direction using the 180-Mev bremsstrahlung. Backgrounds due to cosmic rays varied from about 25% at $\theta=0°$ to about 2% at $\theta=90°$. Other backgrounds, mostly due to accidental coincidences, were determined by measurements below counting threshold at 150 Mev. At $\theta=90°$ these backgrounds were about 25%. At all other angles they were 10% or less. Several runs were taken at each angle. An attempt was made to alternate the runs on each side of $\theta=45°$ to reduce systematic errors. These data have been corrected for the expected geometrical asymmetry discussed above.

The angular distribution of Fig. 4 indicates no marked asymmetry. A least squares fit of the form $f(\theta)=1+a\cos^4\theta$ gives a value of $a=0.025\pm0.090$.

CONCLUSIONS

The results of this experiment do not demonstrate that the spin of the π^0 meson is zero. What they do show is that if the spin of the π^0 should be nonzero, there is no large amount of polarization or alignment of the photoproduced π^0's from carbon.

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Origin and Dynamics of Cosmic Rays*

E. N. PARKER
Enrico Fermi Institute for Nuclear Studies, University of Chicago, Chicago, Illinois
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As a consequence of our inability to observe directly the origin of a cosmic-ray particle, we begin the development with a discussion of the limitations within which we can construct a cosmic-ray accelerator mechanism. We find that we are allowed only the betatron effect and the Fermi mechanism. We review some of the many variations of these mechanisms which are to be found in the literature. Then it is shown that trains of oppositely moving hydromagnetic waves of large amplitude and with sharp crests can accomplish large and continued particle accelerations which are adequate to maintain the observed galactic cosmic-ray field. The large acceleration arises as a consequence of the simple fact that each wave tends to sweep up the cosmic-ray particles before it, so that head-on collisions of particles with waves are much more common than overtaking collisions. It is pointed out that the sharp crests of the waves are a natural consequence of the observed supersonic mass motions. Therefore, the acceleration by oppositely moving waves does not depend upon any special wave form, and we suggest that it is the naturally occuring acceleration process.

By treating the cosmic rays as a gas with relativistic thermal motions, it is shown that the cosmic-ray gas is effectively coupled to the motions of the ordinary matter both parallel and perpendicular to the magnetic field. Thus the effective speed of sound must be computed in the composite cosmic-ray and ordinary gas. It is noted that with this composite speed of sound the irregular mass motions in the galactic disk and halo are approximately Mach one. It is suggested that this represents a general dynamic balance to be found in all sufficiently active regions of space, and explains how it is that we often observe prolonged mass motions in the galaxy and in stellar atmospheres which would otherwise be computed to be highly supersonic and dissipative. The dynamic balance comes about from the fact that increased cosmic-ray density would reduce the effective Mach number below one, allowing the sharp crests of the hydromagnetic waves to degenerate, and thereby halting the production of comsic-ray particles.

I. INTRODUCTION

WITH the large number of astrophysical and geophysical phenomena now attributed to high-speed charged particles, there has been increasing interest in speculations of how electrons and ions might be accelerated up out of the range of thermal velocities by naturally occurring processes: Beginning with thermal velocities one would like to know how protons might be accelerated to 10^3-10^4 km/sec to produce the aurorae[1]; how protons or electrons might achieve 10^5 km/sec to produce some of the solar radio bursts[2]; how electrons might achieve relativistic velocities to produce the radio stars as a consequence of their synchrotron radiation[3–5]; how nuclei might achieve relativistic velocities to become cosmic-ray particles. Acceleration of electrons and nuclei from thermal

* Assisted in part by the Office of Scientific Research and the Geophysics Research Directorate, Air Force Cambridge Research Center, Air Research and Development Command, U. S. Air Force.

[1] J. W. Chamberlain and A. B. Meinel, in *The Earth as a Planet*, edited by G. P. Kuiper (The University of Chicago Press, Chicago, 1954).

[2] Wildt, Roberts, and Murray, Nature **173**, 532 (1954).

[3] H. Alfvén and N. Herlofson, Phys. Rev. **78**, 616 (1950); K. O. Kiepenheuer, Phys. Rev. **79**, 738 (1950).

[4] I. S. Shklovsky, Doklady Akad. Nauk U.S.S.R. **90**, 983 (1953); M. A. Vashakidze, *Abastumani Astr. Circ.* No. 147, 11 (1954); V. A. Dombrosvsky, Doklady Akad. Nauk U.S.S.R. **94**, 1021 (1954); J. H. Oort and J. H. Walraven, Bull. Astron. Soc. Netherlands No. 462 (1956); J. E. Baldwin, Nature **174**, 320 (1954).

[5] A. Unsöld, Phys. Rev. **82**, 357 (1951); V. L. Ginsburg, Doklady Akad. Nauk U.S.S.R. **76**, 377 (1951); I. S. Shklovsky, International Conference on Nuclear Astrophysics, Liège, 1953 (unpublished); Philip Morrison, Revs. Modern Phys. **29**, 235 (1957).

velocities apparently occurs in regions as dense as the solar chromosphere,[6] as active as supernovae,[3–5] and as large-scale and cool as the interstellar medium.[7] Presumably, therefore, the process, or processes, of acceleration are not at all a freak occurrence, but must arise from some naturally and not uncommonly occurring dynamical condition.

Many interesting speculations have been proposed to account for the particle acceleration required to explain the aurorae, solar radio bursts, etc. Unfortunately the actual acceleration processes are not subject to direct observational scrutiny, nor, of course, is it possible to treat the physical situations, in which acceleration occurs, by rigorous mathematical methods in order to deduce what *must* occur according to our knowledge of the basic dynamical properties of matter. Thus, the only approach open is to attempt to write down all possible acceleration mechanisms and decide as best we can which is most likely in light of what we can observe. Hence, the questions at the present time are simply the following:

(a) Which of the acceleration mechanisms that have already been proposed might be expected actually to occur in the physical world? (b) What other acceleration processes might occur within the limitations set down by our contemporary astrophysical knowledge? (c) Which, if any, of the allowed acceleration processes can account for the necessary high speed particles in the various special cases?

We shall, therefore, begin our discussion of the origin of cosmic rays, and of the origin of suprathermal particles in general, by inquiring into the electromagnetic fields theoretically available within the limitations imposed by our present astrophysical knowledge. Obviously the results of such an inquiry suffer at least the same limitations as our present conception of the astrophysical universe.

Then, within the limitations imposed upon the electromagnetic fields, we shall demonstrate what particle acceleration mechanisms are available. In this manner we hope best to answer questions (a) and (b). We will find that many familiar speculations are eliminated, but that there exists a continuous-field version of Fermi's mechanism, presumably commonly occurring in nature, which apparently fulfills the acceleration requirements in most cases, in answer to (c).

But this constitutes only one part of the problem. For the cosmic-ray energy density is not at all negligible, being comparable to the magnetic and turbulence energy densities throughout the galaxy. Thus the remainder of the paper will be devoted to the mutual interaction of the cosmic-ray particles with the electromagnetic and velocity fields throughout the galaxy. We will find that the cosmic rays apparently play an important role in determining the general character of the mass motions in the galaxy and its halo, and perhaps in some stellar atmospheres; in the presence of cosmic rays we find that otherwise immensely supersonic mass motions are possible without excessive dissipation.

[6] E. N. Parker, Phys. Rev. **107**, 803, 926 (1957).

[7] E. Fermi, Phys. Rev. **75**, 1169 (1949); Astrophys. J. **119**, 1 (1954).

II. ELECTROMAGNETIC FIELDS

A. Weak Magnetic Fields

In the presence of magnetic fields sufficiently weak that the cyclotron frequency of a free electron is small compared to its collision frequency, the current density $\mathbf{i}$ is related to the electric field $\mathbf{E}'$ in the frame of reference moving with the matter by the simple scalar form of Ohm's law,

$$\mathbf{i}=\sigma\mathbf{E}'. \tag{1}$$

$\mathbf{E}'$ is related to the electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$ in the fixed frame of reference by the usual Lorentz transformation

$$\mathbf{E}'=\mathbf{E}+(\mathbf{v}/c)\times\mathbf{B} \tag{2}$$

for material velocities, $\mathbf{v}$, small compared to the speed of light. The conductivity σ of a gas at absolute temperature T may be computed from Cowling's approximate numerical expression[8]

$$\sigma\cong 1.8\times 10^7 T^{\frac{3}{2}}/Z \text{ esu}, \tag{3}$$

where Z represents the mean charge on each ion.

Using (1) and (2) to express $\mathbf{i}$ in terms of $\mathbf{E}$ and $\mathbf{B}$, Maxwell's equations,

$$4\pi\mathbf{i}+\partial\mathbf{E}/\partial t=+c\nabla\times\mathbf{B}, \tag{4}$$

$$\partial\mathbf{B}/\partial t=-c\nabla\times\mathbf{E}, \tag{5}$$

may be rewritten

$$\partial\mathbf{E}/\partial t+4\pi\sigma\mathbf{E}=\mathbf{f}(t), \tag{6}$$

where

$$\mathbf{f}(t)=c\nabla\times\mathbf{B}-4\pi\sigma(\mathbf{v}/c)\times\mathbf{B}. \tag{7}$$

We may integrate (6) to obtain the formal result

$$\mathbf{E}(t)=\int_{-\infty}^{t} d\tau\mathbf{f}(\tau)\exp[4\pi\sigma(\tau-t)]. \tag{8}$$

As has been pointed out by Schlüter,[9] using Cowling's formulas,[8] the electrical conductivity σ ranges from 10^{11} esu in the cool H_{I} regions of interstellar space, to 10^{13} or more in a stellar atmosphere, to 10^{18} in a stellar interior. Thus, it is readily seen that σ, which has the dimensions of a characteristic frequency, is large compared to any of the observed characteristic frequencies of macroscopic mass motions.[10] Expanding $\mathbf{f}(\tau)$ about $\tau=t$ we obtain from (8) a series in ascending

[8] T. G. Cowling, in *The Sun*, edited by G. P. Kuiper (University of Chicago Press, Chicago, 1953).

[9] A. Schlüter, Z. Naturforsch. **5a**, 72 (1950).

[10] The only exception to this might be plasma oscillations. However, since plasma oscillations are believed to be caused by (and therefore cannot be a major source of) high-velocity particles, we exclude them from our discussion.

powers of $1/\sigma$,

$$\mathbf{E}(t)=[\mathbf{f}(t)/4\pi\sigma][1+(\partial\mathbf{f}/\partial t)(1/4\pi\sigma\mathbf{f})+\cdots].$$

Then

$$\mathbf{E}(t)=-(\mathbf{v}/c)\times\mathbf{B}+(c/4\pi\sigma)[\nabla\times\mathbf{B} -c^{-2}(\partial/\partial t)(\mathbf{v}\times\mathbf{B})]+O^2(1/\sigma).$$

As already mentioned, the term $(1/4\pi\sigma)(\partial/\partial t)(\mathbf{v}\times\mathbf{B})$ is observed to be always extremely small compared to $\mathbf{v}\times\mathbf{B}$ because $\sigma\geq 10^{11}$ sec^{-1}; it represents the displacement current, which is negligible[11] in all observable astrophysical cases.[12]

The quantity $(c^2/4\pi\sigma)\nabla\times\mathbf{B}$ is small compared to $\mathbf{v}\times\mathbf{B}$ except when the material motions are so sluggish that $\mathbf{v}\cong 0$. Therefore, we write just

$$\mathbf{E}(t)=-(\mathbf{v}/c)\times\mathbf{B}[1+O(1/\mathcal{R})],$$

where $\mathcal{R}$ is the magnetic Reynolds number, defined by Elsasser[12] as

$$\mathcal{R}=Lv\sigma/c^2,$$

and representing the ratio of the magnitude $\mathbf{v}\times\mathbf{B}$ to that of $(c^2/4\pi\sigma)\nabla\times\mathbf{B}$. L is the scale of the material motions. $\mathcal{R}$ is observed to be immensely large: In an interstellar $\mathrm{H_I}$ region where $L\cong 10$ parsecs and $v=10$ km/sec we have $\mathcal{R}=3\times10^{15}$; in the solar photosphere where the smallest observable scale is $L=100$ km and $v\cong 1$ km/sec, we have $\sigma\cong 10^{13}$ sec^{-1} and $\mathcal{R}=10^4$. Therefore, we shall put

$$\mathbf{E}=-(\mathbf{v}/c)\times\mathbf{B}, \tag{9}$$

neglecting all terms of $O(1/\mathcal{R})$.

