Nucleon-Antinucleon Interaction at Intermediate Energies*

JAMES S. BALL AND GEOFFREY F. CHEW
Radiation Laboratory, University of California, Berkeley, California
(Received October 18, 1957)

The Yukawa interaction between nucleon and antinucleon is investigated in the intermediate energy range (50 to 200 Mev) where it is shown that the details of the short-range forces are unimportant; that is, cross sections are primarily determined by the pion exchange potential for distances greater than 10^{-13} cm. Numerical calculations by the WKB method, using two possible forms of the potential, give nucleon-antinucleon cross sections several times larger than the nucleon-nucleon cross sections at corresponding energies. This large difference is due to systematic cancellation effects in the nucleon-nucleon interaction which are removed in the nucleon-antinucleon system. It is shown that irregularities in the annihilation cross section as a function of energy are to be expected as a result of the sudden onset of individual partial waves.

I. INTRODUCTION

THE "large" value of the nucleon-antinucleon cross section in the energy range between 200 and 700 Mev[1,2] has caused surprise and led to speculation that a new mechanism of interaction is present that cannot be understood in conventional terms. Strangely enough, no serious attempt has been made to explore the consequences of the Yukawa theory with regard to antinucleons in spite of the fact that this theory has been successful in explaining many features of the observed nucleon-nucleon and pion-nucleon interactions. The purpose of this paper is to report a preliminary attempt to evaluate the Yukawa interaction between a nucleon and an antinucleon. Although the method is necessarily restricted to moderate energies ($\sim$100 Mev) because the local-potential concept is employed in conjunction with the WKB approximation, the results give such a large ratio between nucleon-antinucleon ($N\bar{N}$) and nucleon-nucleon (NN) cross sections that we believe the observed data at higher energies will *not* require abandonment of the Yukawa picture.

It has for some time been recognized[3] that the Yukawa interaction between nucleon and antinucleon can be split into two parts. The first, due to exchange of pions, is entirely analogous to the NN interaction, except that when an odd number of pions is exchanged the sign of interaction is reversed. (If the mesonic charge of a nucleon is g, that of an antinucleon is $-g$.) The second mechanism of interaction, due to annihilation, has no counterpart in the NN system and is expected to be ineffective outside relative separations of the order of a nucleon Compton wavelength. At small separations, of course, the annihilation mechanism must be of overwhelming importance.

The method of approach we propose to use is motivated by the semiphenomenological technique that has been reasonably successful in describing the NN system.[4] Here one tries to calculate from first principles the outer parts of the interaction due to one and two pion exchanges. The intermediate and inner regions are then treated phenomenologically, and in particular a repulsive "core" is found to be required for the NN interaction with a radius in the neighborhood of $\frac{1}{3}$ of a pion Compton wavelength. (The pion Compton wavelength, 1.4×10^{-13} cm, will be consistently used as a length unit here, while the unit of energy or momentum will be the pion rest mass, 140 Mev.) For the $N\bar{N}$ system we shall replace the "core" by an ingoing-wave boundary condition to represent the overwhelming probability of annihilation if the two particles come close together. Actually for intermediate energies the location of the annihilation boundary is not at all crucial as will be shown later, but we suppose it to be somewhere in the neighborhood of the core radius in the NN system. Outside this boundary we propose to use the same interaction as in the NN system with, of course, appropriate sign changes.

The usual objection to the above approach is that it seems likely to lead to about the same cross sections as are observed in the NN system. We shall show, however, that at least at intermediate energies the change from a reflecting to an absorbing inner boundary together with the change of sign in the one-pion exchange potential is capable of increasing the cross section by a substantial factor. The underlying reason for this effect is that certain cancellations that make the NN cross sections anomalously small are removed in going to the $N\bar{N}$ system. It should be remembered in this connection that if the "radius" of the nucleon were determined by the pion Compton wavelength, the corresponding geo-

* Work done under the auspices of the U. S. Atomic Energy Commission.

[1] Cork, Lambertson, Piccioni, and Wenzel, Phys. Rev. **107**, 248 (1957).

[2] Chamberlain, Keller, Mermod, Segrè, Steiner, and Ypsilantis, University of California Radiation Laboratory Report UCRL-3876, July 22, 1957 (unpublished). See also review by O. Chamberlain in *Proceedings of the Seventh Annual Rochester Conference on High-Energy Nuclear Physics, 1957* (Interscience Publishers, Inc., New York, 1957).

