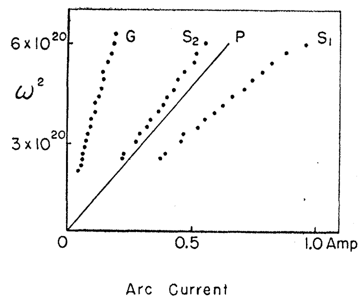

FIG. 3. Estimated plasma frequency *versus* arc current: G, from growing-wave experiment; S_2, minor peak of scattering experiment; S_1, major peak of scattering experiment; P, compilation of probe measurements (from reference 8).

the arc current. Hence we obtain an experimental verification that the maximum rate of growth occurs when the plasma frequency is linearly related to the modulation frequency. While this experiment could be interpreted as a direct measurement of the electron density, it is essential in order to verify the theory that the electron density be measured by another method. This could be accomplished with a Langmuir probe but we chose an alternative microwave method.

The arc column was inserted in a rectangular TE_{10} wave guide so that the axis of the column was perpendicular to both the electric field and the direction of propagation. When the reflection coefficient of the column is plotted *versus* arc current at a fixed frequency, two distinct maxima are found. Double peaks have been observed previously,[6,7] but not as yet explained. The elementary theory predicts only a single resonance at $\omega_p^2/\omega^2=2$ (modified by a straightforward correction for the glass walls of the column, giving $\omega_p^2/\omega^2=2.81$ in this case). It can be shown that with a monotonic density variation radially, reflection should occur at nearly the average density.

Figure 3 is a plot of frequency squared *versus* arc current based on three types of data. The curve labeled P represents data taken from a compilation of probe measurements[8] in mercury arc discharges and gives an average ω_p^2 *versus* I_A if one assumes a uniform plasma distribution throughout the cross section. Curves S_1 and S_2 represent an estimate of ω_p^2 average *versus* I_A based on the wave reflection experiments and obtained by dividing the measured arc current at a given incident frequency, by the reduction factor 2.81. S_1 and S_2 represent the major and minor peaks of the reflection coefficient, respectively. It appears likely that the major scattering peak, S_1, corresponds to the elementary theory. Curve G is from the growing-wave experiments, relates to the density on the axis of the arc, and is a plot of ω_p^2 *versus* I_A for maximum rate of growth. The appreciable discrepancy between the densities indicated by these three methods may possibly be accounted for by radial density variation in the plasma. Work is continuing using the high spatial and frequency resolution of this beam interaction method to attempt to resolve the problem.

Variation of net gain with length and beam current, width of the peaked gain characteristic, and the effect of collisions and thermal velocity distribution on gain, from both theoretical and measured points of view, are being studied and will be reported on in a more detailed article.

It is felt that this amplification experiment verifies previous theoretical efforts of many workers in the field by producing a direct correlation between modulation and plasma frequency as well as demonstrating the laboratory existence of growing plasma waves such as have been postulated as one of many possible sources of solar and other radio astronomical noise. It is suggested that this mechanism may prove useful as a means of measuring radial variations in charge density in low-pressure gaseous discharges as well as other properties, such as characteristics of moving striations.

* Work supported by the Office of Naval Research.
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Electron Resonances with Ultrasonic Waves in Copper*

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AT liquid helium temperatures the electron mean free path l in very pure metals may be considerably greater than the wavelength of readily attainable ultrasonic waves, in which case electron scattering becomes the dominant mechanism of attenuation of the wave. Consequently such measurements can give information about the electrons and, indeed, recently we have found that the attenuation in superconductors leads directly to the temperature dependence of the superconducting energy gap.[1]

The attenuation in the normal state depends on applied magnetic field. Bömmel first reported a minimum in attenuation with magnetic field in tin.[2] More recently we found a series of maxima and minima in an indium polycrystal[3] and reiterated the rather obvious suggestion made earlier[4] that these represent resonance conditions of orbit size to wavelength as the electrons move through the periodic fields associated with the

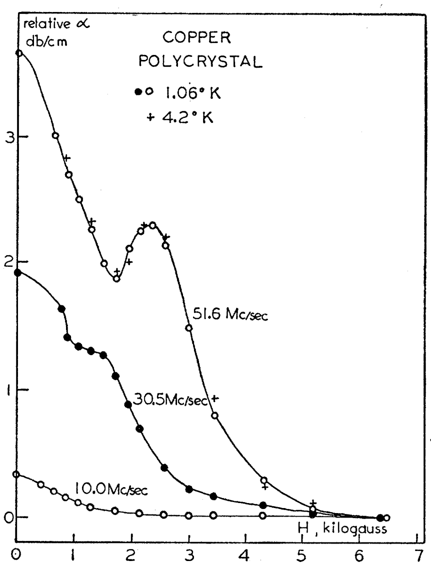

Fig. 1. Longitudinal wave attenuation (relative to that at very high fields) *vs* transverse magnetic field.