Equation (9) is the basic field relation in a conducting gas. Inserting it into (5) yields the familiar hydromagnetic equation,

$$\partial\mathbf{B}/\partial t=\nabla\times(\mathbf{v}\times\mathbf{B}). \tag{10}$$

Including the Lorentz force $(\mathbf{i}/c)\times\mathbf{B}$ exerted by the magnetic field on the fluid, and using (4) without the negligible displacement current term $\partial\mathbf{E}/\partial t$, we readily obtain the equation of motion for the gas as

$$\rho d\mathbf{v}/dt=-\nabla p+(1/4\pi)(\nabla\times\mathbf{B})\times\mathbf{B}. \tag{11}$$

B. Strong Magnetic Fields

In the presence of a magnetic field sufficiently strong that the cyclotron frequency of a free electron is large compared to its collision frequency, one is not justified in writing down (1) because, as is well known, an electric field $\mathbf{E}$ impressed perpendicular to a magnetic field $\mathbf{B}$ produces only a general drift $\mathbf{u}$, where

$$\mathbf{u}=c\mathbf{E}\times\mathbf{B}/B^2 \tag{12}$$

for all free particles regardless of the sign of their charge. Thus no net current results from an externally impressed field $\mathbf{E}$; only a mass motion $\mathbf{u}$ follows. However, if we form the vector product of (12) with $\mathbf{B}$, we find that

$$\mathbf{E}=-(\mathbf{u}/c)\times\mathbf{B}, \tag{13}$$

which is of exactly the same form as (9); if the magnetic field is of large scale compared to the radius of gyration of the thermal motions of the ions and electrons, then $\mathbf{v}=\mathbf{u}$ and (13) is identical with (9). The general macroscopic dynamical equations assume a form similar to (10) and (11), and have been discussed at length in the literature.[6,13–15]

We see that in either extreme of a strong or a weak magnetic field, $\mathbf{E}$ and $\mathbf{B}$ are mutually perpendicular and $|\mathbf{E}|/|\mathbf{B}|$ is of the order of the macroscopic material velocity divided by c.

III. MOTION OF A CHARGED PARTICLE

A. Equations of Motion

Now consider the motion of a particle of mass m, charge q, and velocity $\mathbf{w}$ in electromagnetic fields restricted by (9) or (13). We shall limit ourselves to the nonrelativistic equations of motion,

$$d\mathbf{w}/dt=(q/m)[\mathbf{E}+(\mathbf{w}/c)\times\mathbf{B}], \tag{14}$$

because it is the initial acceleration of particles at nonrelativistic thermal velocities that is basic to our problem of producing high-speed particles, and, what is more, it is at the lowest particle velocities that the losses to the surrounding medium by Coulomb interaction are greatest and acceleration most difficult; we shall regard our nonrelativistic discussion as fundamental.

If $\mathbf{E}$ is related to $\mathbf{B}$ as in (9), then (14) may be written

$$d\mathbf{w}/dt=(q/m)[(\mathbf{w}-\mathbf{v})/c]\times\mathbf{B}.$$

Forming the scalar product with $\mathbf{w}$ we find that the rate of energy gain is

$$\begin{aligned}(d/dt)(\tfrac{1}{2}mw^2)&=-q\mathbf{w}\cdot(\mathbf{v}\times\mathbf{B})/c\\ &=+q\mathbf{v}\cdot[(\mathbf{w}/c)\times\mathbf{B}].\end{aligned} \tag{15}$$

We see from (15) that the particle energy may be increased only by the work which the macroscopic motions $\mathbf{v}$ do against the Lorentz force $q(\mathbf{w}/c)\times\mathbf{B}$ exerted on the particle by $\mathbf{B}$. Hence, whatever accelerating mechanisms may be possible, they must all reduce to the same basic process of $\mathbf{v}$ working against the Lorentz force.

[11] It may also be shown that the acceleration time of the conduction electrons is negligible; thus we ignore plasma oscillations.

[12] W. M. Elsasser, Phys. Rev. **95**, 1 (1954).

[13] L. Spitzer, Astrophys. J. **116**, 299 (1952).

[14] Chew, Goldberger, and Low, Proc. Roy. Soc. (London) **236**, 112 (1956).

[15] K. M. Watson, Phys. Rev. **102**, 12 (1956); K. A. Brueckner and K. M. Watson, Phys. Rev. **102**, 19 (1956).

B. Large-Scale Fields

When the scale L of the magnetic field is large compared to the radius of curvature R of the particle trajectory, and when the characteristic period of the field variations is large compared to the cyclotron period $2\pi/\Omega$ of the particle, one may integrate the equations of motion (14) for arbitrary $\mathbf{E}$ and $\mathbf{B}$ by expanding the coordinates $x^i(t)$ of the particle into a sum

$$x^i(t)=\sum_{n=0}^{\infty} x_n{}^i(t), \tag{16}$$

where $x_n{}^i(t)$ represents terms with a frequency of n times the cyclotron frequency. We use a local moving coordinate system with the $i=3$ axis directed along $\mathbf{B}$ and the origin at the instantaneous center of the circular cyclotron motion [represented by $x_1{}^i(t)$] of the particle; we neglect all terms of $O^2(R/L)$. The coordinates $x_n{}^i(t)$ are of the form

$$x_n{}^i(t)=a_n{}^i(t)\,{\sin \atop \cos}\left[n\int_{-\infty}^{t} d\tau\Omega(\tau)\right], \tag{17}$$

and $n=0, 1, 2, \cdots$.

The variables are readily separated and the integration carried out using the WKB approximation.[16] After considerable elementary manipulation one obtains the familiar result that the particle velocity component w_n perpendicular to the magnetic field varies as the square root of the field density; the velocity component w_s along the field at time t is related to the initial velocity according to

$$w_s(t)=w_s(0)-\frac{w_n{}^2(0)}{2B(0)}\int_0^t d\tau\frac{\partial B(\tau)}{\partial x^3}+O\left(w\frac{R}{L}\right). \tag{18}$$

The component w_n, proportional to $B^{\frac{1}{2}}(t)$, represents the betatron effect; (18) represents the repulsion of a particle along the lines of force away from regions of dense field, and, when the dense region is moving, introduces the energy gain by Fermi's mechanism. The terms $O(wR/L)$ in (18), which we have omitted, are of considerable length and of no particular interest; similar terms appear in the complete expression for w_n.

We see, then, that when the magnetic field is slowly varying in both space and time we have only the familiar betatron effect and Fermi's mechanism. In particular we see that there are no trochoidal orbits leading to large accelerations, as has been speculated by Alfvén.[17]

C. Abruptly Varying Fields

We wonder if something new might appear if we consider abruptly varying fields. In order to have a field which is of large-scale but varying abruptly with time as compared to the cyclotron frequency of the particle, we must have $L\gg R$ and $v/L\gg\Omega$, where v is the material velocity, Ω the cyclotron frequency, R the radius of gyration, and $R=w_n/\Omega$. It follows immediately that v must be large as compared to w. This is obviously not an interesting case because only one collision with a moving magnetic inhomogeneity will make w at least comparable to v by Fermi's mechanism.

We consider the case where the field varies abruptly with position, so that $L\ll R$. Then the variations of the magnetic field may be treated as discontinuities, across which the particle trajectory is continuous. It is readily shown from the hydromagnetic equations (10) and (11) that such discontinuities form hydromagnetic waves with longitudinal and/or transverse material motions.

We have traced the trajectory of a charged particle with velocity $\mathbf{w}$ through a number of hydromagnetic waves with sharp fronts.[18] The procedure is laborious but elementary. A particle in front of an infinite plane shock wave which is moving perpendicular to an initially uniform magnetic field will be engulfed by the wave after several circular passages into and out of the wave front. The ratio of the final particle velocity (perpendicular to $\mathbf{B}$), following engulfing by the wave, to the initial velocity perpendicular to $\mathbf{B}$ is equal to the square root of ratio of the field behind the wave front to the field ahead; hence the final result is $w_n/B^{\frac{1}{2}}=\text{constant}$, as in slowly varying fields.

A shock wave moving perpendicular to $\mathbf{B}$, but with a cylindrical front with axis parallel to $\mathbf{B}$, leads to the same result as a plane front. And again we find the same result when a particle is trapped between two approaching transverse hydromagnetic waves of very large amplitude propagating along a uniform field.

D. Conclusion

We have not considered all possible hydromagnetic wave forms, nor have we considered the intermediate case where the variations of the field are comparable in rate to the circular cyclotron motion of the particle. However, we suspect, on the basis of what calculations we have made, that we probably cannot avoid the relation

$$w_n{}^2/B=\text{constant} \tag{19}$$

in any essential way. It is apparently rather more general than just being the adiabatic invariant in slowly varying fields. We suggest that (19) is intimately related to the fact that the particle energy is increased only when the material velocity $\mathbf{v}$ works against the Lorentz force $q(\mathbf{w}/c)\times\mathbf{B}$, as shown in (15), though we have been unsuccessful in demonstrating with what generality (19) follows directly from (15).

It seems to follow from the calculations discussed in this section that the betatron effect and the Fermi

[16] E. N. Parker (unpublished).

[17] H. Alfvén, Tellus 6, 232 (1954); Phys. Rev. 94, 1082 (1954).

[18] E. N. Parker (unpublished).

mechanism cover all the possibilities contained in (15), and that the two mechanisms are in fact the same process with different geometry. Hence, our study of the acceleration of particles in the astrophysical universe is limited to these two processes. Since other devices for the acceleration of particles have been proposed in the literature, we shall discuss in the next section how they violate the conditions apparently obtaining throughout our galaxy.

IV. EXCLUDED MECHANISMS

A. Betatron Effect

Riddiford and Butler[19] have computed from the relativistic equations of motion the acceleration of a proton, etc., in a field growing linearly with time,

$$B=\alpha t, \tag{20}$$

from zero to several thousand gauss, as one might suppose a sunspot field to do. They find that a proton starting at rest may easily achieve 1000 Bev by the time the sunspot reaches maturity.

In a perfect vaccuum, in which $\sigma=0$, their results are entirely correct. However, if we take into account the fact that $\sigma=10^{13}$ esu in a sunspot, resulting in (9), we find that (20) cannot possibly obtain in the neighborhood of $t=0$. To understand in detail how their result arises, and why (20) is not a possible field density in the vicinity of $t=0$, consider the electric field at a distance $r=10^4$ km from the axis of a hypothetical spot wherein B grows linearly from zero to 3000 gauss in 10^5 seconds (about 30 hours). Then $\alpha=3\times10^{-2}$ gauss/sec and $E=\frac{1}{2}\alpha r/c=0.15$ volt/cm. We see that E does not vanish when $t=0$ but is uniform for $t\geq0$. $E\geq B$ until $t=0.016$ sec, and in that time a proton will fall more or less freely down E a distance s, according to $s=\frac{1}{2}(qE/m)t^2\cong200$ km. By the time $E=B$, then, the proton will have a kinetic energy of the order of 3 Mev. After about 0.06 sec $(1/B)(dB/dt)$ will be smaller than the cyclotron frequency Ω and (19) becomes applicable. $B\cong2\times10^{-3}$ gauss when $t=0.06$ sec and will increase by a factor of 1.5×10^6 before $t=10^5$ sec. We see, then, that it is the first moments of $B=\alpha t$ that are so important to achieving the immense final particle energies; 3 Mev is reached in 0.016 sec.

