Electromagnetic Interaction of the Neutral K Meson*

G. FEINBERG
Brookhaven National Laboratory, Upton, New York
(Received October 17, 1957)

The electromagnetic interaction of a spinless neutral particle, which is different from its antiparticle, is discussed. It is shown that while such particles cannot emit a free photon, or be scattered by an electromagnetic field, their extended charge distribution will undergo a contact interaction with charged particles. The case of the neutral K meson (K^0) is considered as an example. The charge distribution arises from virtual dissociation into charged baryons. One contribution to the interaction is calculated. The interaction leads to a scattering of K^0's by charged particles. The form of this scattering is quite similar to that of the nuclear scattering of the K mesons, while its magnitude is much less, which implies that it will be difficult to detect. Possible other means of detecting the interaction are discussed.

I. INTRODUCTION

According to the present view, first suggested by Gell-Mann and Pais,[1] the neutral K meson (K^0) differs in some properties (e.g., strangeness) from its antiparticle ($\bar{K}^0$), and thus must be represented by a complex field operator ϕ_{K^0}. This suggestion has been verified by the experiments of Lande *et al.*,[2] which show the existence of two lifetimes in the decay of the neutral K meson, and thus the existence of two particles.

It is well known that if a particle is represented by a complex field operator, it is possible to construct a "current" vector (J_μ) for the particle which has the following properties: (1) J_μ is quadratic in the field operator; (2) J_μ is odd under the operation of charge conjugation; (3) J_μ is conserved for the free particle. It is therefore plausible that the neutral K meson will interact with an electromagnetic field through this current. This interaction, which will exist even for a spinless K^0, is not the same as a magnetic moment interaction, whose existence has been suggested if the K meson spin is 1 or more.[3] It is rather analogous to that part of the electromagnetic interaction of a neutron which is due to its charge distribution. In classical terms, the K^0 can be looked on as an extended charge distribution, which varies from positive to negative at different points in space, but whose total charge vanishes. The charge density of the K^0 will arise from its virtual dissociation into charged particles such as baryon pairs, just as the neutron charge density arises from dissociation into π mesons and protons, etc.

In Sec. II of this paper, we shall discuss some of the general properties of the K^0 electromagnetic interaction, and in Sec. III we shall calculate, in weak-coupling approximation, the contribution to this interaction coming from the dissociation of a K^0 into a nucleon and an antihyperon. In Sec. IV we consider some possible experimental consequences of the interaction.

II. ELECTROMAGNETIC INTERACTION OF SPINLESS PARTICLES

A general expression for the interaction of a spin $\frac{1}{2}$ field with an electromagnetic field under the assumptions that the interaction is: (1) linear in the electromagnetic field, (2) quadratic in the spinor field, and (3) Lorentz and gauge invariant, has been given by Foldy.[4] A similar expression may be obtained for a spinless field. Let $\phi(x)$ be the (complex) field operator and

$$J_\mu = i(\phi^\dagger\partial_\mu\phi - \phi\partial_\mu\phi^\dagger) \tag{1}$$

be the current operator.[5] The most general interaction which is linear in A_μ, quadratic in ϕ, and is Lorentz invariant and gauge invariant, is then

$$H(x) = \sum_{n=0}^{\infty} \epsilon_n J_\mu(x)\Box^{2n}A_\mu(x), \tag{2}$$

where $\Box^2 = \partial_\mu\partial_\mu = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2 - \partial^2/\partial t^2$, and the ϵ_n are a set of numbers which characterize the interaction. This form holds if the particle is free before and after emitting the photon. This condition is satisfied in the scattering problem considered. If the incoming or outgoing spinless particle is virtual, a more complicated expression will represent the interaction.

The first term in this expansion, $\epsilon_0 J_\mu A_\mu$, is just the interaction of the total charge of the spinless particle, which is ϵ_0, with the electromagnetic field. A neutral particle is defined by the requirement that this term vanish, or that $\epsilon_0 = 0$. The law of conservation of charge guarantees that ϵ_0 is a number characteristic of the particle in that its value cannot be changed by interactions. We are therefore assured that this term will be absent for the K^0, as we shall see by direct calculation in Sec. III.

The radiation field of a free photon satisfies

$$\Box^2 A_\mu^{\mathrm{rad}} = 0. \tag{3}$$

* Work performed under the auspices of the U. S. Atomic Energy Commission.