[3] See, for example, J. Iwadare and S. Hatano, Progr. Theoret. Phys. (Japan) **15**, 185 (1956); also J. Koba and G. Takeda, Brookhaven National Laboratory report, 1956 (unpublished).

[4] See Supplement to the Progr. Theoret. Phys. (Japan) **3**, (1956); also the review by R. Marshak, in *Proceedings of the Seventh Annual Rochester Conference on High-Energy Nuclear Physics, 1957* (Interscience Publishers, Inc., New York, 1957).

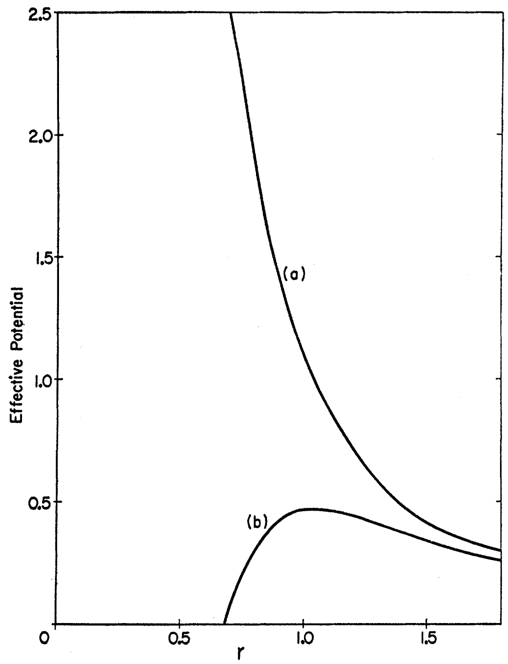

FIG. 1. Typical effective potentials for the $N\bar{N}$ system. The unit of energy is the pion rest energy 140 Mev, while the unit of length is the pion Compton wavelength, 1.4×10^{-13} cm.

metrical area would be 63 mb, and the "black sphere" total cross section 126 mb. Without the NN results for comparison, therefore, there would not be much reason to say that the observed $N\bar{N}$ cross sections ($\sim$100 mb) are unexpectedly large.

Our results fail to be definitive because of uncertainty about the form of the interaction at intermediate distances. It will be shown, however, that two different assumptions about this region both yield $N\bar{N}$ cross sections of the required order of magnitude. So many states contribute that total absorption and scattering cross sections tend to be rather stable. Angular distributions for elastic scattering of course depend sensitively on the details of the force, and these we shall not attempt to discuss at the present time.

II. FORMULATION OF THE PROBLEM: WKB APPROXIMATION

In this investigation, liberal use will be made of the WKB approximation. It has been verified that the errors thereby introduced are not serious for energies in the neighborhood of 100 Mev and the advantages are great, both because computations are simplified and because physical understanding of the essential features of the problem is facilitated.

Consider for example the question of the annihilation region. If one were to integrate the Schrödinger equation exactly, one would have to assign a definite boundary to this poorly defined domain and correspondingly would feel uneasy about the meaning of the result. In WKB terms, however, one quickly sees that the boundary position is not at all crucial. The point is the following: The sum of the outer $N\bar{N}$ potential and the centrifugal barrier for a given angular momentum state characteristically has the shape shown in Fig. 1 by either curve (*a*) or curve (*b*). In the first case, the potential is repulsive or if attractive is too weak to overcome the centrifugal term. Except at high energies the penetration through such a barrier is so small that the annihilation region might as well not be there from the standpoint of a wave incident from the outside. At most, one gets a real phase shift that is determined by the outer part of the potential.

In the second case, where the potential is strongly attractive, the problem from the WKB point of view is that of penetrating or going over the top of a barrier with some reflection from the outside surface. Once part of the wave is over or through one doesn't care how far in it travels before being absorbed. A more precise and complete statement of this principle is that the form of the interaction inside the turning point closest to the origin is unimportant. The WKB approach thus shows that rather little understanding of what happens at small distances is required to perform a plausible calculation of the $N\bar{N}$ interaction at intermediate energies.[5] Ironically one is better off than with the NN system where the radius of the repulsive core is extremely important, as is the behavior of the potential immediately outside the core.

The maximum orbital angular momentum, for which the centrifugal barrier can be overcome at intermediate distances ($r\sim1$) by the Yukawa interaction, seems to be $l=2$. A criterion for the validity of the theory presented here, therefore, is that the important annihilations shall occur in *S*, *P*, or *D* waves but not for higher l values. If one is at such energy that absorption occurs through high angular-momentum barriers which continue to rise right to the annihilation boundary, it is clear that the nature of this boundary and the details of the Yukawa interaction in its neighborhood are important. In practice the restriction to $l\leq2$ limits our discussion to laboratory energies less than about 200 Mev.