But if $\mathbf{v}$ is more or less uniform in time, we see from (10) that at least in the first moments $\mathbf{B}$ must grow exponentially, rather than linearly; in view of (9), $\mathbf{E}$ cannot be as large as $\mathbf{B}$, let alone much greater (when $t<0.016$ sec). Or, to state the matter differently, the conductivity $\sigma\cong10^{13}$ esu will not tolerate a pure electric field of 0.15 volt/cm over a scale of $(10^4\text{ km})^2$.

We suggest that a more realistic calculation of the betatron effect in a sunspot is to apply (19) to a field which grows from one, not zero, gauss to 3000 gauss in 10^5 seconds. Then a particle which initially has 1 ev in the form of thermal energy will achieve only 3 kev after 10^5 sec. And if we take into account the ionization losses, the particle will not succeed in surpassing even a few electron volts.

The betatron effect, being reversible, is difficult to use by itself in a sequence of accelerations. We suggest that with the apparent astrophysical values of σ, it is not of great interest by itself. We shall discuss it further only insofar as it cooperates with acceleration parallel to $\mathbf{B}$ in Fermi's mechanism.

[19] L. Riddiford and S. T. Butler, Phil. Mag. **43**, 447 (1952).

B. Magnetic Beams and Galactic Rotation

It has been suggested[20] that the electric field in the fixed frame of reference arising according to (9) when a magnetic field $\mathbf{B}$ is carried by a more or less uniform velocity $\mathbf{v}$, will accelerate cosmic ray particles: A cloud or beam of material from the sun of width 10^{12} cm [0.15 astronomical unit (a.u.)] carrying 10^{-3} gauss with a velocity of 2000 km/sec yields a potential difference between its faces amounting to about 2×10^9 volts; the rotation of the galaxy, for which $L\cong10^4$ parsecs, $v=200$ km/sec, and $B=10^{-5}$ gauss, yields 2×10^{14} volts. As was pointed out by Swann,[21,22] the error in such speculations is immediately obvious when we note that, if we transform to the coordinate system in which $\mathbf{v}=0$, then $\mathbf{E}$ vanishes and there is no acceleration. The existence of trochoidal orbits, as speculated by Alfvén,[17] in no way alters the situation. The only acceleration that takes place is by Fermi's mechanism, in the jolt the particle receives as it passes into or out of a moving beam.

V. FERMI MECHANISM

A. Isolated Moving Inhomogeneities

In the original version of Fermi's mechanism,[7] wherein the cosmic-ray particles were in a space filled with separate and independently moving clumps of magnetic field, the mean fractional energy gain per collision of a relativistic particle with a moving clump of field, was

$$\Delta W/W=O^2(v/c); \tag{21}$$

W represents the total particle energy, ΔW is the mean energy gain in one collision, and v is the random velocity of the clump. The energy exchange per collision is $\pm O(v/c)$, but the overtaking collisions very nearly cancel the head on collisions, so that the mean is only $O^2(v/c)$.

At least on the surface of it (21) seems to be adequate to accelerate protons to relativistic energies in a solar flare.[6] A field of about 500 gauss is required to explain the energy output of the cosmic-ray flare of February 23, 1957. Putting v equal to the characteristic hydromagnetic velocity resulting from a 500-gauss field in

[20] H. Alfvén, Phys. Rev. **75**, 1732 (1949).
[21] W. F. G. Swann, J. Franklin Inst. **257**, 191 (1954).
[22] W. F. G. Swann, Phys. Rev. **93**, 205 (1954).

the solar chromosphere yields the observed cosmic-ray energy spectrum.

Ginsburg[23,24] has pointed out that (21) seems to be adequate for the acceleration of cosmic rays in novae and supernovae, and in the solar corona.

However, Morrison, Olbert, and Rossi[25] have shown that in order for the interstellar motions to result in the observed energy spectrum as a consequence of (21), as originally suggested by Fermi,[7] it is necessary that the interstellar velocities be of the order of 120 km/sec, reversing their sign every light year. But it is readily shown[26] that the viscous losses of such small-scale and violent motions are prohibitive: The cosmic-ray energy density of 10^{-12} erg/cm^3 with a life of the order of 4×10^6 years requires an energy input of the order of 10^{-26} erg/cm^3/sec; the mechanism of Oort and Spitzer[27–30] may perhaps supply sufficient energy to the interstellar motions for maintaining the input to the cosmic rays, but the mechanism is entirely inadequate to support the viscous losses, which are 100 times greater than 10^{-26} erg/cm^3/sec.

B. Continuous Waves

If regarding moving magnetic inhomogeneities as *isolated* and *separate* units provides inadequate acceleration in interstellar space, our next region of inquiry is into the opposite extreme, wherein we consider the moving magnetic inhomogeneities as *continuous* hydromagnetic waves in an initially uniform field. As is already well known, continuous waves allow many variations on Fermi's basic scheme, and we shall find that $\Delta W/W$ may be made $O(v/c)$ rather than just $O^2(v/c)$.

1. Fermi was the first to consider the acceleration of particles by smooth continuous hydromagnetic waves in a large-scale field. In his second paper[7] he pointed out that a cosmic-ray particle may be trapped between two approaching waves so that all of the reflections of the particle from the waves are head on and $\Delta W/W = +O(v/c)$. The particle eventually escapes from the trap because the direction of its velocity steadily approaches the direction of the large-scale field, allowing penetration of the waves which confine it. Fermi did not show that the particle might not then be caught between two receding waves and decelerated at the same rate, $\Delta W/W = -O(v/c)$ yielding a mean acceleration of second order in v/c, as in (21).

2. Davis[31] has pointed out that as two approaching hydromagnetic waves begin to pass into each other, the field density in the narrowing region between will increase, so that trapped particles will be accelerated by the betatron effect as well as by Fermi's mechanism. The betatron effect, as may be seen from (21), increases the component w_n of the particle velocity perpendicular to $\mathbf{B}$; this is in contrast to the Fermi mechanism which increases only the component w_s parallel to $\mathbf{B}$. Thus the simultaneous operation of the betatron effect may tend to counteract the tendency of the particle velocity to align itself along the magnetic field and thereby escape from between the two waves. Davis showed that the random walk in the cosmic-ray particle energy, resulting from both acceleration and deceleration by the combined betatron and Fermi mechanism, can yield the observed cosmic-ray spectrum if the interstellar motions of 10 km/sec have a scale of 1 light year perpendicular to the large-scale galactic field and 7 light years parallel, and if the net energy input to the cosmic-ray field is zero. In this way the overwhelming viscous losses required by (21) are avoided, but, of course, the injection mechanism must supply the total energy in the form of particles with the mean cosmic-ray energy of about 2 Bev per nucleon.

3. Fan[32] has gone one step further to show that, with suitable wave forms, incorporating the betatron effect may lead to a mean energy gain per collision

$$\Delta W/W = O(v/c), \tag{22}$$

in contrast to (21). Fan assumes that in the region between two hydromagnetic waves approaching each other in a large-scale field, the field density grows in the same manner as the *mean* field density throughout a turbulent field; he takes

$$(1/B)dB/dt = \alpha v/L,$$

where v is the characteristic velocity, L the scale, and α a numerical constant. Hence, between the approaching waves he assumes that

$$B(x,y,z,t) = B_0(x,y,z) + B_1(x,y,z,0)\exp(\alpha vt/L). \tag{23}$$

Thus, both the betatron effect and Fermi's mechanism act to accelerate particles trapped between the approaching waves.

For particles with small angle of pitch θ (θ is the angle between the particle velocity $\mathbf{w}$ and large-scale field $\mathbf{B}_0$) the betatron effect is dominant and θ increases; for large θ the Fermi mechanism dominates and θ decreases. The result is that all the trapped particles remain trapped and undergo considerable acceleration before the waves meet.

Between receding pulses Fan assumes that

$$(1/B)dB/dt = -\alpha v/L,$$

and it follows that θ decreases if it is small and increases if it is large. Hence only a portion of the particles remain trapped; all those with angle of pitch less than

[23] V. L. Ginsburg, Doklady Akad. Nauk U.S.S.R. **92**, 727 (1953) [National Science Foundation translation—207].
[24] V. L. Ginsburg, Suppl. Nuovo cimento **3**, 38 (1956).
[25] Morrison, Olbert, and Rossi, Phys. Rev. **94**, 440 (1954).
[26] E. N. Parker, Phys. Rev. **99**, 241 (1955).
[27] J. H. Oort, Bull. Astron. Soc. Netherlands **12**, 177 (1954).
[28] F. D. Kahn, Bull. Astron. Soc. Netherlands **12**, 187 (1954).
[29] J. H. Oort and L. Spitzer, Jr., Astrophys. J. **121**, 6 (1955).
[30] M. P. Savedoff and J. Greene, Astrophys. J. **122**, 477 (1955).
[31] L. Davis, Phys. Rev. **101**, 351 (1956).
[32] C. Y. Fan. Phys. Rev. **101**, 314 (1956).

some critical value soon find themselves with so small an angle of pitch that they penetrate one or the other of the receding waves and escape the trap. It follows that deceleration by receding waves is much less effective than acceleration by approaching waves, resulting in (22).

Fan's mechanism depends upon the validity of (23), and it is not clear that the field will grow exponentially between approaching waves. If the field were to vary in some other manner, the opposite effect might easily occur. For instance if dB/dt were independent of B, instead of proportional to B, then Fan's first-order effects vanish. If we argue that B increases most where it is weakest, so that dB/dt varies as $(1/B)$ to some positive power, the effect reverses and the mean energy gain is negative. Unfortunately it is not possible at present to establish in just what manner the field at a given point in a turbulent medium will vary with time. Our present objection to Fan's mechanism is that simple variations of the form of (23) do not represent approaching jaws of a trap.

4. We would like to suggest at this point that there is one effect occurring in the presence of continuous hydromagnetic waves which, so far as we are aware, has been overlooked: That is the fact that the cosmic-ray particle density will be systematically greater ahead, as compared to behind, continuous transverse hydromagnetic waves, as a consequence of the relative impenetrability of such waves; significantly more than half of the collisions will be head-on.

To demonstrate this effect quantitatively, consider a uniform magnetic field of density B_0 in an infinitely conducting incompressible fluid medium of density ρ. We put the z axis of our coordinate system parallel to B_0 and introduce at $z=\pm\infty$ plane transverse hydromagnetic waves at intervals of L/C seconds, where C is the characteristic hydromagnetic velocity $B_0/(4\pi\rho)^{\frac{1}{2}}$. Thus we have two sets of waves, traveling in opposite directions along B_0; the waves of each set are separated by a distance L. For convenience we suppose that each individual wave is a narrow pulse of width l, where $l\ll L$.

We suppose that the nth pulse consists of the field $b_n(z,t)$ in the x direction, where

$$b_n(z,t)=-b_0(\xi_n/l)\exp(-\xi_n^2/l^2), \tag{24}$$

and

$$\xi_n=z\pm(nL-Ct). \tag{25}$$

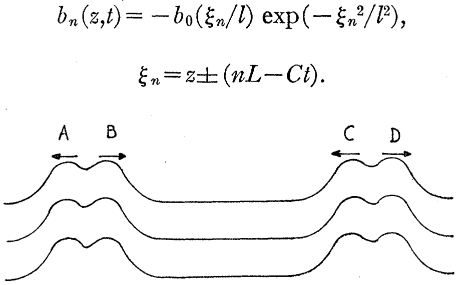

FIG. 1. Schematic diagram of plane transverse hydromagnetic waves in a large-scale magnetic field. The waves B and D are propagating to the right with the usual hydromagnetic velocity $C=B/(4\pi\rho)^{\frac{1}{2}}$; waves A and C are propagating to the left.

The $\pm$ is $+$ for pulses propagating in the positive z direction and $-$ for propagation in the negative z direction. The pulses are shown schematically in Fig. 1.