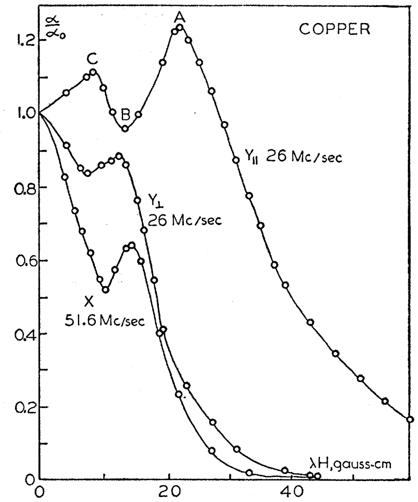

Fig. 2. Normalized attenuation *vs* product of wavelength and magnetic field for longitudinal wave (X) and for shear waves with H perpendicular to polarization ($Y_\perp$) and H parallel to polarization ($Y_{||}$). Attenuation α is that relative to very high fields.

lattice. In this note we report such magnetic field oscillations in copper and show that the simple picture of spatial resonance of the electrons with the sound field seems valid. From it one can calculate very reasonable values for the electronic parameters of copper.

The sample used was a cylinder of polycrystalline copper of a stated purity of 99.999% (American Smelting and Refining Company) and of thickness 1.344 cm. The ultrasonic method was an echo-pulse technique in which one quartz crystal is used for both sending and receiving. Measurements were made at temperatures between 1.06 and 4.2°K and in magnetic fields up to 11 000 gauss.

Figure 1 shows the observed variation of attenuation with transverse magnetic field at three frequencies for a longitudinal wave (velocity: 4.45×10^5 cm/sec). One minimum and one maximum are clearly evident at 51.6 Mc/sec. There is a negligible temperature variation.

Shear waves show a more interesting behavior. These were made for two orientations of the field: the first, which we shall call $Y_\perp$, had the magnetic field perpendicular to both the direction of the wave and plane of polarization; the second, $Y_{||}$, had the magnetic field in the direction of the polarization vector. Normalized results are shown in Fig. 2 for a frequency of 26.0 Mc/sec. The most striking feature is the exchange of maxima and minima as the field is rotated 90°. Also included are the longitudinal wave results scaled according to the wavelength.

The results can be explained partially by a simple classical model in which maxima and minima occur for certain coincidences of the electron orbit diameter ($2r$) and the wavelength. Figure 3 shows the various resonance possibilities for the $Y_\perp$ and $Y_{||}$ cases. (The electric fields associated with the ultrasonic waves are assumed in the direction of polarization.) One can see from Fig. 3 why the maxima shift to minima as the magnetic field is rotated. For $Y_\perp$ the orbits lie in the plane of the wave fields and for the case labeled (A) in Fig. 3, an electron finds the wave fields in the same phase at both extremes of its orbit; for $Y_{||}$, however, case (A) has the wave fields in opposite phase at the orbit extremes. The opposing-phase situation for $Y_\perp$ is the case (B) of Fig. 3. Clearly (A) is the smallest orbit at which there can be a maximum or minimum. Since the highest magnetic field at which a maximum is found experimentally is for $Y_{||}$, the choice of maximum and minimum conditions becomes clear and these identifications are made in Fig. 2. We conclude that the maxima come for both longitudinal waves[5] and the shear case $Y_\perp$ when $2r=n\lambda$ ($n=1, 2, 3, \cdots$); and for $Y_{||}$ when $2r=(n+\frac{1}{2})\lambda$ ($n=0, 1, 2, \cdots$). Therefore, the experiments indicate that attenuation maxima occur when the wave fields have opposite phase at the extremes of the orbit.[6] (The final tailing-off of the attenuation with magnetic field comes about because at high fields the orbit diameter is always less than a half-wavelength, and no resonances are possible.)

