The derivation of the sum rule follows familiar lines and is very simple. At high frequencies, such that $\hbar\omega$ is much greater than any of the binding energies of the electrons in the metal, the absorptive or real part of the conductivity vanishes and the imaginary part becomes [Eq. (B-1) of reference 1]

$$\sigma_2(\omega)=-\frac{2\omega}{\pi}\int_0^{+\infty}\frac{\sigma_1(\omega_1)d\omega_1}{\omega_1{}^2-\omega^2}\sim\frac{2}{\pi\omega}\int_0^{+\infty}\sigma_1(\omega_1)d\omega_1. \quad (2)$$

The sum rule now results from requiring that the electrons and ions all behave as free at these high frequencies, and consequently that $\sigma_2(\omega)$ be the same regardless of whether the metal is in its normal or superconducting state. Introducing $s_1(x)$ and designating the change brought about by the superconducting transition by Δ, we have the sum rule[5]

$$\Delta\int_0^{+\infty}s_1(x)dx=0. \quad (3)$$

At a given frequency the value of the real part of the conductivity can, of course, be different in the superconducting and normal states. The integral, however, of conductivity over frequency is the same for both.

The sum rule (3) will now be used to determine $s_2{}^L(x)$ from Glover and Tinkham's values for $s_1(x)$ shown in Fig. 1. At low frequencies the real part of the conductivity is lower in the superconducting than in the normal state while at high frequencies the two are equal. The sum rule, however, requires that the areas under the two curves $s_1(x)$ and $s_N(x)$ be equal. As indicated schematically in the figure, the necessary additional area is contained in the delta function at the origin associated with the London imaginary conductivity $s_2{}^L(x)$. The formal requirement of the sum rule (3) is that

$$\frac{\pi}{2a}=\int_0^{\infty}[1-s_1(x)]dx. \quad (4)$$

The integral has been evaluated by using the three curves shown in Fig. 1.[6] The center of the three is taken from the original paper[1] while the two outer ones roughly determine maximum and minimum areas which would be compatible with the measurements. The errors given are therefore limits and not probable errors. The resulting value of the Pippard parameter is $a=0.21\pm0.05$. This is to be compared with the value independently determined by Glover and Tinkham from the microwave transmission measurements, $a=0.27\pm0.05$. The agreement is modest but must be considered satisfactory in view of the errors. Two other values of the parameter a are available. Faber and Pippard[3] found the value $a=0.15$ from measurements of the surface resistance of wires. Bardeen, Cooper, and Schrieffer[7] deduce from their theory $a=0.18$. Both of these are closer to the film value obtained with the help of the sum rule from the infrared measurements than to the microwave value. Because of the difficulty encountered in averaging over standing waves in the microwave experiments, there is perhaps also some reason for favoring the infrared value from an experimental point of view.

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[1] R. E. Glover, III and M. Tinkham, Phys. Rev. **104**, 844 (1956); **108**, 243 (1957).

[2] A. B. Pippard, Proc. Roy. Soc. (London) **A216**, 547 (1953).

[3] T. E. Faber and A. B. Pippard, Proc. Roy. Soc. (London) **A231**, 336 (1955).

[4] The factor $\frac{1}{2}$ enters because only positive values of ω are considered.

[5] This is essentially the historical Thomas-Reiche-Kuhn sum rule and is well known in many branches of atomic and nuclear physics. It breaks down in cases where relativistic effects are important. For further discussion see J. S. Toll, Ph.D. thesis, Princeton, 1952 (unpublished), and Gell-Mann, Goldberger, and Thirring, Phys. Rev. **95**, 1612 (1954).

[6] A function suggested by Tinkham [M. Tinkham, Phys. Rev. **104**, 845 (1956)], later discarded[1] but now back in good standing, which fits the data reasonably well is $s_1(x)=1-\gamma^2/x^2$ where γ is the reduced frequency corresponding to the intercept of the curve with the abscissa. On an energy gap picture it corresponds to the width of the gap. Upon using this particular form of $s_1(x)$, the sum rule leads to the result $a=\pi/4\gamma$. The fit to the points in Fig. 1 is moderately good for $\gamma=4$, giving $a=0.20$. This illustrates the point that on a gap picture the strength of the London-type conductivity is connected with the width of the gap. Specific heat measurements made on bulk material suggest a gap of about the same size as that found for thin films. This in turn would require that the $s_2{}^L(x)$ terms be of about the same size, a somewhat surprising result.

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Maser Action in Ruby*

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A three-level maser was proposed by Bloembergen,[1] and first operated by Scovil, Feher, and Seidel.[2] In our endeavor to find paramagnetic materials suitable for maser applications, we have investigated the electron-spin resonance properties of ruby (Al_2O_3:Cr). This note reports briefly the results of our studies.

According to several investigators[3–5] the zero-field splitting in ruby is 0.38 cm^{-1}. The ground state of the trivalent chromium ion, Cr^{+++}, which is responsible for the coloring of ruby, behaves as $S=\frac{3}{2}$. The dependence of the energy levels of ruby on the magnetic field was calculated for the polar angle 54° 44′.[6] Experiment has indicated that the "forbidden" transition $-\frac{3}{2}\rightarrow\frac{1}{2}$ is quite intense for this orientation. Calculations showed that for $H=4200$ gauss, the pumping frequency corresponding to this transition should be 24 kMc/sec, and the signal frequency corresponding to the $-\frac{1}{2}\leftrightarrow\frac{1}{2}$ transition should be approximately 9.3 kMc/sec.

