PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

Rotation of Liquid Helium at Low Reynolds Numbers*

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(Received November 15, 1957)

Previous experiments on the transient and steady-state behavior of the free surface of helium II in a bucket suddenly set into rotation have shown that the normal and superfluid components are in common motion. This contradicts the simple two-fluid equations (neglecting mutual friction) and has been attributed to the fluid's having exceeded some "critical velocity." The principle of dynamical similarity suggests that such experiments should be scaled according to the Reynolds number, and analysis of previous experiments indicates that this parameter has always been rather high ($>3.7\times10^4$). The Reynolds number has been reduced to $\sim1.4\times10^3$ by reducing the radius of the container to 0.0255 cm. The experimental results show that under these conditions the two fluids are still in common motion. The shape of the free surface was much different than in wider containers due to the effects of surface tension, which limit further reduction of the Reynolds number in this manner.

1. INTRODUCTION

OSBORNE first showed[1] that on the basis of the two-fluid hydrodynamical equations (neglecting mutual friction) the shape of the free surface of a steadily rotating bucket of helium II should be given by the expression

$$z=(\rho_n/\rho)\Omega^2r^2/2g, \tag{1}$$

where z is the vertical distance from the origin at the vertex of the parabola to a point on the surface, Ω is the angular velocity of rotation, r is the radial coordinate, g is the acceleration of gravity, and (ρ_n/ρ) the fraction of normal component liquid helium present at the temperature of the experiment. He then showed experimentally that Eq. (1) did not describe the facts and that instead the shape of the free surface is given by the expression for an ordinary liquid,

$$z=\Omega^2r^2/2g, \tag{2}$$

thus providing a striking demonstration of the inadequacy of the simple two-fluid equations.

Andronikashvili and Kaverkin[2] have performed the same experiment (apparently independently) and also verify Eq. (2). Donnelly, Chester, Walmsley, and Lane[3] have extended the studies to include the transient behavior of the liquid upon starting rotation and have concluded that, at the speeds of their experiment, the two fluids always move together.

The failure of these experiments to verify the two-fluid equations has been attributed[4] to the fluid's having exceeded some "critical velocity" below which the equations are supposed to be valid.

If a "critical velocity" does exist in a rotating bucket experiment then one must find just what variable actually becomes critical. The three simplest variables one can use in such an experiment are the angular velocity Ω, the peripheral velocity $v(=\Omega R$, where R is the radius of the container), and the Reynolds number

$$\mathcal{R}\equiv\Omega R^2/\nu, \tag{3}$$

where ν is the kinematic viscosity of the liquid. When the Navier-Stokes equation is applicable (which appears to be the case when the two components of helium II move together) the principle of dynamical similarity requires that the dynamics of the flow be unchanged for changes in variables which preserve the Reynolds number.[5]

TABLE I. Lowest values of v, Ω, and $\mathcal{R}$ obtained in rotating bucket experiments.

Experiment	R(cm)	v(cm/sec)	Ω(rad/sec)	$\mathcal{R}$[a]
Osborne[b]	0.7	35	48	1.2×10^5
Andronikashvili and Kaverkin[c]	1.35	4	3.1	3.7×10^4
Donnelly *et al.*[d]	0.3	28	92	5.2×10^4
This work	0.0766	6.4	84	3.1×10^3
This work	0.0255	8.5	334	1.35×10^3

[a] An average value of $\nu=1.6\times10^{-4}$ cm²/sec (reference 6) has been used for all calculations.
[b] See reference 1.
[c] See reference 2.
[d] See reference 3.

The ranges of these variables covered in the experiments under discussion are summarized in Table I. While fairly low values of Ω and v have been reached, the lowest value of the Reynolds number obtained has been that of Andronikashvili and Kaverkin,[2] $\mathcal{R}=3.7\times10^4$. The object of this paper is to describe an extension of the previous experiments to lower Reynolds numbers. While there is no direct way to compare Reynolds numbers in this geometry to other geometries, the fact that oscillation experiments in helium II have

* Supported in part by a grant from the National Science Foundation.
[1] D. V. Osborne, Proc. Phys. Soc. (London) **A63**, 909 (1950).
[2] E. L. Andronikashvili and I. P. Kaverkin, J. Exptl. Theoret. Phys. (U.S.S.R.) **28**, 126 (1955) [translation: Soviet Phys. JETP **1**, 174 (1955)].
[3] Donnelly, Chester, Walmsley, and Lane, Phys. Rev. **102**, 3 (1956).
[4] J. G. Daunt and R. S. Smith, Revs. Modern Phys. **26**, 182 (1954); see also references 1 and 2.
[5] We are restricting the discussion here to incompressible flow where only forces of friction and inertia are important.

