory by the assumption that the superconducting domains are very thin.

(2) In addition, one has to assume disturbing influences which prevent the perfect alignment of the domains at low values of the longitudinal magnetic field.

One might object that the first assumption leads to unreasonably large values of the surface energy of the domains. This objection is valid as long as one uses equilibrium thermodynamics. However, as soon as the sample is connected to a battery, no matter how small the current drawn, the thermodynamics of irreversible processes should be used rather than the ordinary one. It is interesting to note here that, as shown by Shoenberg (reference 13, page 132), the equations for the current transition of a wire can also be derived from the

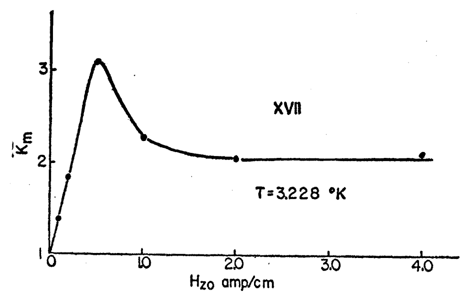

FIG. 10. Dependence of $K_m=\Phi_{max}/\Phi_n$ on the value of H_{z0} for a fixed temperature of $T=3.228$°K for indium sample XVII, 1.94-mm o.d.

condition of a minimum Joule heat, that is, minimum entropy production.

One might further object that the qualitative explanation of the differences between theory and experiment is based upon the assumption that the arrangement of the superconducting domains is substantially the same whether or not a longitudinal magnetic field is present, while the experiments at low values of H_{z0} show to the contrary that the arrangement cannot be quite the same. This objection is considerably more serious than the first one. The question is, however, what constitutes a "substantial difference." A substantial difference certainly exists between the structure with long and thin domains and the double-cone structure proposed by Shoenberg (see reference 1, p. 120, Fig. 40).

TABLE I. Values of the maximum apparent permeability $\tilde{K}_m$ for low values of the superimposed longitudinal field H_{z0}.

Temperature (°K)	H_{z0} (amp/cm)	I (amp)	$H_{\varphi 0}$ (amp/cm)	γ	$\tilde{K}_m$
3.228	0.1	12.8	21	205	1.38
3.398	0.2	0.46	0.756	1.24	1.07
3.388	0.2	0.90	1.48	4.85	1.69
3.377	0.2	1.75	2.88	11.9	2.16
3.228	0.2	12.8	21	105	1.78
3.388	0.5	0.8	1.32	1.62	1.16
3.377	0.5	1.70	2.89	4.58	1.73
3.228	0.5	12.4	20.4	39.8	3.08
3.228	1	12.1	19.9	19.4	2.16
3.228	2	11.8	19.4	9.45	2.05
3.228	4	11.4	18.7	4.55	2.11[a]

[a] This curve has been measured both by ballistic and fluxmetric methods to check for the absence of time constants long enough to falsify the ballistic measurements.

It is, however, not necessary to assume such a drastic difference for the explanation of the vanishing of the paramagnetic effect at low values of H_{z0}. It is fully sufficient to assume that the actual angle which the domains make with the φ-direction fluctuates somewhat, thus reducing the increase of the longitudinal flux. Whether or not this would cause a change in the value of the critical resistance can only be decided after a quantitative calculation has been made.

One is under the impression that the explanation of the constants I_g will be found in connection with the disturbing influences rather than in connection with the difference between the mean magnetic field and the bulk critical field.

Before any quantitative progress can be made, a number of questions must be answered:

(1) What are the principles that govern the size of the superconducting domains?

(2) How can the problems be treated with the use of the thermodynamics of irreversible processes?

(3) What are the disturbing influences and how can they be taken into account?

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