from which it follows upon integration that

$$b_1(z,t) = -t(\partial b_0/\partial z)\times[\langle v_{1z}\rangle + (C/8\pi\gamma p_0) \times(\langle b_0^2\rangle - \tfrac{3}{2}b_0^2)]. \quad (24a)$$

One may then compute $v_{1y}$ from (21a), which becomes

$$\partial v_{1y}/\partial t = (1/4\pi\rho_0)^{\frac{1}{2}}\Bigg\{(\partial b_0/\partial z)[\langle v_{1z}\rangle + C\langle b_0^2\rangle/8\pi\gamma p_0] + Ct\left[\frac{3C}{8\pi\gamma p_0}b_0\left(\frac{\partial b_0}{\partial z}\right)^2 - \left(\langle v_{1z}\rangle + \frac{C}{8\pi\gamma p_0}(\langle b_0^2\rangle - \tfrac{3}{2}b_0^2)\right)\frac{\partial^2 b_0}{\partial z^2}\right]\Bigg\}. \quad (25a)$$

We consider two special cases. If the zero-order wave has a Gaussian form,

$$b_0(z-Ct) = b_0\xi\exp(-\xi^2),$$

where

$$\xi = (z-Ct)/a,$$

then it is readily shown from (24a) that

$$b_1(z,t) = \tfrac{3}{2}b_0\left(\frac{\epsilon}{\gamma}\right)\left(\frac{Ct}{a}\right)\left(\frac{b_0}{B_0}\right)^2\xi^2(1-2\xi^2)\exp(-3\xi^2).$$

The incompressible wave $b_0(z-Ct)$ and the perturbation $b_1(z,t)$ are plotted in Fig. 4 (a). The resulting deformation of a line of force of $\mathbf{B}_0$ is shown in Fig. 4 (b).

If the zero-order wave is an infinitely long train,

$$b_0(z-Ct) = b_0\cos 2\pi\xi,$$

then upon assuming that $\langle v_{1z}\rangle = 0$ we obtain

$$b_1(z,t) = \epsilon b_0\left(\frac{\pi}{4\gamma}\right)\left(\frac{Ct}{a}\right)\left(\frac{b_0}{B_0}\right)^2(\sin 2\pi\xi - 3\sin 6\pi\xi).$$

The waves $b_0(z-Ct)$ and $b_1(z,t)$ are plotted in Fig. 5(a), and the deformation of a line of force in Fig. 5(b).

The steepening of the wave front is obvious in both Figs. 4(b) and 5(b).

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# Isotropy of Pion Emission at 6 Bev*

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Observations have been made of the angular distribution of energetic electron pairs in nuclear emulsions which had been exposed to the internal 6.3-Bev proton beam of the Bevatron. The method by which the pairs were found is discussed. The angular distribution of the pairs is reasonably similar to that of the neutral pions originating in proton-nucleus collisions in the emulsion. Examination of the angular distributions in terms of multiple meson production in nucleon-nucleon collisions indicates that for these observations the emission is consistent with isotropy in the center-of-mass system.

## INTRODUCTION

A LARGE body of evidence[1] has been brought forward, largely from experiments with the cosmic radiation, in support of the hypothesis of multiple meson production in nucleon-nucleon collisions. These observations have led to discussions of the angular distribution of the emitted particles in the center-of-mass (c.m.) system of the colliding nucleons. An analysis by Fermi[2] shows that, for off-center collisions of very great energy, the c.m. angular distribution of the emergent pions departs notably from isotropy. Several experiments with particles of energy in the 100-Bev region, where the assumptions of the Fermi theory are valid, support this conclusion.[3–8] On the other hand, observations in the 10–50 Bev region suggest isotropic emission.[9,10] A recent paper,[11] however, reports that the median angles of showers caused by 6-Bev protons are subject to wide fluctuations, and are likely to be lower than expected for isotropic emission. The premises of the Fermi theory hardly apply at this energy for collisions of any impact parameter. It is therefore of interest to examine the emission angles of the neutral pions arising from 6-Bev stars.