[1] M. Gell-Mann and A. Pais, Phys. Rev. **97**, 1387 (1955).
[2] Lande, Booth, Impeduglia, Lederman, and Chinowsky, Phys. Rev. **103**, 1901 (1956).
[3] M. L. Good, Phys. Rev. **105**, 1120 (1957).
[4] L. L. Foldy, Phys. Rev. **87**, 688 (1952).
[5] We take $\hbar = c = 1$. $\partial_\mu = (\partial/\partial x, \partial/i\partial t)$. A_μ is the electromagnetic four-potential. The dagger indicates Hermitean conjugate.

Therefore, all of the higher order terms in the expansion (2) vanish for free photons. As a result of this a neutral K meson cannot emit a real photon, or be scattered by an electromagnetic field. The interaction (2) will manifest itself only as a contact interaction of the K^0 with charged particles. This can be seen by using the field equation for the electromagnetic field in the presence of sources,

$$\Box^2 A_\mu = J_\mu^{\text{ext}}, \tag{4}$$

where J_μ^{ext} is the current vector of the source of the field A_μ. Substitution of this expression into (2) gives

$$H(x)=\sum_{n=0}^{\infty} \epsilon_{n+1} J_\mu(x) \Box^{2n} J_\mu^{\text{ext}}(x). \tag{5}$$

The form of this interaction is quite similar to the correction to the interaction between charged particles coming from vacuum polarization.[6] It can be seen directly from Eq. (5) that if a K^0 passes through a region of space where there is an electromagnetic field, but no matter, it will be unaffected.

The current J_μ of the K^0 can be rewritten in terms of the fields ϕ_1, ϕ_2 which create the particles θ_1 and θ_2 which are eigenstates of charge conjunction. If we write

$$\phi=(\phi_1+i\phi_2)/\sqrt{2}, \quad \phi^\dagger=(\phi_1-i\phi_2)/\sqrt{2}, \tag{6}$$

then

$$J_\mu=\phi_2\partial_\mu\phi_1-\phi_1\partial_\mu\phi_2. \tag{7}$$

The fact that J_μ contains only cross terms between ϕ_1 and ϕ_2 follows from the requirement that it be odd under charge conjunction, since ϕ_1 and ϕ_2 go into themselves under charge conjugation, but with a relative minus sign.

If charge conjugation or time reversal is conserved in the K^0 decay the particles θ_1, θ_2 will have well-defined lifetimes.[1,7] In this case the interaction (5) will lead to transitions between the long- and short-lived K^0's in the presence of matter, and, for example, a regeneration of the 2π decay mode at large distances from the place of production of the K^0. It follows from our previous comments that for spinless K^0, no such regeneration will occur in an electromagnetic field.

III. K^0 CHARGE DENSITY

In this section we discuss the contribution to the K^0 charge density which comes from the virtual dissociation of the K^0 into a proton-antiΣ pair. We assume the K^0 has spin zero and even parity, while the hyperon has spin $\frac{1}{2}$ and is defined to have the parity of the proton. This calculation is only of illustrative interest, for the following reason. The charge distribution coming from the process discussed here will be distributed over a region with dimensions of the order of a nucleon Compton wavelength. However, there will be higher order radiative corrections to the charge density involving pions. These will give rise to a charge distribution which is spread over a larger region. Such terms are likely to give the major contribution to (5) for low-energy K^0's. The term we shall calculate will therefore be useful only to verify the general properties discussed in Sec. II, and to give an idea of the order of magnitude of the effect.

For simplicity, we take the $p\Sigma K^0$ interaction to be a direct coupling,

$$g\bar{\psi}_p\psi_\Sigma\phi_{K^0}+h.c.$$

The two Feynman diagrams for the K^0—electromagnetic interaction through a proton-antiΣ pair are given in Fig. 1. We shall write the contribution of the diagrams to the S matrix in coordinate space, as the properties that we want to exhibit are more evident in that form.

$$\begin{aligned} S=-eg^2\int\int\int d^4x d^4y d^4z\, P[&\bar{\psi}_p(x)\psi_\Sigma(x)\phi_{K^0}(x) \\ &\times\{\bar{\psi}_p(y)\gamma_\mu\psi_p(y)A_\mu(y)+\bar{\psi}_\Sigma(y)\gamma_\mu\psi_\Sigma(y)A_\mu(y)\} \\ &\times\bar{\psi}_\Sigma(z)\psi_p(z)\phi_{K^0}^\dagger(z)]. \end{aligned} \tag{8}$$

Here $P[\;]$ is the time-ordered product and the operators without daggers annihilate the particles with whose symbol they are labeled.