It is readily shown that an infinite plane transverse wave, such as we are considering, involves none of the nonlinear terms in the hydromagnetic equations (10) and (11). Hence the pulses do not interact, and pass through each other unchanged. The fluid velocity associated with the nth pulse is readily shown to be $v_n(z,t)$ in the x direction, where

$$v_n(z,t)=\mp b_n(z,t)/(4\pi\rho)^{\frac{1}{2}}. \tag{26}$$

The densest field in a pulse occurs where

$$\xi_n=\pm l/\sqrt{2},$$

and has the value

$$B_{\max}=B_0[1+(b_0/B_0)^2\tfrac{1}{2}\exp(-1)]^{\frac{1}{2}}. \tag{27}$$

We consider a charged particle with velocity $\mathbf{w}$ whose radius of curvature in B_0 is small compared to the pulse width l. It follows from (19) that in the frame of reference in which a given pulse is stationary, the angle of pitch θ (the angle between $\mathbf{w}$ and the total magnetic field) varies in the well-known manner[7]:

$$\sin^2\theta/B=\text{constant}. \tag{28}$$

Penetration of the pulse occurs when θ is sufficiently small that $\theta\leq\theta_c$ in the uniform field between pulses, where

$$\sin^2\theta_c=B_0/B_{\max}. \tag{29}$$

Hence all particles for which θ is greater than the critical angle will be reflected by the pulses; all particles for which $\theta<\theta_c$ pass freely through the pulses with no net change in θ or in speed w.

At the moment when two pulses have just passed through each other, as have A and B in Fig. 1, all the particles will be contained between approaching pulses, such as B and C. All collisions will be head on. The particle velocities parallel to B_0 will increase, and θ for each particle will decrease to θ_c. Penetration of the approaching pulses B and C follows, with all particles ultimately between the receding pulses, A and B, and C and D (undergoing deceleration and increasing θ) when B meets C. Thus all particles begin with head on collisions and end with overtaking collisions during each cycle, of duration $L/2C$.

We will now show that, if θ_c is sufficiently small, penetration of the pulse does not occur for the average particle until the wave B has nearly arrived at C; the particles spend a significantly greater portion of their time between approaching waves making head on collisions, than between receding waves. What is more, the particles are crowded together in greatest density by approaching pulses when the pulses are closest and acceleration is most rapid. The particles are not between receding pulses which have just passed through each other and are close together, when deceleration would be most rapid. Because our wave configuration is periodic, it is equivalent to the single cell shown in Fig. 2 where a single pulse starts from the reflecting surface $z=0$ at time $t=0$, and proceeds with velocity $C'=2C$ to the reflecting surface $z=L$ at time $t=L/C'$.

Consider a particle ahead of the pulse, between $z=C't$ and $z=L$. We suppose that the particle velocity is large compared to C'. The particle collides with the pulse every Δt seconds, where

$$\Delta t=2(L-C't)/w_s.$$

Reflection from the pulse results in an increase in w_s of

$$\Delta w_s=2C',$$

and

$$\Delta w_n=0.$$

Noting that

$$\tan\theta=w_n/w_s,$$

we find that

$$\frac{d\theta}{dt}=-\frac{C'}{L-C't}\sin\theta\cos\theta; \tag{30}$$

we have written $d\theta/dt$ in place of $\Delta\theta/\Delta t$ because with the particle velocity w large compared to C', $\Delta\theta/\theta$ and $\Delta t/(L/C')$ become small quantities. We may also show that

$$\frac{dw}{dt}=+\frac{C'}{L-C't}w\cos^2\theta. \tag{31}$$

For a particle behind the pulse, in $0<z<C't$, we have

$$d\theta/dt=+(1/t)\sin\theta\cos\theta, \tag{32}$$

$$dw/dt=-(1/t)w\cos^2\theta. \tag{33}$$

Equation (29) is readily integrated to give the angle of pitch of a particle initially at θ_0; we obtain

$$\theta(\theta_0,t)=\arctan\{[(L-C't)/L]\tan\theta_0\}. \tag{34}$$

We see that $\theta(\theta_0,t)$ is a monotonically decreasing function of time, as a consequence of $\Delta w_s>0$, $\Delta w_n=0$. This brings us to the question of the redistribution of particle velocities, first discussed by Fermi.[7]

If θ becomes less than θ_c, defined in (29), the particle no longer interacts with the pulse, and acceleration ceases. Fermi suggested that sharp kinks in the magnetic field may sometimes be encountered by the cosmic-ray particle, reorienting the field so suddenly that the particle cannot follow, and effectively scattering θ from $\theta<\theta_c$ to larger values where acceleration may again take place. In a later section of this paper we will show that all hydromagnetic disturbances of sufficiently large amplitude are expected to have sufficiently sharp crests to accomplish the reorientation of θ. Therefore, we shall for the present calculation suppose that the pulse, $z=C't$, has a sufficiently sharp crest to scatter the angle of pitch of those particles escaping (with $\theta=\theta_c$) through the pulse from the region ahead of the pulse. We shall assume that the probability of a particle being scattered into $(\theta,\theta+\delta\theta)$ from θ_c is $\psi(\theta)\delta\theta$. We suppose that the scattering takes place without much change in the speed of the particle, so that if $w_n=w_{nc}$ and $w_s=w_{sc}$ before the scattering, we have

$$w_s=w_{sc}\cos\theta/\cos\theta_c,\quad w_n=w_{nc}\sin\theta/\sin\theta_c$$

following the scattering.

We consider, then, a particle with an initial speed w_0 and angle of pitch θ_0 (at time $t=0$). It is trapped ahead of the pulse until its angle of pitch, $\theta(\theta_0,t)$ has decreased to θ_c, at time t_c; it is readily shown from (34) that

$$t_c=(L/C')[1-\tan\theta_c/\tan\theta_0].$$

The component of the velocity perpendicular to the large-scale field remains unchanged up to $t=t_c$, so that it is just $w_0\sin\theta_0$. The component parallel to the field is, therefore, just $w_0\sin\theta_0\cot\theta_c$ when $t=t_c$.

Following penetration of the pulse, and scattering to some new pitch, say $\theta=\theta_1$, the particle is decelerated behind the pulse during the remaining time $t_c<t<L/C'$. The perpendicular velocity w_n remains unchanged from $w_{nc}\sin\theta_1/\sin\theta_c$. The angle of pitch increases according to (32), or

$$\theta(\theta_1,t,t_c)=\arctan[(t/t_c)\tan\theta_1].$$

The parallel velocity w_s decreases from $w_{sc}\cos\theta_1/\cos\theta_c$ at time $t=t_c$ to

$$\begin{aligned}w_s(\theta_1,t,t_c)&=w_n\cot\theta(\theta_1,t,t_c)\\&=w_{nc}\cot\theta(\theta_1,t,t_c)\sin\theta_1/\sin\theta_c.\end{aligned}$$

at some subsequent time t. Thus, by the time that $t=L/C'$, we have a total particle velocity of

$$\begin{aligned}w&=w_0\sin\theta_0\sin\theta_1/\sin\theta_c\sin\theta(\theta_1,L/C',t_c)\\&=w_0\frac{\sin\theta_0\cos\theta_1}{\sin\theta_c}\left[\left(1-\frac{\tan\theta_c}{\tan\theta_0}\right)^2+\tan^2\theta_1\right]^{\frac{1}{2}}.\end{aligned}$$

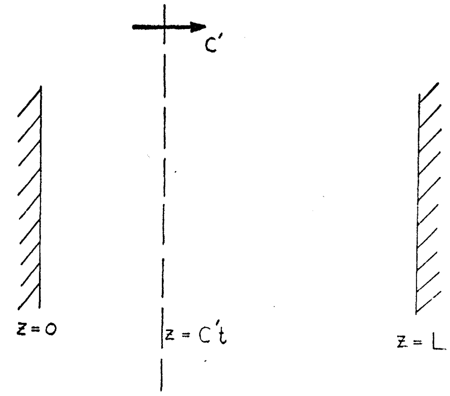

FIG. 2. Equivalent acceleration cell. Between the reflecting barriers at $z=0$ and $z=L$ we have the plane transverse hydromagnetic pulse at $z=C't$. Penetration of the pulse is possible only for particles with angle of pitch θ less than θ_c.

The mean of the velocity over θ_1 is

$$\langle w\rangle(\theta_1)=w_0\frac{\sin\theta_0}{\sin\theta_c}\int_{\theta_c}^{\pi/2}d\theta_1\cos\theta_1\psi(\theta_1)\times\left[\left(1-\frac{\tan\theta_c}{\tan\theta_0}\right)^2+\tan^2\theta_1\right]^{\frac{1}{2}}.$$

If $f(\theta_0)$ is the initial distribution of the angles of pitch (when $t=0$), then the mean final velocity is

$$\langle w\rangle(\theta_0,\theta_1)=\frac{w_0}{\sin\theta_c}\int_{\theta_c}^{\pi/2}d\theta_0 f(\theta_0)\sin\theta_0\times\int_{\theta_c}^{\pi/2}d\theta_1\psi(\theta_1)\cos\theta_1\times\left[\left(1-\frac{\tan^2\theta_c}{\tan^2\theta_0}\right)^2+\tan^2\theta_1\right]^{\frac{1}{2}}. \quad (35)$$

Thus far we have said nothing of the particles initially in $\theta<\theta_c$, or of the particles scattered into $\theta<\theta_c$ as they penetrate the pulse. We shall avoid this unnecessary complication by restricting ourselves to a pulse of large amplitude. Then $\theta_c\ll\pi/2$ and the number of particles in $\theta<\theta_c$, being $O^2(\theta_c)$, may be made vanishingly small.

If we suppose that the initial distribution is isotropic, and that the scattering by the pulse is also isotropic, so that

$$f(\theta)=\psi(\theta)=\sin\theta,$$

then

$$\langle w\rangle(\theta_0,\theta_1)\sim\frac{w_0}{\sin\theta_c}\int_0^{\pi/2}d\theta_0\sin^2\theta_0\int_0^{\pi/2}d\theta_1\sin\theta_1$$

in the limit as $\theta_c\to 0$. This follows from the fact that $(1-\tan\theta_c)$ approximates to unity everywhere in $(0,\pi/2)$ except in the vicinity of $\theta_0=\theta_c$; but there the contribution to the integral is negligible because of the factor $\sin^2\theta_0$. Thus

$$\langle w\rangle(\theta_0,\theta_1)\sim\frac{\pi w_0}{4\sin\theta_c}. \quad (36)$$

The mean kinetic energy has increased by the factor $\pi^2/16\theta_c^2$ during the passage of a pulse of large amplitude, $\theta_c\ll 1$. The final velocity distribution has a mean angle of pitch which is *larger* than the $\langle\theta\rangle=1$ for the initial isotropic distribution, because θ increases following scattering to isotropy when $t=t_c$.

We have demonstrated that oppositely moving trains of sharp crested hydromagnetic waves yield large and repeated acceleration of the cosmic-ray particles contained within. The acceleration is effectively first order in v/c, as in (22). Since the acceleration does not depend upon any subtle property of the wave forms, requiring only that they be of large enough amplitude to develop sharp crests (discussed below), we suggest that our acceleration model is a naturally occurring process in any sufficiently perturbed hydromagnetic region of space, and that it dominates other versions of the Fermi mechanism.

We have, in effect, established Fermi's suggestion in his second paper[7] (that the observed cosmic-ray particles are accelerated primarily when trapped between advancing waves) by demonstrating a simple example where the overtaking collisions (when trapped between receding waves) do not cancel the major portion of the head-on collisions.

VI. EFFECTIVENESS OF FERMI MECHANISM

To demonstrate that the continuous-wave version of the Fermi mechanism is sufficiently potent that the observed interstellar motions in the vicinity of the sun may perhaps account for the observed cosmic-ray energy spectrum, let us suppose after Morrison, Olbert, and Rossi[25] that the cosmic-ray particles undergo random walk through interstellar space with a mean step λ across the galactic magnetic field until they escape out of the galactic disk, of thickness h. If the mean fractional energy gain per step is $\Delta W/W$, they were able to show that $\Delta W/W$ must be as large as

$$\Delta W/W=0.6\lambda^2/h^2$$

in order to account for the observed energy spectrum.