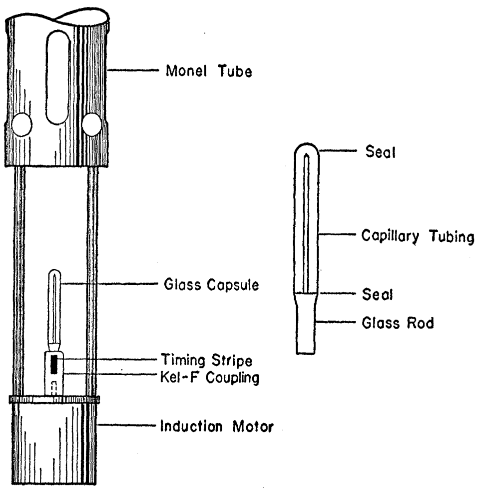

FIG. 1. Rotating capillary apparatus. All the parts shown are submerged in a liquid helium bath. The detail illustrates the construction of the glass capsule.

critical velocities at low Reynolds numbers,[6] led to the present experiments.

2. APPARATUS

The limiting factor in this experiment is the smallest depth of meniscus which can be observed (a few microns). Since the total depth of the meniscus is $\Omega^2R^2/2g=v^2/2g$ [Eq. (2)], one can see that this implies a certain minimum peripheral velocity of the container. Equation (3) may be written $\mathcal{R}=vR/\nu$ so that it is clear that the only parameter at our disposal to reduce $\mathcal{R}$ is the radius R of the container. Since film flow would rapidly alter the level of the free surface in a narrow container, sealed capillary tubes were used to eliminate film flow completely. Sufficient helium gas was enclosed at high pressure to provide a depth of several millimeters of liquid in the capillary upon cooling to liquid helium temperatures.

The method of making the capsules may be understood from the detail in Fig. 1. The body of the capsule is a length of precision-bore Pyrex capillary tubing. Since the bore in such tubing is rarely concentric with the outside wall, the tube was placed on a piece of drill rod, ground concentric, and polished. The seal which joins the capillary tube to its glass rod holder was made by polishing both surfaces optically flat and sealing with a small hydrogen flame. The tube was then placed in a pressure bomb with the open end down. The bomb was pumped out and refilled with helium gas to a pressure of about 2200 lb per sq in. The final seal was made by passing current through a few turns of nichrome wire surrounding the end of the tube. An enclosure of refractory material about the glass tube prevented excessive loss of heat from about the seal by convection of the gas.

[6] R. J. Donnelly and A. C. Hollis Hallett, Ann. Phys. (to be published).

TABLE II. Observed shifts of the center of the meniscus on setting the capillary into rotation.

R(cm)	T°K	Ω(rad/sec)	$\langle\Delta z\rangle$(cm)[a]	$(\rho_n/\rho)\langle\Delta z\rangle$(cm)
0.0766	2.15	84	0.0021[b]	...
	1.40	84	0.0020	0.00015
0.0255	2.14	334	0.0014[b]	...
	1.51	334	0.0014	0.00016

[a] Estimated maximum error ±20%, mainly due to uncertainty in the position of the center, and to convection in the liquid nitrogen surrounding the helium Dewar.
[b] $\langle\Delta z\rangle$ not significantly different just above the λ point.

The completed apparatus is shown in the figure. The capsule is attached to the drive shaft of the motor by means of a Kel-F plastic coupling. Particular care was exercised to ensure smooth rotation of the capillary tube. The surface of the liquid could be seen equally well when rotating or stationary. The entire apparatus shown is operated submerged in the liquid helium bath. Details of the motor have been published elsewhere.[7] The meniscus was observed through a cathetometer and was illuminated by light from a stroboscope. The speed of rotation was measured by synchronizing the stroboscope with a black stripe painted on the plastic coupling. The speed was manually regulated by slight adjustments of the oscillator powering the motor. Because of the cylindrical distortion of the tube and Dewar system only the position of the tip of the meniscus was recorded. The liquid nitrogen shielding the helium Dewar was cooled to stop boiling and convection which made observation of the small meniscus impossible. This was done by drawing some liquid nitrogen through a needle valve and some 20 feet of copper tubing coiled in the nitrogen Dewar.

3. EXPERIMENTAL RESULTS AND DISCUSSION

(i) Equilibrium Measurements

The observed shifts of the center of the meniscus on setting the capillary into rotation are shown in Table II. The average of many observations is denoted as $\langle\Delta z\rangle$ and is seen to be independent of temperature. This supports Eq. (2) rather than Eq. (1). The value which would have supported Eq. (1) is shown as $(\rho_n/\rho)\langle\Delta z\rangle$ and clearly does not fit the observed facts.