The expression (8) is evaluated by standard methods. We shall call its matrix element between a state containing one K^0 and a state containing a K^0 and a photon, $\langle S\rangle$. Then

$$\langle S\rangle=\frac{-ie}{(2\pi)^4}\int\int d^4y d^4z\, \mathcal{S}[J_\mu(z)] \times A_\mu(y)\int d^4q\, e^{iq(y-z)} f(q^2). \tag{9}$$

Here $\mathcal{S}[\;]$ is the normally ordered product. The function $f(q^2)$ is given by

$$f(q^2)=g_1(q^2)-g_2(q^2),$$

$$\begin{aligned} g_1(q^2)=\frac{g^2q^2}{4\pi^2}\Bigg[\int_0^1 dz\int_{-z}^{z} d\omega &\times\left\{\frac{\omega^2 z}{16}+\frac{z^2(z-2)}{8}+\frac{[\frac{1}{32}m^2z-\frac{1}{16}mm'(1-z)]}{m^2z+m'^2(1-z)}\right\} \\ &\times\{m^2z+m'^2(1-z)+\tfrac{1}{4}q^2(z^2-\omega^2)\}^{-1}\Bigg], \end{aligned} \tag{10}$$

where m is the nucleon mass and m' the Σ mass, and $g_2(q^2)$ is obtained from g_1 by interchanging m and m'. The K-meson mass has been taken as zero in this expression.

The two terms g_1 and g_2 come from the two Feynman diagrams in Fig. 1, corresponding to the cases where

[6] J. Schwinger, Phys. Rev. **75**, 651 (1949).
[7] H. W. Wyld and S. B. Treiman, Phys. Rev. **106**, 169 (1957); R. Gatto, Phys. Rev. **106**, 168 (1957).

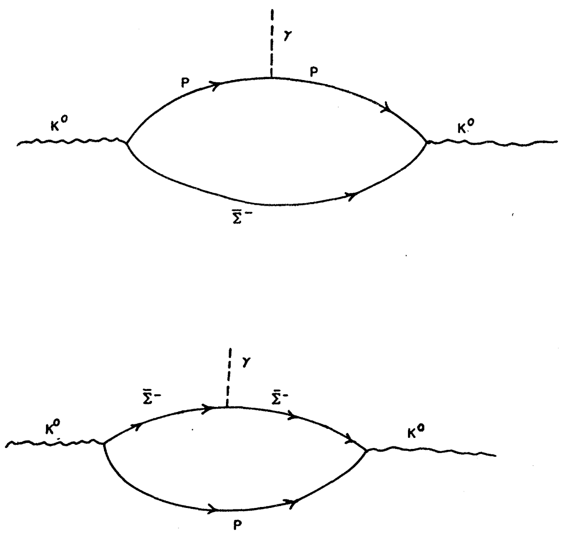

FIG. 1. Feynman diagrams for the emission of a photon by a K^0 via an intermediate proton-antiΣ pair. By $\bar{\Sigma}^-$ we mean the antiparticle of Σ^+.

the proton or the hyperon emit the photon. The minus sign in $f(q^2)$ is present because the heavy particles have opposite charge.

From the form of $f(q^2)$, we can note 2 properties of $\langle S\rangle$.

1. Since $f(0)=0$, there is no term in $\langle S\rangle$ of the form

$$\epsilon_0\int d^4xJ_\mu(x)A_\mu(x).$$

This is consistent with our discussion of Sec. II, as such a term would contribute a term $\epsilon_0 J_\mu A_\mu$ to Eq. (2).

2. If we neglect the mass difference between nucleon and hyperon, $g_1=g_2$, and therefore F and $\langle S\rangle$ vanish. This happens because in the order to which we are calculating, the nucleon and hyperon are distinguished from each other only by their masses, and therefore if the masses are taken equal, the charge densities of the two are the same except for sign, and cancel everywhere in space.