We have been able to show that in the presence of trains of hydromagnetic waves with velocity v the fractional energy gain is $O(v/c)$: If all collisions were head on, it is readily shown[7] that $\Delta W/W=2v/c$; we shall put $\Delta W/W=\eta v/c$ and guess that η, which ultimately appears only to the one-half power, may be of the order of 0.5. In a large-scale continuous magnetic field of scale L, a particle with velocity w and radius of gyration R will drift perpendicular to the field along a contour of constant field density with a velocity of the order of wR/L. In a time L/w between reflections from hydromagnetic waves the total drift is $\lambda=O(R)$. It follows that in our case λ is a function of particle energy and the analysis of Morrison *et al.* is not strictly applicable. However, if as an upper limit λ were taken equal to the radius of gyration of a 10^{15}-ev proton in a field of 10^{-5} gauss, *viz.*, 0.3×10^{18} cm or 0.3 light year, we deduce that the usual interstellar velocities of 10 km/sec would suffice to produce the observed cosmic-ray energy spectrum, provided h were 60 light years. Thus, so far as escape out the sides of the galactic disk is concerned, there should be no problem in producing the observed cosmic-ray energy spectrum with the observed interstellar motions.

VII. HYDROMAGNETIC REDISTRIBUTION

Now consider whether the hydromagnetic waves to be found in nature can be expected to develop sufficiently sharp crests as to redistribute the cosmic-ray particle velocities in the manner assumed above.

It is necessary that the magnetic lines of force be deflected appreciably in distances which are less than the Larmor radius fo the cosmic-ray particles; only then can we violate the adiabatic relation (28) and scatter the angles of pitch of the individual particles away from the small values toward which the Fermi acceleration causes them to drift.

Consider the formation of kinks in the magnetic lines of force in a highly conducting medium such as the interstellar gases or a stellar atmosphere. Obviously a sharp angular bend in a large-scale magnetic field is not an equilibrium configuration and represents a hydromagnetic shock wave of one form or another. For simplicity we shall discuss only two special classes of waves: those which contain only longitudinal motions,[26,33,34] wherein the mass motions are all parallel to the direction of propagation of the shock front; and those which contain only transverse motions,[26,33,34] wherein the material flow is essentially incompressible and represents an Alfvén-type hydromagnetic wave.

A. Longitudinal Waves

So far as the purely longitudinal wave is concerned, the reader is referred to the rather extensive literature[35–37] for a description of their macroscopic properties. So far as the tendency to grow a sharp front is concerned, one may think of such a shock wave as an ordinary hydrodynamic shock, which incidentally carries with it whatever magnetic fields are present according to the usual hydromagnetic equation,

$$\partial\mathbf{B}/\partial t=\nabla\times(\mathbf{v}\times\mathbf{B}),$$

in a fluid medium with velocity $\mathbf{v}$. Any compressional wave will eventually steepen into a shock front simply because the velocity of propagation is slightly higher in the warmer gases behind the wave front.

Petschek[38] has pointed out that in a tenuous plasma the thickness of the front of such a hydromagnetic shock wave propagating more or less perpendicular to the large-scale magnetic field $\mathbf{B}$ will be of the same order as the radius of gyration of the thermal ions in $\mathbf{B}$. Thus the shock front will have a thickness of the order of the radius R, where

$$R=Mu_nc/ZeB,$$

rather than the much larger mean free path. Here M is the ion mass and u_n is the component of the mean thermal velocity, u, perpendicular to $\mathbf{B}$. Since any ion with an energy very much in excess of the thermal energy will have a greater radius of curvature in B than will a thermal ion, and hence a radius greater than the thickness of the shock front, such shock waves will probably always represent kinks in $\mathbf{B}$ which are sufficiently sharp as to redistribute the angle of pitch of accelerated particles.

To see how this works out numerically, we note that the radius of curvature of a 10^4 °K hydrogen ion (16 km/sec) in a galactic arm field of 10^{-5} gauss is 160 km, whereas the mean free path is of the order of 0.01 light year, or 10^{11} km (if the collision radius is 10^{-8} cm and the density is about $N=1$ hydrogen atom/cm^3). The radius of curvature of a cosmic-ray proton exceeds the mean free path of 0.01 light year only if its energy is in excess of 3.3×10^4 Bev; but any proton moving faster than 16 km/sec ($\sim$1 ev) has a radius of curvature in excess of the shock front thickness, $O(R)$.

Shock waves propagating *exactly* parallel to $\mathbf{B}$ are of no interest because they have thick fronts (of the order of the mean free path), and because they do not yield any variation in $\mathbf{B}$. They contribute neither to the acceleration and decrease of θ, nor to the opposing redistribution over θ. We shall not mention them again in our discussion.

B. Transverse Waves

Now a single *transverse* hydromagnetic pulse propagating along a large-scale magnetic field in a highly conducting, incompressible fluid possesses no dispersion and propagates without changing its form.[26] If the wave is of sufficiently small amplitude, its form is preserved even upon meeting and passing through an oppositely moving wave of comparable amplitude. It is observed[39] that the plane of polarization of starlight deviates from the direction of the galactic arm (and hence from the direction of the galactic arm field) by only about ±0.2 radian, suggesting that the distortions of the galactic arm field are hydromagnetic waves of small amplitude.[40,41] However, though it is possible that they may be waves of small amplitude so far as the distortion of the magnetic field is concerned, their 7-km/sec material velocities[42,43] are often rather supersonic; in the observed cool H_I regions the temperature may be of the order of 100°K, corresponding to a thermal velocity of only 1.6 km/sec. Therefore, compressibility effects are probably not negligible.

The propagation of an infinite, plane, transverse, hydromagnetic wave in a slightly *compressible* medium is treated by perturbation methods in the Appendix. As one would expect, the slight rarefaction in the region of

[33] C. F. Sommerfeld, *Mechanics of Deformable Bodies* (Academic Press, Inc., New York, 1950), p. 147.
[34] H. Lamb, *Hydrodynamics* (Dover Publications, New York, 1945), pp. 31, 202.
[35] F. de Hoffmann and E. Teller, Phys. Rev. **80**, 692 (1952).
[36] H. L. Helfer, Astrophys. J. **117**, 177 (1953).
[37] R. Lüst, Z. Naturforsch. **8a**, 277 (1953); **10a**, 125 (1955).
[38] H. E. Petschek, *Third Symposium on Cosmical Gas Dynamics*, June 24–28 (1957) Smithsonian Institution, Astrophysical Observatory, Cambridge, Massachusetts (to be published).
[39] W. A. Hiltner, Astrophys. J. **109**, 471 (1949); **114**, 241 (1951).
[40] L. Davis, Phys. Rev. **81**, 890 (1951).
[41] S. Chandrasekhar and E. Fermi, Astrophys. J. **118**, 113 (1953); **118**, 116 (1953).
[42] A. Blaauw, Bull. Astron. Soc. Netherlands **11**, 405 (1952).
[43] H. C. van de Hulst, *Third Symposium on Cosmical Gas Dynamics*, June 24–28 (1957) Smithsonian Institution, Astrophysical Observatory, Cambridge, Massachusetts (to be published).

strongest field leads to a higher velocity of propagation within the wave and a steepening of the wave front. Again it is not the mean free path but the radius of curvature of the thermal ion trajectories which limits the sharpness of the kink.

Since, as will be discussed later, the Mach number of the observed material motions in regions active enough to perhaps accelerate cosmic rays (galactic halo, novae, solar flares, etc.) is at least as large as unity, we suggest that the magnetic kink associated with each wave usually involves bending the lines of force through an angle of the order of unity.

Thus in both the longitudinal and transverse cases we expect to find the waves possessing magnetic kinks, which are sharp as compared to the radius of curvature of the trajectories of accelerated particles. We expect that the waves have sufficiently sharp fronts or crests to accomplish the necessary redistribution of the cosmic-ray particle velocities.

VIII. COSMIC-RAY GAS

A. Effective Mach Number

One may think of the cosmic-ray particles in our galaxy as the atoms of an extremely hot gas. This cosmic-ray gas is so hot that its thermal motions are relativistic, and it is unable to cool except by nuclear collision with the ordinary non-cosmic-ray matter.[44] Pickelner and Shklovsky[45] have pointed out that infinite plane shock fronts propagating in a direction perpendicular to the magnetic field must compress the cosmic-ray gas (and also the magnetic field), in addition to the ordinary gas. Thus the effective Mach number of such a shock wave must be computed using the speed of sound in the composite gas, consisting of the ordinary gas, of density ρ and thermal velocity u, together with the cosmic-ray gas of pressure p. The composite speed of sound is of the order of $[(\rho u^2+p)/\rho]^{\frac{1}{2}}$, and is in many cases very much larger than the ordinary thermal velocity u. Therefore the effective Mach number may be very much smaller than one would estimate using the speed of sound in the ordinary matter; the expected dissipation of the shock wave is very much reduced by the high speed of sound in the composite gas.

The basis for the suggestion of Pickelner and Shklovsky is the inference[45,46] from 21-cm radio observations that the cosmic-ray gas pressure p in the halo is about the same as in the galactic arm, *viz.*, $p\cong 10^{-12}$ dyne/cm^2, and that much of the ordinary hydrogen is un-ionized and near 100°K. Taking a mean halo density of $N=0.01$ hydrogen atom/cm^3, we obtain the velocity of sound in the ordinary matter as 2 km/sec. But for propagation perpendicular to the magnetic field one uses the speed of sound in the composite gas, which is of the order of 80 or 100 km/sec. Thus the observed 100-km/sec mass motions in such regions are not immensely supersonic when perpendicular to the magnetic field, and their dissipation may not, therefore, be as excessive as the conventional estimate of Mach 40 or 50 might indicate.

Now one observes continued violent disordered mass motions, far in excess of the thermal velocities, in the atmospheres of Wolf-Rayet stars, class B emission stars, etc.; in the envelopes of novae and supernovae such as the Crab Nebula and elsewhere[47]; in the Coma cluster and in the halos of $M31$, $M33$, etc.[48]; one infers continued disordered mass motions of 100 km/sec in the halo $\mathrm{H_I}$ regions of our own galaxy. We wish to extend the suggestion of Pickelner and Shklovsky[45] that the coupling exists in the two directions perpendicular to the magnetic field. We would like to suggest that in such regions the disordered motions of the ordinary matter are coupled to the cosmic-ray gas in all *three* directions.

Now it is certainly true that a purely longitudinal shock wave of ordinary matter propagating *exactly* parallel to a static uniform magnetic field will not be coupled to the cosmic-ray gas. But the disordered mass motions produce disordered magnetic fields, and motions are never exactly longitudinal, with the result that this condition is never realized.

The disordered mass motions may be looked upon as randomly moving hydromagnetic waves of large amplitude, involving *both* longitudinal and transverse motions. The interaction of cosmic-ray particles with transverse hydromagnetic waves in the ordinary matter has been discussed quantitatively in Sec. V: The cosmic-ray particles are reflected from the denser magnetic fields carried by the transverse waves; the transverse hydromagnetic waves are cosmic-ray barriers which tend to sweep the cosmic-ray gas ahead of them as they move. Therefore, in any region where there are large numbers of disordered hydromagnetic waves, the cosmic-ray gas is strongly coupled along $\mathbf{B}$ to the ordinary matter making up the waves. The cosmic-ray gas is thus coupled to the ordinary mass motions in all *three* directions.

If we care to look further into the details of the interaction of the cosmic-ray gas and the ordinary matter parallel to the magnetic field, then we may use the results derived elsewhere[6] for the motion of

[44] It is interesting to note, though probably of no importance, that we, armed with our electronic particle counters, assume the unaccustomed role of Maxwell demons in the tenuous, relativistic cosmic-ray gas.

[45] S. B. Pickelner and I. S. Shklovsky, *Third Symposium on Cosmical Gas Dynamics*, June 24–28 (1957) Smithsonian Institution, Astrophysical Observatory, Cambridge, Massachusetts (to be published).