The effect of the decay in flight of a neutral pion beam emitted at polar angle $\theta$ in the laboratory (lab)

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system is a distribution of photons of marked forward collimation, one-half of the photons lying within ten degrees of the pion direction when the pion total energy is $4M_\pi c^2$. However, a fraction of the photons exceeding one-half appears at polar angles $\theta_\gamma > \theta$. The mean spread of photon angles $\theta_\gamma > \theta$ is, moreover, larger than for $\theta_\gamma < \theta$. The resultant effect of a neutral pion angular distribution $R(\theta)$ is therefore a photon distribution $R'(\theta_\gamma)$ formed by flattening $R(\theta)$ and smearing it out to larger angles. For example, neutral pions emitted at a polar angle of 30° (lab) with total energy $4M_\pi c^2$ give rise to a photon distribution of median angle about 35°.

In comparing our observations on pion production in proton-nucleus collisions with predictions based on a nucleon-nucleon interaction, we consider a number of factors. For any c.m. emission distribution $F(\theta_C)$ reasonably close to isotropic and with pion velocities in that system close to the velocity of light, the angular distribution in the lab system is insensitive to a spread of energy in the c.m. system. With an average pion multiplicity between three and four and an available energy for pion production in the c.m. system exceeding $14M_\pi c^2$, we assume a single pion emission velocity $0.94c$, corresponding to a total energy of $3.0M_\pi c^2$ in the c.m. system. Consideration of the internal motions of the target nucleons shows that the velocity of the nucleon-nucleon center of mass is spread into a small interval and exerts a minor effect. If an incident nucleon were to interact simultaneously with more than one target nucleon with appreciable probability, a characteristic distortion of the angular distribution in the lab system would be found. Similarly, if any considerable fraction of the emergent pions were subject to secondary scattering processes, the effect on the observations would be a shift in intensity toward backward angles.

Bearing these considerations in mind, we have evaluated the relationship $\theta_L = f(\theta_C)$ connecting the polar angle of pion emission $\theta_C$ in the c.m. system and the consequent polar angle $\theta_L$ observed in the laboratory. For the 6-Bev nucleon-nucleon collision, mesons with velocity in the c.m. system less than that of the c.m. system itself do not appear at any angle $\theta_L$ exceeding 72° in the laboratory no matter how far backward $\theta_C$ is. The quantity $\Delta\theta_C/\Delta\theta_L$ is found as a function of $\theta_L$, and employed in deriving the differential laboratory distribution

$$\Delta N(\theta_L) = F(\theta_C)\,\sin\theta_C(\Delta\theta_C/\Delta\theta_L)$$

which would be expected for emission function $F(\theta_C)$ in the c.m. system.

## EXPERIMENTAL

The exposures to the internal 6.3-Bev proton beam of the Bevatron[12] resulted in an average beam intensity of about $10^4\ \mathrm{cm}^{-2}$ in the middle of the plates. Electron pairs were found by random selection of a thin flat track, and following it in either direction. In this way many beam tracks, and tracks at all directions to the beam, were followed both in the forward and backward directions. The selection of tracks of small dip angle resulted in the finding of most of the pairs lying close to the emulsion plane, since the majority were found more than once. The emulsion-plane and dip angles relative to the forward beam direction were recorded for each pair. The number of pairs within the intervals $\pm 5°$ of dip angle and $(\theta_L \pm 5)°$ of emulsion-plane angle is then proportional to the lab system intensity per unit solid angle at $\theta_L$ for $\theta_L \gtrsim 5°$. The results of measurements on 207 pairs, gained from two exposures, are shown in Fig. 1. The individual results from the two exposures are similar.