Another great advantage of the WKB method is its simplification of the tensor-force problem. As pointed out by Christian and Hart,[6] the finding of eigenstates becomes an algebraic task only, and one is led naturally to "effective potentials" which act in each eigenstate separately. There is no way to calculate mixing parameters, but these are needed only for angular distributions, not for total-scattering and absorption cross sections.

[5] It is clear that to describe the very low-energy region, where a $1/v$ absorption law sets in, a more exact treatment of the problem is required; according to WKB the *S* wave would be completely absorbed, leading to a $1/v^2$ behavior asymptotically. The WKB approximation of course breaks down at very low energies.

[6] R. Christian and E. Hart, Phys. Rev. **77**, 441 (1950).

Without further ado we write down now the generalization of the Christian-Hart effective potential[6] for arbitrary J and parity, including a possible spin-orbit interaction. Suppose that in the triplet spin state for a definite value of total isotopic spin (singlet or triplet) the nuclear interaction energy is of the form[7]

$$V_c+\mathbf{L}\cdot\mathbf{S}V_{LS}+S_{12}V_T. \tag{1}$$

Then the effective potentials in the three eigenstates of total angular momentum J, *including centrifugal repulsion*, are

$$V(l=J\pm1)=V_c-\tfrac{3}{2}V_{LS}-V_T+\frac{J(J+1)+1}{Mr^2}\pm\left[\left(\frac{2J+1}{Mr^2}-\frac{2J+1}{2}V_{LS}-\frac{3V_T}{2J+1}\right)^2+\frac{36J(J+1)}{(2J+1)^2}V_T^2\right]^{\frac{1}{2}}, \tag{2}$$

$$V(l=J)=V_c-V_{LS}+2V_T+\frac{J(J+1)}{Mr^2}.$$

With the help of these formulas, we can construct separate potentials for each eigenstate and calculate for each the penetration coefficients as well as the phase shift of the reflected wave. For the WKB penetration formula we use

$$T=(1+e^{2\phi})^{-1}, \tag{3}$$

where ϕ is the phase integral between the two turning points of the barrier. This formula is recommended by Miller and Good, as being more accurate than the slightly more complicated conventional result.[8] Also we follow the standard rule of replacing $l(l+1)$ by $(l+\frac{1}{2})^2$ as recommended by Langer.[9] Actually, as seen below, our results are not sensitive to such fine points as these.

If the penetration factor for a particular J, l, S is denoted by T_{JlS}, then the absorption cross section is

$$\sigma_{\text{abs}}=\frac{\pi}{4k^2}\sum_{J=0}^{\infty}\sum_{S=0}^{1}\sum_{l=J-S}^{J+S}(2J+1)T_{JlS}, \tag{4}$$

where k is the wave number in the barycentric system. To calculate the scattering we need also the phase shifts, which will be designated by δ_{JlS}. The total cross section (scattering plus absorption) is then

$$\sigma_{\text{total}}=\frac{\pi}{2k^2}\sum_{J,l,S}(2J+1)[1-(1-T_{JlS})^{\frac{1}{2}}\cos2\delta_{JlS}]. \tag{5}$$

The formulas (4) and (5) are of course completely general, although one might perhaps use the concept of a complex phase shift rather than a penetration factor if one were not thinking in WKB terms.

The isotopic spin situation is well known.[10] The proton-antiproton and neutron-antineutron systems are each 50–50 mixtures of triplet and singlet while the neutron-antiproton and proton-antineutron systems are pure triplet.[11] The formulation of our problem is thus complete. All we have to do now is specify spin-triplet potentials of the form (1) for the two isotopic spin states and corresponding central spin-singlet potentials. This specification of the interaction, of course, is the essential part of the task.