From the measurements one can estimate the average electron momentum at the Fermi surface ($\bar{p}_f$) with very few assumptions. It is clear that in going from a maximum to an adjacent minimum the change in r, Δr, is $\frac{1}{4}\lambda$; between adjacent maxima it is $\frac{1}{2}\lambda$. We observe the following fields corresponding to the points of Fig. 2: (A) 1980 gauss; (B) 1160 gauss; (C) 790 gauss. At 26.0 Mc/sec, $\lambda=1.12\times10^{-2}$ cm. Considered on a strictly classical basis, we measure $\bar{p}_\perp$, the average momentum perpendicular to the field H, given by

$$\bar{p}_{\perp}=(e\lambda/4c)H_1H_2/(H_2-H_1),$$

where H_1 and H_2 are the fields for adjacent maxima and minima (for adjacent maxima the right side has a factor of 2). If we assume a spherical shell of electrons of radius $\bar{p}_f$, then $\bar{p}_{\perp}=0.82\bar{p}_f$. Calculating $\bar{p}_f$ for the three combinations AB, AC, and BC shown in Fig. 2, one obtains an average value of $\bar{p}_f=1.45\times10^{-19}$ g-cm/sec. If the number of conduction electrons per unit volume is calculated from the general expression $n=(8\pi/3)(\bar{p}_f/h)^3$, one obtains the value 1.02 for the number of conduction electrons per atom. This agrees remarkably well with what one would expect for copper.

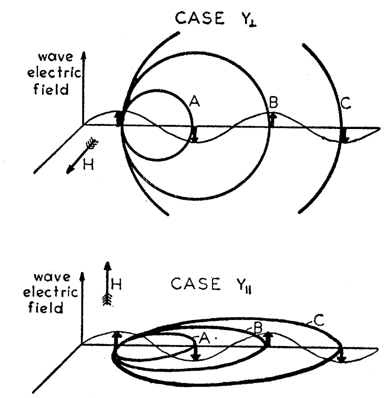

FIG. 3. Various wavelength-orbit resonant possibilities for the two shear-wave cases. A, B, and C are identified with maxima and minima of Fig. 2.

One can also get an estimate of $\bar{p}_f$ from the value of the longitudinal attenuation at zero field. From phonon-electron scattering theory, one expects[7]

$$\alpha=(m^*C)^2\nu/(2\rho_0 v_l^2\hbar^3),$$

where ρ_0 is the metal density, v_l the wave velocity, and C the lattice-electron interaction constant.[8] For long waves, C is $\frac{2}{3}$ of the Fermi energy and so α is proportional to $(\bar{p}_f)^4$. If for α we take the entire change in α between $H=0$ and that extrapolated for $H=\infty$, then we measure $\alpha=0.43$ cm^{-1} at 51.6 Mc/sec. This gives $\bar{p}_f=1.34\times10^{-19}$ g-cm/sec which is in fair agreement with the value calculated above. However, it is not clear that a strong magnetic field reduces the electronic attenuation to zero. In superconductors we have found that a magnetic field can remove only a fraction of the attenuation removed by superconductivity.

One naturally wants to extend such measurements to single crystals and this we are doing in tin which shows a more complicated behavior than copper, as is to be anticipated. Finally it should be observed that ultrasonic resonances of this type are inherently easier to do than cyclotron resonances, since ultrasonic resonances require $2\pi l/\lambda>1$ and not $\omega\tau>1$. Moreover, here there is no problem having the fields penetrate the metal since they are carried by the lattice.

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Dielectric Anomalies and Cyclotron Absorption in the Infrared: Observations on Bismuth

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OBSERVATIONS of intraband magneto-optical effects in bismuth at infrared frequencies and at room temperature have recently been reported.[1] As these authors have pointed out, these effects are related to cyclotron absorption, which has also been observed at microwave frequencies in bismuth.[2–4] We have made similar observations at liquid helium temperatures, and it is the purpose of this note to present these observations as well as an interpretation which shows their relationship to cyclotron absorption.

We now discuss the normal incidence of circularly polarized radiation on the plane surface of a metal with only one isotropic charge carrier in the presence of a magnetic field normal to the surface. The complex conductivity for such a wave if the plane is infinite is[5]

$$\sigma_+=ne^2\tau/\{m/[1+j(\omega-\omega_c)\tau]\}, \quad (1)$$

where n is the number of charge carriers per unit volume, τ the relaxation time, e the charge, m the effective mass of the charge carriers, and $\omega_c=eH/mc$. Equation (1) shows that we have a singularity in σ_+ at the cyclotron resonance frequency $\omega_c=\omega$. The position of the points for which this condition is satisfied are shown by the solid line in Fig. 1. The depolarizing factor is zero for this sample. This line is distorted as shown by the dotted line in Fig. 1 if depolarizing effects enter.[6] In this case the condition becomes

$$\omega_c=(\omega^2-\omega_p{}^2)/\omega, \quad (2)$$