It is instructive to compare the result for the K^0 with that for the π^0, which is represented by a real field, and therefore cannot have an electromagnetic interaction linear in A_μ.[8] For the π^0, it is easy to see that for each Feynman diagram representing a virtual dissociation into two charged particles, and emission of a photon, there is another diagram, related to the first by charge conjugation, which exactly cancels it. For the K^0, on the other hand, the charge-conjugate diagram refers to another process, the electromagnetic coupling of a K^0, and therefore no cancellation is possible. Instead, the invariance of the interaction under charge conjugation implies that the charge distribution of the $\bar{K}^0$ is the negative of that for the K^0.

The integral (10) is difficult to evaluate for arbitrary q^2. We therefore calculate only the first term in an expansion in q^2. Since, in a scattering process, q^2 is related to the momentum transfer, this is a good approximation if the momentum transfer is small compared to the nucleon mass. It is easy to see that this approximation is equivalent to taking the second term in Eq. (2).

The first term in the expansion in q^2 is obtained by setting $q^2=0$ inside the square bracket in Eq. (10). We then get for $f(q^2)$:

$$f(q^2)=\frac{g^2q^2}{4\pi^2}\Bigg[\frac{1}{(m^2-m'^2)^3}\times\left\{-\frac{3}{16}m^4+\frac{5}{8}m^2m'^2-\frac{3}{16}m'^4+\frac{mm'(m^2+m'^2)}{8}\right\}+\frac{1}{(m^2-m'^2)^4}\ln\left(\frac{m^2}{m'^2}\right)\left\{\frac{5}{48}m^6-\frac{11}{48}m^4m'^2-\frac{11}{48}m^2m'^4+\frac{5}{48}m'^6-mm'\left(\frac{m^4}{24}+\frac{m^2m'^2}{6}+\frac{m'^4}{24}\right)\right\}\Bigg]. \quad (11)$$

Using the experimental value of m,m', we obtain approximately

$$f\approx -0.05\frac{(m'-m)}{m'^3}g^2\frac{q^2}{4\pi^2} \equiv\lambda q^2. \quad (12)$$

Upon substitution of (12) into (9), we obtain

$$\langle S\rangle=-ie\lambda\int d^4x\mathcal{S}[J_\mu(x)]\square^2A_\mu. \quad (13)$$

The S matrix element is related to the effective Hamiltonian by

$$\langle S\rangle=-i\int H_{\mathrm{eff}}d^4x.$$

Therefore, the contribution of the diagrams considered to the interaction (5) is

$$H(x)=e\lambda\mathcal{S}[J_\mu(x)]J_\mu^{\mathrm{ext}}(x). \quad (14)$$

Since the negatively charged antihyperon is heavier than the proton, the charge distribution is such that the K^0 will be repelled by an external negatively charged particle.

In Fig. 2 we give a Feynman diagram for one of the radiative corrections to the interaction. An approximate evaluation of this diagram indicates that it will con-

[8] We are interested only in an electromagnetic coupling arising from strong interactions, which are invariant under charge conjugation. It is easy to see that no quantities which contain even powers of a real field can be odd under charge conjugation, and thus no quantity can be constructed which when multiplied by A_μ gives a result even under charge conjugation. For a real spinless field, the absence of an interaction of this kind follows from gauge invariance alone.

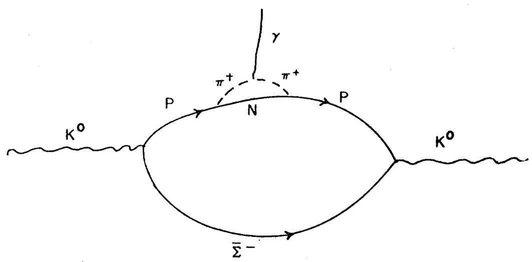

FIG. 2. Feynman diagram for one of the radiative corrections to the emission of a photon by a K^0.