III. THE NUCLEON-ANTINUCLEON POTENTIAL

The determination of the $N\bar{N}$ interaction due to pion exchange is subject to the same considerations as that for the NN system.[4] In particular, at large distances the one-pion contribution (note the negative sign)

$$V_{1\pi}=-f^2\tau_1\tau_2\boldsymbol{\sigma}_1\cdot\nabla\boldsymbol{\sigma}_2\cdot\nabla(e^{-r}/r)$$
$$=-f^2\frac{\tau_1\cdot\tau_2}{3}\left[\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+S_{12}\left(\frac{3}{r^2}+\frac{3}{r}+1\right)\right]\frac{e^{-r}}{r}, \tag{6}$$

with $f^2\approx0.08$, is guaranteed by general principles to be asymptotically correct. However, in the neighborhood of $r=1$ two-pion exchange is already important in the central force, and corrections due to nucleon recoil may be of the same order of magnitude. It is fortunate that at least the tensor force continues to be dominated by single-pion exchange down to quite short distances because the proper method of calculating two-pion contributions and recoil effects is not yet clear.[4]

At present, then, the interaction to use for the intermediate region $0.7<r<1.5$, which unfortunately is important for determining total cross sections, is somewhat uncertain; nonetheless we believe it is already possible to understand why $N\bar{N}$ cross sections should systematically be larger than those for the NN system. As further progress is made in the theory of the NN force, our grasp of the $N\bar{N}$ situation will becomes correspondingly firmer.

The first potential to be considered here is that of Gartenhaus,[12] with the spin-orbit term added by Signell and Marshak.[13] (We call this the GSM potential.) The

[7] Our notation is the same as that used by J. L. Gammel and R. M. Thaler, Phys. Rev. **107**, 291 (1957).

[8] S. C. Miller, Jr., and R. H. Good, Jr., Phys. Rev. **91**, 179 (1953).

[9] R. Langer, Phys. Rev. **51**, 669 (1937).

[10] T. D. Lee and C. N. Yang, Nuovo cimento **3**, 749 (1956).

[11] The formula for the charge-exchange scattering cross section is

$$\sigma_{\text{ex}}=\frac{\pi}{4k^2}\sum_{J,l,S}(2J+1)\times\left|\frac{(1-T^3{}_{JlS})^{\frac{1}{2}}\exp(2i\delta^3{}_{JlS})-(1-T^1{}_{JlS})^{\frac{1}{2}}\exp(2i\delta^1{}_{JlS})}{2}\right|^2,$$

where the superscripts 3 and 1 designate isotopic triplet and singlet states, respectively.

[12] S. Gartenhaus, Phys. Rev. **100**, 900 (1955).

[13] P. Signell and A. Marshak, Phys. Rev. **106**, 832 (1957); also see review by R. Marshak in *Proceedings of the Seventh Annual Rochester Conference on High-Energy Nuclear Physics, 1957* (Interscience Publishers, Inc., New York, 1957).

TABLE I. Transmission coefficients and phase shifts for $N\bar{N}$ scattering at 140 Mev according to the GSM potential. The NN phase shifts at 150 Mev as calculated by Signell and Marshak are shown for comparison.

State	Transmission coefficient	$N\bar{N}$ phase shift at 140 Mev (lab) (deg)	NN phase shift at 150 Mev (lab) (deg)
$^3S_1{}^1$ [a]	1	...	+23
$^3P_0{}^1$	1	...	...
$^3P_1{}^1$	0	−41	...
$^3P_2{}^1$	1	...	...
$^3D_1{}^1$	0	0	−23
$^3D_2{}^1$	0	−17	+26
$^3D_3{}^1$	1(0.83)	...	+10
$^1S_0{}^1$	0	−36	...
$^1P_1{}^1$	0	+6	−22
$^1D_2{}^1$	0	+6	...
$^3S_1{}^3$	1	...	...
$^3P_0{}^3$	0	−33	+16
$^3P_1{}^3$	1	...	−17
$^3P_2{}^3$	1	...	+12
$^3D_1{}^3$	0	−13	...
$^3D_2{}^3$	0	+6	...
$^3D_3{}^3$	0	+2	...
$^1S_0{}^3$	1	...	+19
$^1P_1{}^3$	1	...	...
$^1D_2{}^3$	0	0	+6

[a] The right-hand superscript designates the isotopic spin.

reasons for this choice are: (1) The GSM potential has the correct asymptotic form (6); (2) at intermediate distances it is not in conflict with meson theoretical ideas, although it cannot really be said to be "derived" therefrom; and (3) it gives quantitative agreement with NN experiments up to 150-Mev lab energy.[13]

We propose simply to reverse the sign of the single-pion exchange part of the GSM potential and to leave the remainder untouched. To the extent that the remainder is due to two-pion exchange, this recipe is theoretically sound. Of course the spin-orbit term is phenomenological and of unknown origin, so our handling of this particular part is open to question.