[46] G. R. Burbidge, Phys. Rev. **101**, 907 (1956).

[47] R. Minkowski, *Third Symposium on Cosmical Gas Dynamics*, June 24–28 (1957) Smithsonian Institution, Astrophysical Observatory, Cambridge, Massachusetts (to be published).

[48] D. S. Heeschen, Astrophys. J. **124**, 662 (1956); *Third Symposium on Cosmical Gas Dynamics*, June 24–28, 1957, Smithsonian Institution, Astrophysical Observatory, Cambridge, Massachusetts (to be published).

the ions of a tenuous gas along a large-scale magnetic field. In general we expect, in the presence of a magnetic field, no purely longitudinal motions, but rather always a mixture with the transverse. The transverse motions represent variations in the magnetic field, and, except in trivial special cases, will always couple to the cosmic-ray gas as just discussed. But suppose, just to be contrary, that somehow we have on our hands a purely longitudinal (acoustical) wave, given by

$$\mathbf{v}=(\mathbf{k}/k)v_0 f(\mathbf{k}\cdot\mathbf{r}-\omega t),$$

where $\mathbf{k}$ is the wave vector, ω is the angular frequency, $C(=\omega/k)$ is the velocity of sound (not the hydromagnetic velocity as in earlier sections) in ordinary matter, and $\mathbf{r}$ is the position vector. Suppose further that this acoustical wave is of small amplitude, i.e., $v_0\ll C$, and that it is of smaller scale than the larger hydromagnetic waves which, as pointed out above, may form cosmic-ray barriers. Finally we shall suppose that the scale of the acoustical wave, though small compared to the large-scale field, is nonetheless large compared to the radius of gyration of the cosmic-ray particles in the large-scale field; thus the cosmic-ray particles move adiabatically through the wave. Then, if this acoustical wave in the ordinary matter can interact strongly with the cosmic-ray gas, we have reason to believe that so will most any other wave in the ordinary matter.

Since the acoustical wave is of small scale, we may regard the unperturbed magnetic field as uniform across the wave. We set up a local Cartesian coordinate system with z axis along the large-scale field

$$\mathbf{B}_0=\mathbf{e}_z B_0,$$

and orient the x and y axes so that the wave vector reduces to

$$\mathbf{k}=k(\mathbf{e}_y\sin\chi+\mathbf{e}_z\cos\chi).$$

χ is the angle between $\mathbf{B}$ and $\mathbf{v}$; the unit vectors along the coordinate axes are $\mathbf{e}_x$, $\mathbf{e}_y$, and $\mathbf{e}_z$. It is readily shown from

$$\partial\mathbf{B}/\partial t=\nabla\times(\mathbf{v}\times\mathbf{B})$$

that as a result of the acoustical wave the magnetic field becomes

$$B_y=-B_0(v_0/C)\sin\chi\cos\chi f(\xi),$$
$$B_z=+B_0[1+(v_0/C)\sin^2\chi f(\xi)],$$

when one omits terms of second order in v_0/C. We have put

$$\xi=ky\sin\chi+kz\cos\chi-\omega t.$$

To the same order, it is readily shown that the magnitude of the total field $\mathbf{B}$ is

$$B(y,z,t)=B_0[1+(v_0/C)\sin^2\chi f(\xi)].$$

Thus the cosmic-ray gas sees only a small variation in field density amounting to $B_0 O(v_0/C)$.

The velocity of sound C is, of course, extremely small (only 2 km/sec in $\mathrm{H_I}$ regions) as compared to the velocity (of the order of c) of the cosmic-ray particles. Thus we may in first approximation neglect the motion of the acoustical wave and limit our inquiry to the effect of the variation in field density on the component of the cosmic-ray motion parallel to $\mathbf{B}$. We let θ represent the angle between $\mathbf{B}$ and the individual cosmic-ray particle velocity. Neglecting relativistic effects to obtain an order-of-magnitude result, we find[6] that if $F(z,\theta)$ is the velocity distribution function per unit volume for the cosmic-ray particles spirally along the lines of force of $\mathbf{B}$, then $F(z,\theta)$ may be constructed from a linear sum of terms of the form $[B(0,y,t)/B(z,y,t)]^{\frac{1}{2}(\alpha-1)}\sin^\alpha\theta$. Noting that $\alpha=1$ represents isotropy, and supposing that the effective α is probably somewhat less than unity in a region where cosmic rays are being accelerated, we see that the cosmic-ray gas density tends to increase in regions where $\mathbf{B}$ is denser (since the converging lines of force tend to concentrate the particles spiraling along them). The cosmic-ray gas density increases as $B^{\frac{1}{2}(1-\alpha)}$ or as $[1+\frac{1}{2}(1-\alpha)(v_0/C)\sin^2\chi f(\xi)]$. The density of ordinary matter increases by the factor $[1+(v_0/C)f(\xi)]$. Thus, except for the special cases that $\alpha=1$ and $\chi=0$, the perturbation in the cosmic-ray gas is of the same order as in the ordinary gas. Even small and purely longitudinal waves of small amplitude perturb the motion of the cosmic-ray particles *along* $\mathbf{B}$ to such an extent that the cosmic-ray gas density is affected to the same order as the ordinary gas density. The correct treatment of such a wave, therefore, must be carried out in the composite ordinary and cosmic-ray gas.

Now the cosmic-ray gas pressure is of the order of 10^{-12} dyne/cm^2 in the galactic arms and halo.[45,46] The ordinary gas pressure in an interstellar $\mathrm{H_I}$ region, where the temperature is 100°K, and the density is $N\cong10$ hydrogen atoms/cm^3, is 0.14×10^{-12} dyne/cm^2. In the galactic halo $\mathrm{H_I}$ regions, where the temperature is perhaps no more than 100°K and $N\cong0.01$ hydrogen atom/cm^3, the ordinary gas pressure is only 0.0014 $\times10^{-12}$ dyne/cm^2. Therefore, the cosmic-ray gas pressure p is the dominant gas pressure throughout many regions of the galaxy. It may also be the dominant pressure in sufficiently active stellar atmospheres, etc. which we will discuss more generally a little farther on. Our point in this section is that the cosmic-ray gas pressure in active regions of space (coupled in all three dimensions to the ordinary matter) is, in many cases, the dominant pressure and must be included in any dynamical calculations.

In particular the cosmic-ray gas must be included in calculations of effective Mach number; the effective speed of sound is of the order of $[(\rho u^2+p)/\rho]^{\frac{1}{2}}$. Many otherwise temendously supersonic mass motions become sonic, with corresponding reduction in the theoretical dissipation. Thus it need not be paradoxical that we seem to observe turbulent mass motions in excess of the ordinary thermal velocities.

B. Irreversible Compression

We have suggested that, in about the same length of time as it takes two approaching hydromagnetic waves to sweep up and compress the cosmic-ray gas caught between them, there will be a redistribution of the cosmic-ray particle velocities by the sharp crests of the waves. Thus, on the whole, the cosmic-ray velocity distribution will not differ by a whole order of magnitude from isotropy, but, on the other hand, we do expect to find some not insignificant deviations from isotropy (see Sec. V). Therefore, the compression and expansion of the cosmic-ray gas, as it is alternately caught between approaching and receding hydromagnetic waves, is significantly irreversible, which, as is well known, leads to a marked heating of the gas. Thus the continuous wave version of the Fermi mechanism, developed in Sec. V, is equivalent to heating by irreversible compression and expansion.

C. Viscosity

The viscosity of the galactic cosmic-ray gas is not negligible.[49] Let us take the expression, $Nmul$ from elementary kinetic theory as a measure of the viscosity. Here m is the mass of the individual gas atom or molecule, u the thermal velocity, and l the mean free path. N is the number of atoms per unit volume, so that Nm is the total mass density of the gas. The mean free path of a cosmic-ray gas particle is essentially the distance between hydromagnetic waves. We take this to be 20 parsecs[42,43] or more in the galactic arm and 100 parsecs in the halo.[46] Putting $u=c$, $m=1.66\times10^{-24}$ g, and $N=10^{-9}$ cosmic-ray particle/cm^3 throughout the galaxy and its halo, we have a viscosity of the order of at least 3×10^{-3} g/sec cm in the galactic arms and 15×10^{-3} in the halo. By comparison we note that the same expression $Nmul$ yields about 1.0×10^{-3} for ordinary hydrogen at 100°K and a collision cross section of 3×10^{-16} cm^2. Thus the cosmic-ray gas is not negligible and may, in many cases, be a not insignificant form of dissipation of mass motions.

We note that the cosmic-ray energy density is of the order of 10^{-12} erg/cm^3. A material density of $N=1$ particle/cm^3 (in the galactic arms) yields a life of about 10^7 years for the heavy nuclei, requiring an energy input of 3×10^{-27} erg/cm^3 sec. A density of $N=0.01$ particle/cm^3 (the mean density of matter throughout the halo) requires an input of 3×10^{-29} erg/cm^3. Most of the input may come from irreversible compression, but on the other hand the heating of a gas of viscosity η due to shearing velocities v with a scale of L is of the order $\eta v^2/L^2$. Using the previously estimated values for η, we find an input of 1.6×10^{-30} erg/cm^3 sec in the galactic arm where $v\cong7$ km/sec and $L\cong10$ parsecs, and 1.6×10^{-29} in the halo where $v=100$ km/sec and $L\cong100$ parsecs.[6,16] Thus, at least in the halo, the acceleration of cosmic rays by the viscous heating may not be negligible. And perhaps the heating by irreversible compression and expansion may not need to be much greater than the viscous heating to supply the necessary total 3×10^{-29} erg/cm^3.

[49] The author is indebted to R. Schlüter on this point.

IX. MACH-ONE EFFECT

From the radio observations[48] of the halos of other galaxies, and from general theoretical considerations of the halo of our own,[45,46] we conclude that the galactic halo possesses disordered mass motions of the order of 100 km/sec; the composite sound velocity in the cosmic-ray and ordinary halo gas is of the order of 80 km/sec. In the galactic arm the mass motions have a mean value of 7 or 8 km/sec[42,43]; the composite sound velocity at 100°K with $N\cong1$ particle/cm^3 is readily shown to be about 8 km/sec. Thus we see that throughout the galaxy, in both the halo and the arms, the mass motions are of the order of Mach one.

We speculate that Mach one may not be a coincidence, or simply a matter of limitation of the motions by shock dissipation. We suggest that it represents a dynamical balance common to any region of space in which the tenuous matter is stirred sufficiently violently. Suppose that in some large enclosed region of space there is a tenuous conducting gas bearing a magnetic field. We introduce large amounts of energy in the form of disordered mass motion. The motions throughout the space will be immensely supersonic, quickly going over into hydromagnetic shock wave phenomena. Acceleration of cosmic rays will begin and the intensity of the comsic-ray field will increase without bound so long as the hydromagnetic waves can maintain sharp fronts or crests. If the gas is sufficiently tenuous, then there is no stopping of cosmic-ray particles, and sooner or later the cosmic-ray gas pressure will be increased to where the composite speed of sound becomes comparable to the mass velocities. Then a delicate dynamical balance may be set up, wherein an increase in cosmic-ray gas pressure reduces the effective Mach number below one and the sharp crests disappear. The cosmic-ray acceleration ceases until the Mach number is restored to one by the decaying cosmic-ray gas pressure. Thus a balance at Mach one is obtained *no matter how large is the energy input* to the mass motions.

There are many expections to this principle, of course. Obviously it will not obtain in a medium which is so dense that cosmic-ray particle acceleration is not possible; that is why we specified a region filled with tenuous matter. Obviously it will not obtain in a small active region from which cosmic-ray particles can leak too readily; that is why we specified a closed region of large extent. But we suggest that at least the galaxy as a whole (and perhaps many smaller objects such as a solar flare, the atmosphere of an active star, or a supernova shell, etc.) is sufficiently large and sparsely filled that the principle is operative.