The magnitude of $\langle\Delta z\rangle$ however, requires some comment. If the surface were described by Eq. (2) then it would be a parabola of height $\Omega^2R^2/2g$, and a simple calculation shows that the shift of the tip of the meniscus from its plane equilibrium position to its position in steady rotation would be $\Omega^2R^2/4g$. For the smaller tube (of radius 0.0255 cm) $\Omega^2R^2/4g=0.0186$ cm to be compared with an observed shift of 0.0014 cm. For the larger tube (of radius 0.0766 cm) we have a calculated shift of 0.0106 cm to be compared with an observed shift of 0.0020 cm. Because of the lack of temperature-dependence of $\langle\Delta z\rangle$ this discrepancy is

[7] R. J. Donnelly, Rev. Sci. Instr. 28, 351 (1957).

unlikely to have anything to do with the peculiar properties of helium II but rather with surface tension. The surface of the liquid in the tube is probably spherical when the tube is at rest and when in rotation its shape will depart from parabolic since the forces of surface tension act according to the curvatures at every point in the surface. The calculation of the shape of the free surface of a liquid with surface tension in a rotating bucket does not seem to have appeared in the literature so that a quantitative comparison with theory cannot easily be given. However, an estimate of the relative importance of centrifugal and surface tension forces can be obtained from the magnitude of the dimensionless combination[8]

$$\rho\Omega^2 R^3/8T, \tag{4}$$

where T is the surface tension in dynes/cm. This is $\sim$0.1 for the small tube and $\sim$0.2 for the large tube, showing that surface tension forces are very important.[9,10] It may be seen that surface tension places a limit on this technique for studying the rotation of liquid helium at low Reynolds numbers.

(ii) Transient Measurements

In the presence of such relatively strong surface tension forces, the calculation of the characteristic relaxation time for the liquid quoted in reference 3 (which ignored surface tension) will not be applicable. Nevertheless, from dimensional considerations it follows that if the relaxation time is independent of temperature (i.e., shows no variation as strong as ρ_n/ρ) the motion will probably be classical. With the small dimensions of the containers used in this experiment, it was not possible to make quantitative measurements of the development of the meniscus. Therefore, estimates were made of the apparent time it took the meniscus to reach its equilibrium depth. The measurements were complicated by the fact that the motor took about 2 sec to reach its equilibrium speed, as judged by synchronism with the stroboscope. The time for the motor and the time for the meniscus to reach equilibrium are recorded in Table III. It is apparent that there is no strong temperature variation of the transient time and we conclude that at the Reynolds numbers used the motion cannot be distinguished from that of a classical liquid, in particular, helium I.

TABLE III. Estimated time for meniscus to reach its equilibrium depth.

R(cm)	T°K	Ω(rad/sec)	t motor (sec)	t meniscus (sec)[a]
0.0766	2.14	84	2.3	6.2
		168	2.3	5.8
		336	2.3	6.3
	1.43	84	2.0	7.1
		168	1.7	6.1
		336	1.8	6.8
0.0255	2.14	334	2.9	4.2
	1.51	334	2.9	4.8

[a] These times are only estimates and should not be taken as more than an indication of the magnitude.

4. CONCLUSIONS

The results of this experiment together with those quoted in Table I show that the transient and steady-state motion of helium II in a rotating bucket cannot be distinguished from a classical liquid down to peripheral velocities of 4 cm/sec, angular velocities of 3 radians per sec, and Reynolds numbers of 1.4×10^3.

It has been shown that the effects of surface tension limit further reduction in the Reynolds number by the technique of reducing the diameter of the container. It also appears that even in wider containers free surface observations are of limited value unless the effects of surface tension can be minimized by the correct choice of parameters.

ACKNOWLEDGMENTS

I wish to thank Dr. N. L. Balazs for discussions on the effects of surface tension on the free surface of a rotating liquid. I am also indebted to Mr. Paul Dolmer and Mr. Nicholas Froio for preparing the precision glass parts for the experiments.

[8] Lord Rayleigh, *Scientific Papers* (Cambridge University Press, New York, 1920), Vol. 6, p. 258.

[9] It is appropriate to note here that the effects of surface tension are not necessarily negligible in a wide tube since the forces due to surface tension increase as the surface becomes more curved. The low surface tension of liquid helium makes this situation more favorable than with most other liquids. For example, the ratio (4) has the value $\sim$13 in the transient experiments of Donnelly *et al.*,[3] showing that the effects of surface tension are much less pronounced than in the case discussed here.

[10] An approximate calculation by N. L. Balazs (private communication) valid for a very narrow tube accounts satisfactorily for the data obtained with the smaller tube. Details of this calculation will be published at a later date.