In Table I the WKB penetration coefficients and phase shifts for the various partial waves are tabulated for the GSM $N\bar{N}$ potential at a relative kinetic energy of 0.5 (lab energy of 140 Mev). Note that the penetration coefficients are listed always as either 0 or 1. This result is of course not exact, but is a good approximation because the centrifugal barriers, even when rendered finite in height by a strong nuclear attraction, are smooth and thick as shown by the examples of Fig. 2. Only when the energy is close to the top of the barrier, either just above or just below, does the penetration coefficient differ appreciably from the classical values of 0 or 1. In the $^3D_3{}^1$ state (see Fig. 2, curve b), we are only slightly above the barrier at the energy in question and integration of the Schrödinger equation in this case yields a penetration coefficient of 0.83 (shown in parenthesis in Table I), a result which could no doubt also be obtained by the WKB method if one continued the penetration formula across the top of the barrier. [The WKB penetration just at the top of the barrier, according to Eq. (3) is 0.5. Miller and Good have shown that continuation of this formula to energies above the barrier gives a more accurate result than a sudden jump to complete penetration, the normal consequence of WKB.[8]]

TABLE II. Plane wave absorption, scattering (including charge exchange), and total cross sections for proton-antiproton and for neutron-antiproton systems at 140 Mev, calculated from Table I. Orbital angular momenta greater than 2 are neglected.

System	σ_{abs}(mb)	σ_{sc}(mb)	σ_{total}(mb)
$p\bar{p}$	72(69)	96(85)[a]	168(154)
$n\bar{p}$	69	79	148

[a] The charge-exchange scattering contribution, $p+\bar{p}\rightarrow n+\bar{n}$, is 22 mb.

Small deviations from complete penetration, while they have little effect on the absorption cross section,

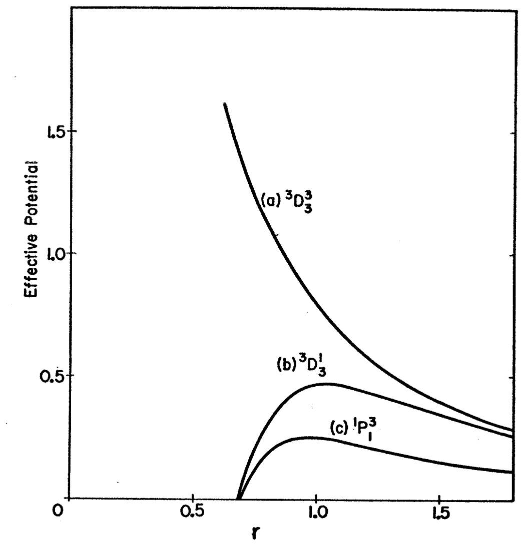

FIG. 2. Examples of effective potentials according to Eq. (2). In each case the nuclear potential is attractive in the intermediate region, but in (a) the centrifugal barrier is not overcome and in (b) the barrier is barely surpassed by a kinetic energy of 0.5 (140 Mev). The units are the same as in Fig. 1.

may substantially reduce the scattering and correspondingly the total cross section, as shown by formulas (4) and (5). For example, the 17% reduction of the $^3D_3{}^1$ penetration coefficient mentioned above produces a 64% reduction of the corresponding partial-wave scattering cross section. Because our potential certainly cannot be accurate to a few percent, we must anticipate a substantial uncertainty in scattering cross sections if many states lie in the marginal range. Also, if the annihilation region does not absorb perfectly, scattering cross sections will be systematically reduced. Absorption cross sections, on the other hand, are relatively stable. Table II shows the absorption, scattering, and total cross sections calculated on the basis of Table I. Only the $^3D_3{}^1$ state is marginal here but the figures in parentheses illustrate the effect described.

Now let us compare our results to those for the NN

system with the corresponding potential, as calculated at 150-Mev lab energy by Signell and Marshak.[13] Table I lists the NN-scattering phase shifts and Table III compares the "total" cross sections in the various partial waves for the NN and $N\bar{N}$ systems.[14] It is seen that the NN partial-wave cross sections are always much smaller than the value corresponding to complete absorption (i.e., "twice-geometrical") while a substantial number of $N\bar{N}$ partial waves are completely absorbed. The plane-wave total cross sections for the $N\bar{N}$ system calculated from formula (5) are correspondingly much larger than for the NN system, as shown in Table IV.

What is the underlying reason for the small NN cross sections? A strong potential of well-defined range is expected to produce phase shifts that average about 45° for l values smaller than kR, if k is the wave number

TABLE III. Comparison of the partial-wave $N\bar{N}$ and NN "total" cross sections calculated from Table I and expressed in units of the cross section corresponding to complete absorption of the partial wave ("twice-geometrical").