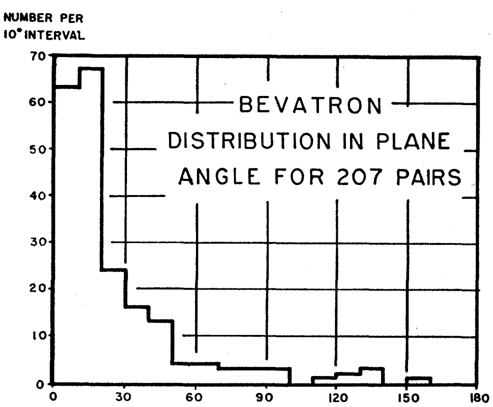


FIG. 1. The distribution in emulsion-plane angle relative to the forward beam direction for 207 electron pairs of small dip angle. Nuclear emulsion exposure to the 6.3-Bev proton beam of the Bevatron.

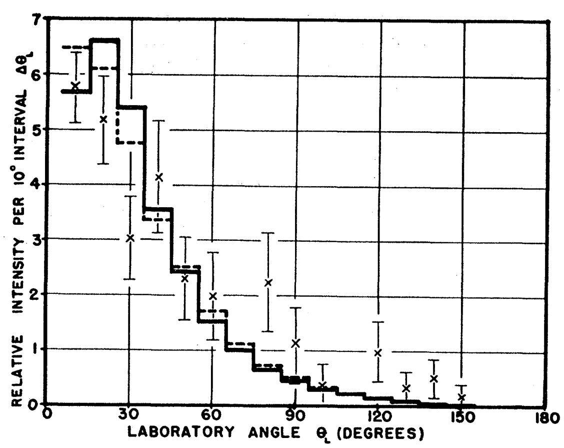


FIG. 2. The differential distribution in the lab system. The line histogram indicates the theoretical pion distribution if emission in the c.m. system is isotropic; the dashed histogram is the theoretical prediction for $1+0.5\cos^2\theta_C$ emission in the c.m. system. The crosses show the experimental points, for electron pairs, with limits of error determined from the standard deviations.

[12] We are grateful to Dr. E. J. Lofgren, Dr. W. W. Chupp, and the Bevatron staff for their courtesy in making the exposures.

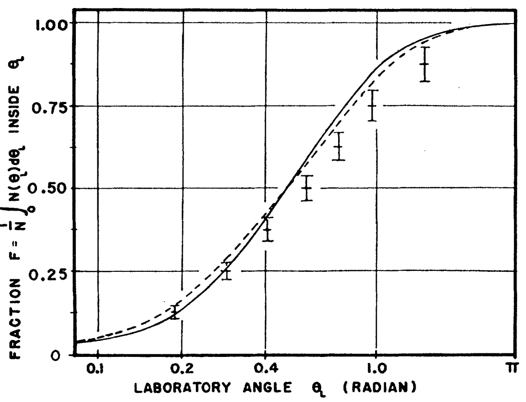


FIG. 3. The integral angular distribution in the lab system. The smooth curve indicates the theoretical distribution if c.m. emission is isotropic; the dashed curve correspondingly for $1+0.5\cos^2\theta_C$ emission in the c.m. system. The crosses show the experimental material. The poor agreement above $\theta_L \sim 20°$ is evidently due in part to the properties of decay photons from neutral pions.

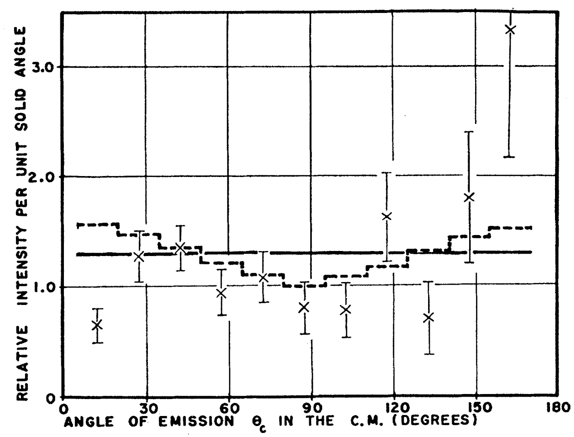


FIG. 4. The differential distribution in the c.m. system. The dashed histogram indicates $1+0.5\cos^2\theta_C$ emission, in contrast with isotropy (plain line). The experimental points, shown by crosses, are consistent with isotropy. The first interval is subject to systematic error because it includes material between laboratory angles 1.3° and 5.2°.