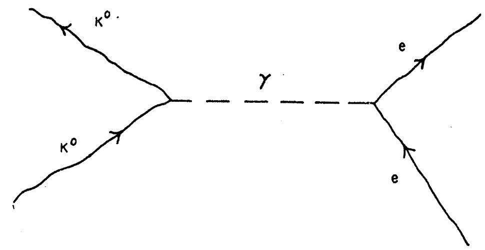

FIG. 3. Feynman diagram for the scattering of K^0's by electrons by exchange of a photon.

tribute terms to $f(q^2)$ which are of order $q^2/m^2 \ln m^2/\mu_\pi^2$ rather than q^2/m^2, and which do not depend on the nucleon-hyperon mass difference. These features may not hold rigorously, since one should also consider diagrams in which the hyperon emits the π meson, which may cancel the terms independent of the mass difference. One might attempt to treat a large class of the radiative corrections by replacing the baryon-electromagnetic vertex $\gamma_\mu A_\mu$ by an interaction which includes the anomalous magnetic moments and spread-out charge distributions of the baryons. However, theoretical calculations of these have not been very successful, and since the virtual baryons are far off the mass shell, it is not likely to be a good approximation to use the charge distributions observed in the electron-nucleon scattering.

IV. CONCLUSIONS

As we have emphasized, the interaction (5) will appear as a contact interaction of the K^0 with a charged particle. It will therefore lead to scattering of K^0's by such particles, or to regeneration of θ_1's in a beam of θ_2 in the presence of matter. The latter effect is probably easier to observe. To illustrate the process, we calculate the electromagnetic contribution to scattering of K^0's by spin $\frac{1}{2}$ particles.

The Feynman diagram for the scattering is given in Fig. 3. The K^0-electromagnetic coupling is taken from Eq. (13). This coupling leads to a factor in momentum space matrix elements:

$$e\lambda(p_\mu+p_\mu')(p-p')^2 a_\mu(p-p'), \tag{15}$$

where p and p' are the 4-momenta of the K^0 before and after emitting the photon and a_μ is the photon creation operator.

The matrix element for scattering is therefore

$$M=e^2\lambda(p_\mu+p_\mu')(p-p')^2 \times\frac{(-i)^3}{(p-p')^2}\bar{U}_{K'}\gamma_\mu U_K\frac{1}{(2\pi)^2}\frac{1}{(4\omega_{p'}\omega_p)^{\frac{1}{2}}}, \tag{16}$$

where the K and K' are the initial and final momenta of the spin $\frac{1}{2}$ particle. We note that the factor $(p-p')^2$ coming from the factor $\Box^2 A_\mu$ in (13) cancels the $1/(p-p')^2$ in the photon propagator. This means that the K^0-charged particle differential cross section will not diverge at small angles like the Coulomb scattering of two charged particles. This makes it difficult to distinguish the electromagnetic scattering from the purely nuclear scattering, if the target particle is a nucleon, which may have a similar form.[9] If the constant λ is not too small, it may be possible to detect the presence of electromagnetic scattering of K^0's by looking for deviations from charge independence in the scattering of K^0's by neutrons and protons, since only the proton will scatter the K^0 electromagnetically in the approximation considered.

If it is possible to observe scattering of K^0's by electrons, as in the case of neutrons,[10] it would be easier to find the electromagnetic scattering, since (6) will represent the total matrix element for the scattering of K^0's by electrons. Even in this case one can only observe interferences between the nuclear scattering and the electron scattering,[10] so the amplitude (16) is more relevant than the cross section. However, the cross section corresponding to (16) is easily calculated, and we give it here in the center-of-mass system.

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{c.m.}}=\frac{e^4\lambda^2}{8\pi^2}\frac{1}{(E_e+E_K)^2}[2k^2E_eE_K+2E_K^2E_e^2 -\mu^2(E_K^2-\mu^2)+\cos\theta(k^4+k^2E_K^2+2k^2E_eE_K)].$$

Here k is the momentum in the c.m. system, E_e the electron energy, and E_K the K^0 energy, while μ and m are the masses of the K^0 and electron, respectively. If $\sqrt{\lambda}$ is of the order of the pion Compton wavelength, the total cross section will be approximately 10^{-30} cm^2.

It would therefore seem that while conventional field theory predicts qualitatively the existence of a K^0-electromagnetic coupling, it is difficult to make quantitative calculations which should represent the interaction accurately. Since it also appears to be difficult to measure this interaction experimentally, it is not very suitable for probing the details of the K-meson-baryon interaction.

ACKNOWLEDGMENT

I would like to thank Dr. L. M. Lederman for an interesting discussion.

[9] See, e.g., R. Spitzer, University of California Radiation Laboratory Report UCRL-3604 (unpublished).

[10] E. Fermi and L. Marshall, Phys. Rev. **72**, 1139 (1947); Havens, Rabi, and Rainwater, Phys. Rev. **72**, 634 (1947).