State	$\sigma_{N\bar{N}}$	σ_{NN}
$^3S_1{}^1$	1	0.30
$^3P_0{}^1$	1	...
$^3P_1{}^1$	0.86	...
$^3P_2{}^1$	1	...
$^3D_1{}^1$	0	0.31
$^3D_2{}^1$	0.17	0.38
$^3D_3{}^1$	1(0.59)	0.06
$^1S_0{}^1$	0.69	...
$^1P_1{}^1$	0.02	0.28
$^1D_2{}^1$	0.02	...
$^3S_1{}^3$	1	...
$^3P_0{}^3$	0.59	0.15
$^3P_1{}^3$	1	0.17
$^3P_2{}^3$	1	0.08
$^3D_1{}^3$	0.10	...
$^3D_2{}^3$	0.02	...
$^3D_3{}^3$	0	...
$^1S_0{}^3$	1	0.22
$^1P_1{}^3$	1	...
$^1D_2{}^3$	0	0.02

and R the radius of the interaction.[15] That our potential is strong is demonstrated by its ability to overcome the centrifugal barrier in the $^3D_3{}^1$ state of the $N\bar{N}$ system; why then does it give phase shifts consistently much smaller than 45° for the NN system? The answer to this question lies in the detailed structure of the NN interaction; to know its average strength and range is not enough.

The origin of some of the small NN phase shifts lies in a kind of "Ramsauer effect," that is, in a cancellation of the effect of the repulsive core against an attractive outside region. The S-phase shifts actually change sign near 200 Mev as a result of this cancellation, according to most analyses.[16] The $N\bar{N}$ system, with the repulsive core replaced by a black hole, does not suffer from the same disadvantage and makes full use of the outside potential. A further unusual effect occurs in triplet odd states where the NN scattering has long been recognized as anomalously weak.[17] Here the underlying cancellation appears to be between the repulsive long-range one-pion exchange potential and the shorter-range attractive two-pion interaction. The reversal of sign of the one-pion part in the $N\bar{N}$ system removes this cancellation, giving a strong over-all average attraction that accounts for a large part of the total plane-wave cross section. There may be other systematic effects of this kind that are more difficult to pinpoint, but in any event the final effect is clear: The GSM potential gives cross sections much larger for the $N\bar{N}$ system than for the NN, at least in the neighborhood of 150 Mev. Let us consider now whether other reasonable potentials lead to the same result.

TABLE IV. Comparison of plane-wave total cross sections for nucleon-nucleon and nucleon-antinucleon scattering, calculated from Table III for a lab energy of 140 Mev. Orbital angular momenta higher than 2 are neglected.

	Type of scattering			
	$p\bar{p}$	$n\bar{p}$	pp	np
σ_{total}(mb)	168(154)	148	29	60

A completely phenomenological potential which fits the known facts about the NN system has been produced by Gammel and Thaler,[16] but although this interaction has many features in common with the GSM potential, it does not have the detailed asymptotic form [Eq. (6)] and thus is not easily converted to the $N\bar{N}$ problem. Recently a new field-theoretical calculation of the two-pion exchange potential, including multiple-pion scattering, has been carried out by Konuma, Miyazawa, and Otsuki[18] as well as by Younger, Pearlstein, and Klein.[19] This potential (which we call the KMO potential) can of course be readily adapted to our problem but it has not been tested against NN experiments at the energies considered here. Nevertheless, to illustrate the reliability of our conclusion that the Yukawa mechanism is capable of producing a strong $N\bar{N}$ interaction, we present in Table V the absorption cross sections calculated for the KMO potential. The single-pion exchange part of the KMO interaction is of course the same as that of GSM, but KMO contains no spin-orbit term and has a substantially different central force due to the multiple

[14] Because of the Pauli principle, only half as many states occur in the NN system as in the $N\bar{N}$. Nevertheless the measured plane-wave total cross sections would be equal if the average of those partial-wave cross sections that do occur were the same for both systems.

[15] An average phase shift of 45° for $l \leq kR$ gives the "twice geometrical" cross section $2\pi(R+\lambda\!\!\!^{-})^2$.

[16] J. M. Gammel and R. M. Thaler, Phys. Rev. **107**, 291 (1957); and Phys. Rev. **108**, 163 (1957).

[17] For example the Serber potential, with no odd-state force at all, has often been used for qualitative considerations.