In terms of the statistical velocity distribution function, the Mach one effect manifests itself as a double-humped or camel-backed distribution function, as sketched in Fig. 3. The bunching at low velocities represents the thermal and mass motions in the ordinary matter; the maximum just below c represents the cosmic-ray gas. The Mach one effect states that the kinetic energies contained under the two maxima are approximately equal. Presumably the effect could be formulated from the classical Boltzmann equation for sufficiently tenuous active regions of space.

In order to understand where we might expect to find the Mach-one effect, consider the critical conditions which allow the initiation of cosmic-ray velocities from the ordinary thermal velocity distribution to be expected in an active region of space. It was estimated elsewhere[6] that the distance λ over which a proton of mass M and velocity w will lose about half its energy through Coulomb interaction with the surrounding plasma,[50] of N hydrogen ions per unit volume and thermal velocity u, is of the order of

$$\lambda(w)\cong 3M^2w^6/\{32\pi Ne^4u^2 \ln[M^{\frac{3}{2}}w^2u/4(3\pi N)^{\frac{1}{2}}e^3]\}. \quad (37)$$

If we hope to start cosmic rays from near the thermal velocity u, where the loss to an accelerated particle is greatest, we must have an energy input sufficient to overcome the losses corresponding to a relaxation distance $\lambda(u)$.

In the presence of hydromagnetic waves of large amplitude propagating with velocity C, it was shown in Sec. V that the mean fractional energy gain per collision is of the order of C/w (for a single nonrelativistic head on collision it is $4C/w$). Thus in order that acceleration be effective it is sufficient to require that the distance l between collisions be not more than the critical value l_c, where

$$l_c/\lambda(w)=O(C/w).$$

For simplicity we suppose that as a consequence of shock phenomena the ordinary thermal velocity is of the same order as C. Then, very roughly, we must have $w=u$ and

$$l_c=\lambda(u).$$

Numerically this becomes

$$l_c\cong 1.5\times 10^{-12}(u^4/N)[\ln(u^3/N^{\frac{1}{2}})-20.3]^{-1}. \quad (38)$$

An H_{II} halo region, where $u\cong 16$ km/sec (10^4 °K) and $N\cong 0.01$ particle/cm^3, leads to $l_c\cong 4\times 10^{13}$ cm or 2 a.u.

In an interstellar H_I region where $T\cong 100$°K and the degree of ionization is extremely low, it is probably more correct to use the elementary free path $1/(\pi r^2 N)$,

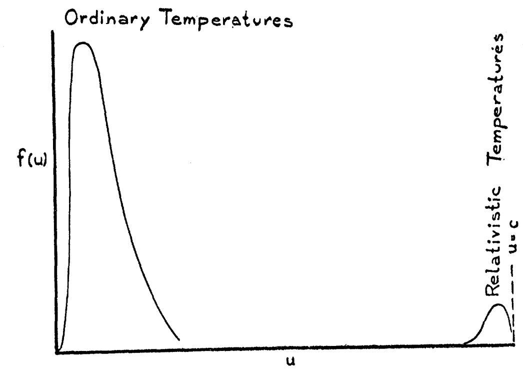

FIG. 3. A schematic velocity distribution function $f(u)$ *versus* the particle velocity u when the Mach one effect is operative. The maximum at lower velocities represents the thermal motions and the turbulent mass motions. The maximum at relativistic velocities represents the cosmic-ray gas. The Mach-one effect states that the kinetic energy under the two maxima should be approximately equal.

where r is the collision radius and $\pi r^2\cong 10^{-16}$ cm^2. Then $l_c\cong 10^{16}/N$ cm. For an H_I region in the galactic arm, where $N\cong 1$, l_c is 0.01 light year. An H_I region in the halo, where $N\cong 0.01$/cm^3, yeilds $l_c\cong 1$ light year.

All these distances are rather less than the expected scale of the hydromagnetic waves. But, on the other hand, 1000 km/sec in the expanding shell of a supernova where $N=10^4$ particles/cm^3 gives $l_c=5\times 10^9$ km or 30 a.u., and 1000 km/sec in an active stellar atmosphere where $N=10^{10}$ requires that $l_c=6000$ km. Therefore, it appears that we may expect cosmic-ray particles to originate in the vicinity of active stars,[6,23,24,51] since we may expect hydromagnetic waves with scales less than l_c. We may not expect to find them originating elsewhere.

As cosmic-ray particles are being accelerated from the initial thermal velocities of the ordinary matter up into the relativistic range, the dissipation length increases rapidly (as the sixth power of the particle velocity) until it reaches an asymptotic value of the order of $10^{26}N$ cm for nuclear collisions. The distance $\lambda(w)$ then has only a small effect, and acceleration proceeds easily in most any active region of space. The heating of the cosmic-ray gas begins, therefore, with irreversible compression in stellar activity of one kind or another, and upon reaching near relativistic temperatures may be supplemented by general galactic acceleration (irreversible compression and perhaps viscous dissipation).

We conclude, therefore, that large cosmic-ray pressures are to be looked for in the atmospheres of very active stars and novae, or in any larger, active, more or less closed, system (such as the galactic arms and halo) containing a sufficient number of violent stars for injection. It is in such closed active regions that cosmic-ray gas pressures will build up, limiting the effective Mach number to the order of unity, as discussed above. In this manner we can perhaps

[50] An extensive treatment of the general interactions which take place between a charged particle and the plasma through which it is moving has been carried out by S. Hayakawa and K. Kitao [Progr. Theoret. Phys. Japan 16, 139 (1956)].

[51] S. Hayakawa, Progr. Theoret. Phys. Japan 15, 111 (1956).

understand the observation that in the atmospheres of extremely active stars, in interstellar space, and in the galactic halo there are turbulent mass motions very much in excess of the ordinary speed of sound.

ACKNOWLEDGMENTS

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APPENDIX

Consider the wave solution of the hydromagnetic equations

$$\partial\mathbf{B}/\partial t=-(\mathbf{v}\cdot\nabla)\mathbf{B}+(\mathbf{B}\cdot\nabla)\mathbf{v}-\mathbf{B}\nabla\cdot\mathbf{v}, \tag{1a}$$

$$\partial\mathbf{v}/\partial t=-(\mathbf{v}\cdot\nabla)\mathbf{v}-(1/\rho)[\nabla(p+B^2/8\pi)-(\mathbf{B}\cdot\nabla)\mathbf{B}/4\pi], \tag{2a}$$

in an inviscid, infinitely conducting fluid containing a uniform magnetic field parallel to the z axis,

$$\mathbf{B}_0=\mathbf{e}_zB_0. \tag{3a}$$

$\mathbf{e}_x$, $\mathbf{e}_y$, and $\mathbf{e}_z$, represent unit vectors parallel to the x, y, and z axes, respectively. We suppose that p_0 is the uniform hydrostatic pressure when the system is unperturbed, and that the fluid satisfies the equation of state

$$p=p_0(\rho/\rho_0)^\gamma. \tag{4a}$$

We define the parameter ϵ as

$$\epsilon=B_0^2/8\pi p_0, \tag{5a}$$

and consider solution of (1) and (2) for a single hydromagnetic wave pulse propagating along $\mathbf{B}_0$ (with velocity $C=B_0/(4\pi\rho_0)^{\frac{1}{2}}$), neglecting all terms $O^2(\epsilon)$ and smaller. Thus, we shall consider the compressibility of the medium as a perturbation on the rigorous incompressible hydromagnetic wave

$$\mathbf{B}=\mathbf{B}_0+\mathbf{b}_0, \tag{6a}$$

$$\mathbf{v}=\mathbf{v}_0, \tag{7a}$$

where

$$\mathbf{b}_0=\mathbf{b}_0(x,y,z\pm Ct), \tag{8a}$$

$$\mathbf{v}_0=\pm\mathbf{b}_0/(4\pi\rho_0)^{\frac{1}{2}}. \tag{9a}$$

Note that (8a) and (9a) constitute rigorous solutions of (1a) and (2a) regardless of whether $|\mathbf{B}_0|/|\mathbf{b}_0|$ is large or small. It is the compressibility, and not the wave amplitude, which we are treating as a small perturbation.

Substituting (8a) and (9a) into (2a), taking the divergence, and remembering that the zero-order incompressible velocity satisfies $\nabla\cdot\mathbf{v}_0=0$, we obtain the auxiliary condition that

$$\nabla^2(p+B^2/8\pi)=0.$$

Since we regard p and B uniform at infinity with the values p_0 and B_0, it follows that $p^2+B^2/8\pi$ is uniform everywhere. We let p_1 represent the local variation in the pressure as a consequence of the hydromagnetic wave, obtaining

$$p_1+(\mathbf{b}_0\cdot\mathbf{b}_0+2\mathbf{B}_0\cdot\mathbf{b}_0)/8\pi=0. \tag{10a}$$

We see that p_1 is of the order of ϵp_0. We shall now compute how the density variations ρ_1 arising from the pressure fluctuation p_1 modify the propagation of the wave.

We let

$$\begin{aligned}\mathbf{B}&=\mathbf{B}_0+\mathbf{b}_0+\mathbf{b}_1+\cdots,\\ p&=p_0+p_1+p_2+\cdots,\end{aligned} \tag{11a}$$

etc., where the subscript denotes the order in ϵ. Equation (4a) becomes

$$p_1=\gamma p_0(\rho_1/\rho_0)[1+O(\epsilon)]. \tag{12a}$$

The equation of continuity becomes

$$\partial\rho_1/\partial t+\mathbf{v}_0\cdot\nabla\rho_1+\rho_0\nabla\cdot\mathbf{v}_1=O^2(\epsilon). \tag{13a}$$

Using (10a) and (12a) to eliminate ρ_1 and p_1 from (13a) leads to

$$\left[\frac{\partial}{\partial t}\pm\frac{\mathbf{b}_0\cdot\nabla}{(4\pi\rho_0)^{\frac{1}{2}}}\right](\mathbf{b}_0\cdot\mathbf{b}_0+2\mathbf{B}_0\cdot\mathbf{b}_0)=8\pi\gamma p_0\nabla\cdot\mathbf{v}_1. \tag{14a}$$

We have used (9a) to replace $\mathbf{v}_0$ by $\mathbf{b}_0$, and shall continue this practice whenever $\mathbf{v}_0$ appears below. The left-hand side of (14a) involves only the zero-order wave solutions, and is presumed known. Hence (14a) serves to determine $\nabla\cdot\mathbf{v}_1$.

Substituting (11a) into (1a) and (2a), and making use of the fact that

$$\partial\mathbf{b}_0/\partial t=\pm C\partial\mathbf{b}_0/\partial z,$$

we ultimately obtain

$$\begin{aligned}\partial\mathbf{b}_1/\partial t=&\mp(\mathbf{b}_0\cdot\nabla)\mathbf{b}_1/(4\pi\rho_0)^{\frac{1}{2}}-(\mathbf{v}_1\cdot\nabla)\mathbf{b}_0\\&+B_0\partial\mathbf{v}_1/\partial z+(\mathbf{b}_0\cdot\nabla)\mathbf{v}_1\pm(\mathbf{b}_1\cdot\nabla)\mathbf{b}_0/(4\pi\rho_0)^{\frac{1}{2}}\\&-(\mathbf{B}_0+\mathbf{b}_0)\nabla\cdot\mathbf{v}_1+O^2(\epsilon),\end{aligned} \tag{15a}$$

$$\begin{aligned}\partial\mathbf{v}_1/\partial t=&\mp[(\mathbf{b}_0\cdot\nabla)\mathbf{v}_1+(\mathbf{v}_1\cdot\nabla)\mathbf{b}_0]/(4\pi\rho_0)^{\frac{1}{2}}\\&+(1/4\pi\rho_0)\{-\nabla(\mathbf{B}_0\cdot\mathbf{b}_1+\mathbf{b}_0\cdot\mathbf{b}_1+p_2)\\&+[B_0\partial\mathbf{b}_1/\partial z+(\mathbf{b}_0\cdot\nabla)\mathbf{b}_1+(\mathbf{b}_1\cdot\nabla)\mathbf{b}_0]\\&-(\rho_1/\rho_0)[(\mathbf{B}_0+\mathbf{b}_0)\cdot\nabla]\mathbf{b}_0\}+O^2(\epsilon).\end{aligned} \tag{16a}$$

Using (10a) and (12a) to express ρ_1 in terms of $\mathbf{B}_0$ and $\mathbf{b}_0$, we see that (15a) and (16a) are the general linearized equations for the first-order perturbation fields, $\mathbf{v}_1$ and $\mathbf{b}_1$.