A cylindrical cavity was designed and built so as to excite the TE_{114} and TE_{011} modes, respectively, at the

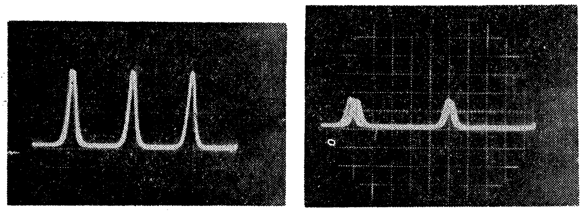
(a) (b)

FIG. 1. X-band (9.22 kMc/sec) pulses emitted by ruby under constant H, fixed pumping frequency, and no external X-band signal. Figure 1(a) shows that the pulse interval is about 0.3 msec corresponding to maximum power available. Figure 1(b) shows the effect of reduced pumping power.

above frequencies. A ruby crystal, with about 0.1% chromium concentration, was placed at the center of the cavity on the end of an axially located quartz rod. The crystal was mounted so as to make the c axis normal to the cavity axis. A selected Varian VA-96 klystron, rated at 120 mw, was used for pumping.

At room temperature, K- and X-band absorption lines characteristic to ruby were observed, and no interaction of any kind between the two bands was detected. The initial evidence of stimulated microwave emission in ruby was obtained at liquid helium temperature (4.2°K), with a sample of about three cubic millimeters in volume. Subsequently, the volume of the sample was increased to approximately two tenths of a cubic centimeter. Evidence of oscillations and amplification was obtained with the latter sample.

Figures 1(a) and 1(b) demonstrate the dependence of emitted X-band power on pumping power in the absence of an external X-band signal. It is interesting to note that both the pulse-height and the repetition rate decrease with decreased pumping power. The pulse interval was found to be approximately 0.3 millisecond for maximum K-band power at our disposal. The radiated frequency was 9.22 kMc/sec for $H=4230$ gauss and pumping frequency of 24.2 kMc/sec.

Figures 2(a) and 2(b) show the effects of amplification. The traces were taken before and after application of K-band power, respectively. The small downward pips in Fig. 2(a) indicate the position of cavity resonance. Net gain up to 20 db has been observed. For this presentation, a small frequency-modulated X-band signal was applied to the cavity.

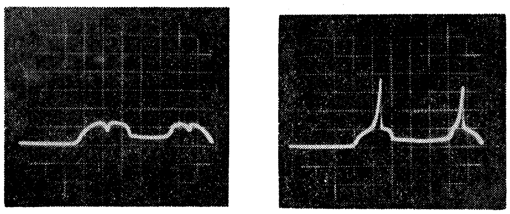
(a) (b)

FIG. 2. The traces (a) and (b) were obtained before and after application of pumping power, respectively, which was maintained below oscillation level. To observe amplification, a small frequency-modulated X-band signal was applied to the cavity.

Details of this study will be published at a later date. In the meantime we should like to point out that ruby possesses a number of physical properties which contribute to its usefulness as a maser medium, such as very high chemical stability, good thermal conductivity, and low dielectric losses.

We wish to thank R. Ager and M. Bair for their capable technical assistance during the progress of this investigation.

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[5] J. E. Geusic, Phys. Rev. **102**, 1252 (1956).

[6] $\theta=\cos^{-1}(1/\sqrt{3})$.

Decay of the π Meson and a Universal Fermi Interaction

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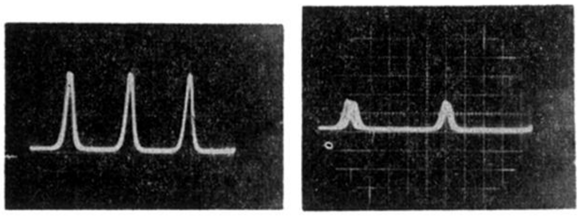
IT is well known that π mesons rarely decay into electrons, with or without the emission of a γ ray. Experimental upper limits for the frequencies of such decay modes as compared to the more usual π-μ decay are[1]

$$\rho=(\pi\rightarrow e+\nu)/(\pi\rightarrow\mu+\nu)<10^{-5}, \tag{1}$$

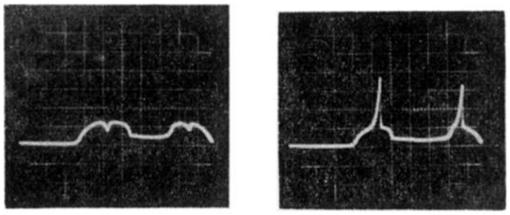
$$\rho_\gamma=(\pi\rightarrow e+\gamma+\nu)/(\pi\rightarrow\mu+\nu)<10^{-5}. \tag{2}$$

Based upon the values of the coupling constants in nuclear β decay as accepted a few years ago, various workers[2] have expressed the belief that the smallness of (1) and (2) cannot be understood in terms of a universal Fermi interaction. The purpose of this note is to point out that this belief is no longer necessary in the light of recent experiments in nuclear β decay. On the contrary, in the framework of a universal Fermi interaction, the conditions (1) and (2) determine a set of universal coupling constants which is not ruled out by existing experiments. Such a set will be exhibited below. The calculations leading to these results are similar to those made by Treiman and Wyld,[2] and some of the results have already been obtained by them. In the following, we employ units in which $\hbar=c=m_\pi=1$, where m_π is the mass of the π meson. We state the results as follows.



(a) (b)

FIG. 1. X-band (9.22 kMc/sec) pulses emitted by ruby under constant H, fixed pumping frequency, and no external X-band signal. Figure 1(a) shows that the pulse interval is about 0.3 msec corresponding to maximum power available. Figure 1(b) shows the effect of reduced pumping power.



(a) (b)

FIG. 2. The traces (a) and (b) were obtained before and after application of pumping power, respectively, which was maintained below oscillation level. To observe amplification, a small frequency-modulated X-band signal was applied to the cavity.