[18] Konuma, Miyazawa, and Otsuki, Progr. Theoret. Phys. (Japan) (to be published).

[19] Younger, Pearlstein, and Klein, University of Pennsylvania (private communication).

TABLE V. Comparison of $N\bar{N}$ absorption cross sections for the GSM, KMO, and single-pion exchange potentials at 140 Mev.

Potential	$p\bar{p}$ cross section (mb)	$n\bar{p}$ cross section (mb)
GSM	69	69
KMO	65	69
Single-pion exchange	44	32

scattering corrections ignored by Gartenhaus. In spite of these differences the $N\bar{N}$ plane-wave absorption cross sections are seen to be not much altered. It was not felt worthwhile to calculate the KMO elastic scattering in detail because it cannot be drastically different from that for GSM so long as we maintain our assumption that complete absorption results once the centrifugal barrier has been overcome.

This last aspect or our model is probably the weakest feature. Perfect absorption by the annihilation region is unreasonable, and we have seen that small deviations from complete absorption can cause a drastic reduction of the scattering cross section. Nevertheless, there seems no way at present even to estimate these deviations, because they depend on the detailed nature of the interaction at short distances. We must resign ourselves to some uncertainty in the scattering part of the cross section and pay relatively more attention to the absorptive part in comparing theory with experiment.

As a final example we have calculated the $N\bar{N}$ absorption cross sections resulting from the single-pion exchange interaction [Eq. (6)] acting *alone*. As mentioned earlier, this interaction supplies most of the tensor force but only a small part of the central force; nevertheless, one sees in Table V that it produces a quite respectable amount of annihilation.

IV. ENERGY DEPENDENCE OF THE $N\bar{N}$ CROSS SECTIONS, DISCUSSION, AND COMPARISON WITH EXPERIMENT

We have shown that the Yukawa interaction seems likely to lead to $N\bar{N}$ absorption cross sections in the neighborhood of 70 mb at a laboratory energy of 140 Mev, with scattering cross sections of the same order of magnitude. What can theory say about the energy dependence of these cross sections. To apply our potential model at much higher energies is unreasonable, but we have ventured to calculate the annihilation cross sections in the range from 50 to 200 Mev laboratory energy for the GSM and KMO potentials, with the results shown in Fig. 3. Penetration factors have always been taken as either zero or unity, depending on whether or not the top of the potential barrier is reached, so the sharp breaks in the plotted curves are not to be interpreted literally. Between breaks the dependence is proportional to the reciprocal of the energy as a result of the $1/k^2$ factor in formula (4). It seems likely that the experimentally observed energy dependence will show irregularities due to the onset of individual partial waves even though there will not be sharp discontinuities.

It has been emphasized that because of incomplete absorption the elastic scattering may not actually be as large as indicated in Table II. However, it cannot be completely negligible because the force is sometimes repulsive and consequently there must be some states with zero barrier penetration and large negative phase shifts. In this connection one may reflect on the classical significance of our model. We have a small black hole surrounded sometimes by an attractive well and sometimes by a wall. If particles are trapped by the well they eventually roll into the hole at the center and are absorbed, but if they hit the wall they are merely scattered. It is surprising, in view of this picture, that according to Table I the GSM potential produces only 24 mb of "classical" elastic scattering for the $p\bar{p}$ system at 140 Mev and only 10 mb for the $n\bar{n}$ system. The bulk of the elastic scattering is, after all, due to diffraction and therefore subject to substantial reduction if individual partial waves are incompletely absorbed.

As the energy rises, one may expect this latter effect to become more important. At 140 Mev it is hard to see how the Yukawa interaction can give a really small