The material in Fig. 1 leads directly to the lab differential distribution of Fig. 2. In this figure the line histogram represents the emission probability per unit interval of laboratory polar angle that would be expected for isotropic emission in the c.m. system; the dashed histogram is the corresponding quantity expected for the c.m. emission function $F(\theta_C)=1+0.5\cos^2\theta_C$. Both histograms, normalized to the experimental data, are consistent with the observations, for which the indicated limits of error are standard deviations arising only from the statistics. Presented in this way, with the majority of the experimental material in the first two intervals, the data are clearly not in their most useful form. The integral angular distribution in the lab system is shown in Fig. 3. The smooth and dashed curves again represent emission functions $F(\theta_C)=1$, and $F(\theta_C)=1+0.5\cos^2\theta_C$, respectively, in the c.m. system.

The cumulative experimental material is in accord with both theoretical curves for small values of $\theta_L$, but systematically drifts away from both for large $\theta_L$. This is evidently due to the broadening and shift of the decay photon distribution relative to that for the neutral pions, but also reflects the occurrence of secondary scattering processes. The median angle is 32.2°, which is to be compared with the theoretical value 27.3° for pions emitted from the 6.3-Bev proton-nucleon interaction. Study of Fig. 2 indicates that, if the discrepancies between the experimental and theoretical results are attributed to secondary scattering in proton-nucleus collisions, then roughly 30% of the emergent pions arise from secondary interactions. The figure of 30% represents an upper limit because of the photon spread effect toward large $\theta_L$. The anisotropy coefficient,

$$(\theta_{\frac{3}{4}}-\theta_{\frac{1}{4}})_{\text{experimental}}/(\theta_{\frac{3}{4}}-\theta_{\frac{1}{4}})_{\text{isotropic}},$$

has, for the experimental data, the value 1.37. This result is certainly too large as it is sensitive to the amount of secondary scattering. The corresponding quantity for the dashed curve of $1+0.5\cos^2\theta_C$ anisotropy is 1.13.

The differential angular distribution in the c.m. system represents the emission intensity per unit solid angle as a function of $\theta_C$, and enables the experimental data to be spread out more evenly. For this purpose the distribution $\Delta N'(\theta_L)$ of our material in angular intervals $\Delta\theta_L$, corresponding to uniform intervals $\Delta\theta_C=15°$, is determined, using the relation between $\theta_L$ and $\theta_C$. We then find $\Delta N/\Delta\omega_C=\Delta N'(\theta_L)(\sin\theta_L)/(\sin\theta_C)$, as shown in Fig. 4, with indicated limits of error computed only from the statistics. The straight line normalized to the data represents isotropy; the dashed histogram correspondingly indicates the emission function $\Delta N/\Delta\omega_C \propto 1+0.5\cos^2\theta_C$. Again the experimental material is consistent with either histogram. The effects of secondary nuclear scattering and photon spread are evident at large backward angles, causing an apparent asymmetry. A further systematic error arises from our failure to detect some pairs which lie within one or two degrees of the beam direction. For this reason the data in the first five degrees of both differential distributions are not included.

We conclude that with the present limited material it is not possible to distinguish the detailed form of the emission function. The data are, however, consistent with isotropic emission. We are indebted to Dr. R. D. Present for many helpful discussions. The majority of the observations were made by P. B. Burt and C. L. Sachs, and the emulsions were processed by W. M. Bugg and R. L. Childers.