However, though (15a) and (16a) are linear, they are by no means elementary. It will be sufficient for our purposes to consider the special case of a plane transverse wave for which the zero-order field is in the y direction and independent of x and y. Then we let

$$\mathbf{b}_0=\mathbf{e}_y b_0(z-Ct). \tag{17a}$$

It is obvious that $\partial/\partial x=\partial/\partial y=0$ and $\mathbf{b}_1=\mathbf{e}_y b_1$, $\mathbf{v}_1=\mathbf{e}_y v_{1y}+\mathbf{e}_z v_{1z}$ in (15a) and (16a). Hence (14a) reduces immediately to

$$\begin{aligned}8\pi\gamma p_0 \partial v_{1z}/\partial z&=\partial b_0^2/\partial t\\&=-C\partial b_0^2/\partial z.\end{aligned} \tag{18a}$$

Intergrating (18a) over z, we find that v_{1z} is proportional to b_0^2 plus a constant. In order to evaluate the constant we integrate over z, obtaining

$$v_{1z}=\langle v_{1z}\rangle+(C/8\pi\gamma p_0)(\langle b_0^2\rangle-b_0^2), \tag{19a}$$

where the angular brackets denote the mean value over z.

The z component of (16a) serves to determine the second order pressure variation, p_2. The z component of (15a) yields the physically obvious result that $\partial b_{1z}/\partial t=0$. The y components of (15a) and (16a) reduce to

$$\partial b_1/\partial t=B_0\partial v_{1y}/\partial z-(\partial/\partial z)(b_0 v_{1z})+O^2(\epsilon), \tag{20a}$$

$$\begin{aligned}\partial v_{1y}/\partial t=[1/(4\pi\rho_0)^{\frac{1}{2}}][v_{1z}\partial b_0/\partial z+C\partial b_1/\partial z\\-C(\rho_1/\rho_0)\partial b_0/\partial z]+O^2(\epsilon).\end{aligned} \tag{21a}$$

Eliminating v_{1y} between these two equations, using (10a) and (12a) to express ρ_1 in terms of $\mathbf{b}_0$, and noting that v_{1z} and b_0 are both functions only of $z-Ct$, so that $\partial/\partial t$ is equivalent to $-C\partial/\partial z$, we obtain the inhomogeneous wave equation

$$\partial^2 b_1/\partial t^2-C^2\partial^2 b_1/\partial z^2 =C\frac{\partial}{\partial z}\left\{\frac{\partial b_0}{\partial z}\left[2\langle v_{1z}\rangle+\frac{C}{8\pi\gamma p_0}(2\langle b_0^2\rangle-3b_0^2)\right]\right\}. \tag{22a}$$

We have used (19a) to eliminate v_{1z}.

Assuming that the first-order perturbation field vanishes initially, we expect b_1 to grow linearly with time. Consequently we seek a solution of (22a) of the form

$$b_1(z,t)=t\beta(z-Ct). \tag{23a}$$

Substituting into (22a) yields

$$\frac{\partial\beta}{\partial z}=-\frac{1}{2}\frac{\partial}{\partial z}\left\{\frac{\partial b_0}{\partial z}\left[2\langle v_{1z}\rangle+\frac{C}{8\pi\gamma p_0}(2\langle b_0^2\rangle-3b_0^2)\right]\right\},$$

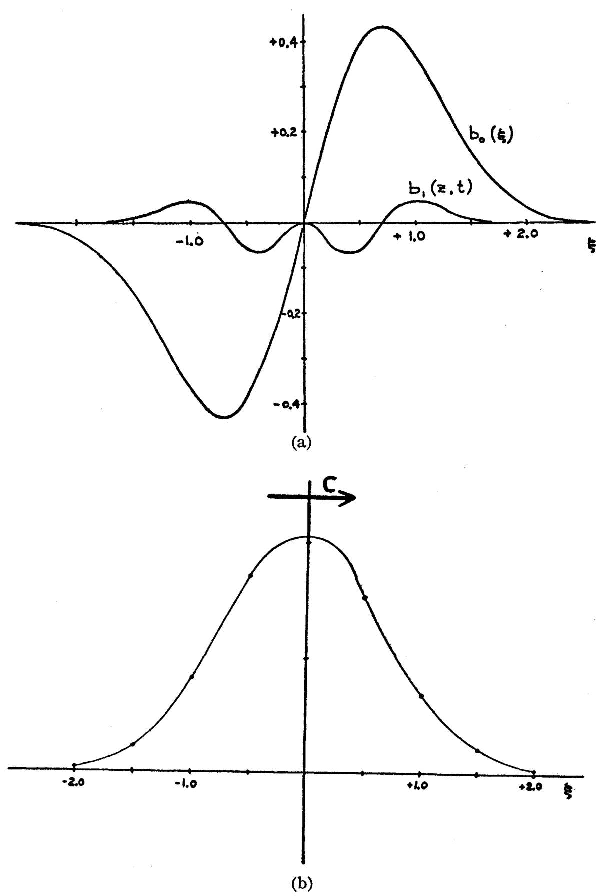

FIG. 4. (a) The incompressible wave field $b_0(\xi)=\xi\exp(-\xi^2)$, and the first-order compressional perturbation field, $b_1(z,t)=\xi^2(1-2\xi^2)\exp(-3\xi^2)$ at time $t=(2a\gamma/3\epsilon C)(B_0/b_0)^2$. (b) A line of force through the wave $b_0(\xi)+b_1(z,t)$.

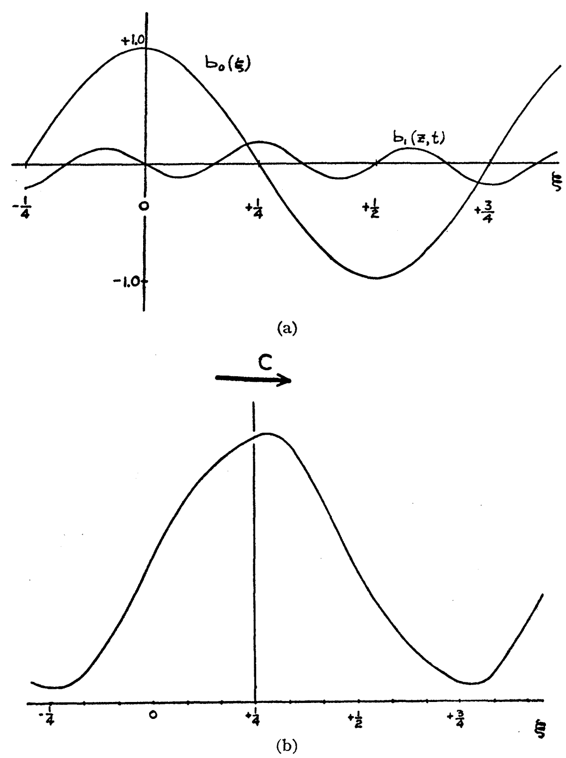

FIG. 5. (a) The incompressible wave field $b_0(\xi)=\cos 2\pi\xi$ and the first-order compressional perturbation field $b_1(z,t)=0.05\times[\sin 2\pi\xi-3\sin 6\pi\xi]$ and time $t=(a\gamma/5\pi\epsilon C)(B_0/b_0)^2$. (b) A line of force through the wave train $b_0(\xi)+b_1(z,t)$.

from which it follows upon integration that

$$b_1(z,t) = -t(\partial b_0/\partial z)\times[\langle v_{1z}\rangle + (C/8\pi\gamma p_0) \times(\langle b_0^2\rangle - \tfrac{3}{2}b_0^2)]. \quad (24a)$$

One may then compute v_{1y} from (21a), which becomes

$$\partial v_{1y}/\partial t = (1/4\pi\rho_0)^{\frac{1}{2}}\Bigg\{(\partial b_0/\partial z)[\langle v_{1z}\rangle + C\langle b_0^2\rangle/8\pi\gamma p_0] + Ct\Bigg[\frac{3C}{8\pi\gamma p_0}b_0\left(\frac{\partial b_0}{\partial z}\right)^2 - \left(\langle v_{1z}\rangle + \frac{C}{8\pi\gamma p_0}(\langle b_0^2\rangle - \tfrac{3}{2}b_0^2)\right)\frac{\partial^2 b_0}{\partial z^2}\Bigg]\Bigg\}. \quad (25a)$$

We consider two special cases. If the zero-order wave has a Gaussian form,

$$b_0(z-Ct) = b_0\xi\exp(-\xi^2),$$

where

$$\xi = (z-Ct)/a,$$

then it is readily shown from (24a) that

$$b_1(z,t) = \tfrac{3}{2}b_0\left(\frac{\epsilon}{\gamma}\right)\left(\frac{Ct}{a}\right)\left(\frac{b_0}{B_0}\right)^2\xi^2(1-2\xi^2)\exp(-3\xi^2).$$

The incompressible wave $b_0(z-Ct)$ and the perturbation $b_1(z,t)$ are plotted in Fig. 4 (a). The resulting deformation of a line of force of $\mathbf{B}_0$ is shown in Fig. 4 (b).

If the zero-order wave is an infinitely long train,

$$b_0(z-Ct) = b_0\cos2\pi\xi,$$

then upon assuming that $\langle v_{1z}\rangle = 0$ we obtain

$$b_1(z,t) = \epsilon b_0\left(\frac{\pi}{4\gamma}\right)\left(\frac{Ct}{a}\right)\left(\frac{b_0}{B_0}\right)^2(\sin2\pi\xi - 3\sin6\pi\xi).$$

The waves $b_0(z-Ct)$ and $b_1(z,t)$ are plotted in Fig. 5(a), and the deformation of a line of force in Fig. 5(b).

The steepening of the wave front is obvious in both Figs. 4(b) and 5(b).

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Isotropy of Pion Emission at 6 Bev*

D. T. King
Department of Physics and Astronomy, The University of Tennessee, Knoxville, Tennessee
(Received September 23, 1957)

Observations have been made of the angular distribution of energetic electron pairs in nuclear emulsions which had been exposed to the internal 6.3-Bev proton beam of the Bevatron. The method by which the pairs were found is discussed. The angular distribution of the pairs is reasonably similar to that of the neutral pions originating in proton-nucleus collisions in the emulsion. Examination of the angular distributions in terms of multiple meson production in nucleon-nucleon collisions indicates that for these observations the emission is consistent with isotropy in the center-of-mass system.

INTRODUCTION

A LARGE body of evidence[1] has been brought forward, largely from experiments with the cosmic radiation, in support of the hypothesis of multiple meson production in nucleon-nucleon collisions. These observations have led to discussions of the angular distribution of the emitted particles in the center-of-mass (c.m.) system of the colliding nucleons. An analysis by Fermi[2] shows that, for off-center collisions of very great energy, the c.m. angular distribution of the emergent pions departs notably from isotropy. Several experiments with particles of energy in the 100-Bev region, where the assumptions of the Fermi theory are valid, support this conclusion.[3–8] On the other hand, observations in the 10–50 Bev region suggest isotropic emission.[9,10] A recent paper,[11] however, reports that the median angles of showers caused by 6-Bev protons are subject to wide fluctuations, and are likely to be lower than expected for isotropic emission. The premises of the Fermi theory hardly apply at this energy for collisions of any impact parameter. It is therefore of interest to examine the emission angles of the neutral pions arising from 6-Bev stars.

The effect of the decay in flight of a neutral pion beam emitted at polar angle θ in the laboratory (lab)

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