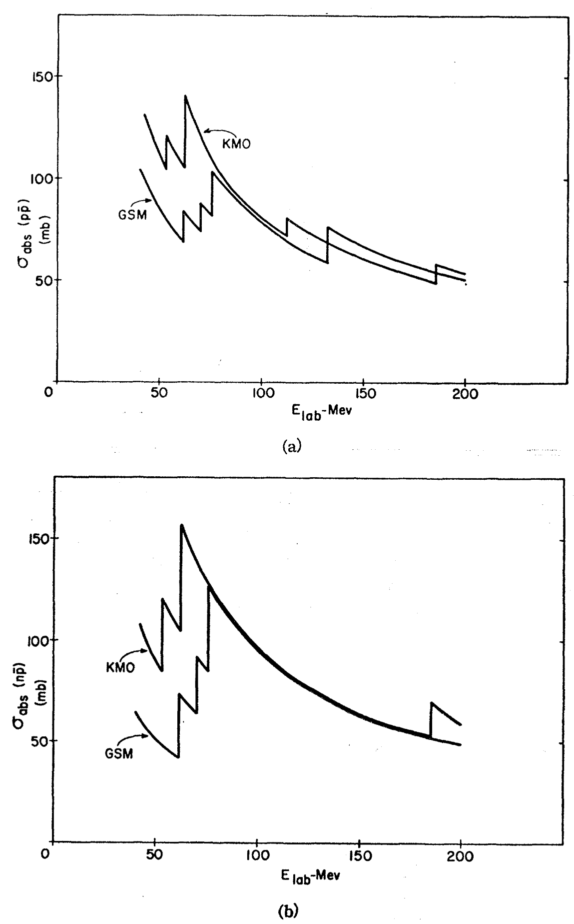

FIG. 3. $N\bar{N}$ absorption cross sections calculated as a function of energy from the GSM and KMO potentials. S, P, and D waves only are included. (a) Proton-antiproton; (b) neutron-antiproton.

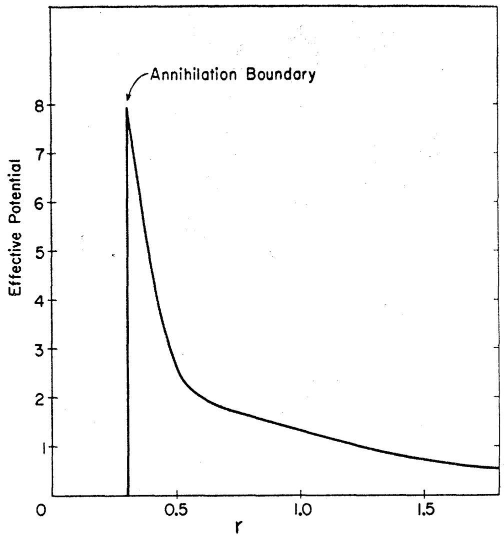

FIG. 4. Typical effective potential for high angular momentum, illustrating how barrier becomes narrow for high kinetic energy.

scattering-to-absorption ratio, but as noted above eventually one reaches energies where the important partial waves have $l>2$ so that there is no maximum in the effective potential (including centrifugal repulsion). Then it will be the annihilation boundary that terminates the barrier, and one will be dealing with narrow "peaks" (see Fig. 4), in contrast to the situation at intermediate energies where the important barriers are smooth and thick. As a result partial penetration can be expected to be commonplace at high energies, and diffraction scattering correspondingly smaller. We see no way at present to attack the high-energy problem quantitatively because not only does the whole potential approach break down at small distances, but the detailed position and nature of the annihilation boundary becomes important.

In the limited range where the present theory makes contact with experiment the agreement is satisfactory. At 190 Mev, Cork, Lambertson, Piccioni, and Wenzel have found a total $p\bar{p}$ cross section of 136±16 mb,[1] whereas theory predicts for this energy an absorption cross section of ~55 mb [Fig. 3(a)] with a scattering cross section of the same order of magnitude or perhaps a little larger. In the energy range between 40 and 200 Mev, emulsion experiments give an average elastic $p\bar{p}$ scattering cross section of (75_{-32}^{+50}) mb.[20] Unfortunately at the only energy (450 Mev) where both scattering and absorption measurements are currently available,[2] partial waves higher than $l=2$ play a large role, and the approach of this paper is not valid.

Since the theory of the short-range parts of the Yukawa interaction promises to be extremely difficult, whereas that for intermediate distances may be under control in the foreseeable future, experimental emphasis on energies below 150 Mev seems desirable. It will be particularly interesting to see if "bumps" that can be identified with individual partial waves are observed in the cross section *vs* energy curve. As our understanding of the Yukawa interaction in the intermediate region becomes more refined, it might be possible to use the position and magnitude of such irregularities to check the details of the theory.

ACKNOWLEDGMENTS

The authors are indebted to Dr. H. P. Noyes for pointing out the simplicity of the tensor force problem in the WKB approximation and to Professor E. Segrè for a careful reading of the manuscript. We also wish to thank Dr. Gartenhaus and Dr. Miyazawa for furnishing us with numerical tables of their potentials.

[20] Goldhaber, Kalogeropoulos, and Silberberg, in Physics Division Quarterly Report, University of California Radiation Laboratory Report UCRL-3914, August 20, 1957 (unpublished).