Electric-Monopole Directional-Correlation Experiments*

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Angular-correlation experiments involving conversion electrons are sensitive to a small admixture of electric-monopole ($E0$) transitions. The coefficient of $P_2(\cos\theta)$ in the correlation function involving a mixed $E0+M1+E2$ transition contains an interference term between the $E0$ and $E2$ electrons, which is proportional to the ratio of the $E0$ and $E2$ matrix elements. Measurement of this coefficient provides a sensitive means for determining both the magnitude of the monopole matrix element and its phase relative to the $E2$ gamma-ray matrix element. The effect on the directional-correlation function between such K-conversion electrons and a coincident gamma ray is calculated explicitly. Specific numerical results are given for Pt ($Z=78$) over a range of energies, and the feasibility of the method is illustrated with reference to the particular case of the $2+\rightarrow2+\rightarrow0+$ cascade in Pt^{196}.

INTRODUCTION

TRANSITIONS between nuclear levels proceed by the competing processes of gamma-ray emission and internal-conversion-electron ejection.[1] It has long been understood that in the lowest approximation, the internal-conversion mode of de-excitation involves the same nuclear matrix elements as the gamma-ray mode, and therefore contains no further information about nuclear structure.

It has recently[2,3] been emphasized, however, that the rate of internal-conversion-electron ejection is not completely determined if the rate of gamma-ray emission is known. In fact, nuclear matrix elements enter into the expression for the rate of internal conversion which are different from those which determine the rate of gamma-ray emission. The appearance of these new matrix elements for internal conversion is directly related to the finite size of the nucleus, and occurs as a consequence of the fact that the converting electron can penetrate within the nuclear charge and current distributions. These new nuclear matrix elements carry distinctive information about nuclear structure, and are, therefore, as informative and important as the more familiar gamma-ray matrix elements. In the limit of a point nucleus, the rates of gamma-ray emission and internal conversion become strictly proportional to one another. In this case, the proportionality constant is the internal conversion coefficient computed by Rose *et al.*[4] Sliv *et al.*,[5] on the other hand, have recently computed internal-conversion coefficients for a finite nucleus, for particular nuclear charge and current distributions.[6] In their calculations, the model-dependent assumptions which they were required to make in order to obtain results in closed form implicitly equated the new conversion matrix elements to their corresponding gamma-ray matrix elements. The explicit dependence of Sliv's conversion coefficients on the new nuclear matrix elements has recently been discussed elsewhere.[3]

Perhaps the most striking effect of the new matrix elements for internal conversion is the existence of the

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[1] For transition energies greater than $2m_ec^2$, pair production is also possible. We do not consider this mode of de-excitation in the present paper.

[2] E. Church and J. Weneser, Phys. Rev. **100**, 943 (1955); **100**, 1241 (1955); **103**, 1035 (1956). J. Weneser and E. Church, Bull. Am. Phys. Soc. Sec. II, **1**, 181 (1956).

[3] E. Church and J. Weneser, Bull. Am. Phys. Soc. Ser. II, **1**, 330 (1956); Phys. Rev. **104**, 1382 (1956).

[4] Rose, Goertzel, Spinrad, Harr, and Strong, Phys. Rev. **83**, 79 (1951) and Rose, Goertzel, and Perry, Oak Ridge National Laboratory Report ORNL-1023, 1951 (unpublished), give values of K-shell conversion coefficients computed for a point nucleus without screening. M. E. Rose, in *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (Interscience Publishers, Inc., New York, 1955), gives some K-shell and L-shell coefficients for a point nucleus including the effects of screening. More complete tabulations have been circulated by Rose, Goertzel, and Perry, as well as unscreened point-nucleus values of the M-subshell conversion coefficients.

[5] L. A. Sliv, Zhur. Eksptl. i Teoret. Fiz. **21**, 770 (1951); L. Sliv and M. Listengarten, Zhur. Eksptl. i Teoret. Fiz. **22**, 29 (1952); L. Sliv and I. Band, J. Eksptl. i Teoret. Fiz. **31**, 134 (1956) [translation: Soviet Physics JETP **4**, 133 (1957)]; L. Sliv and I. Band, Leningrad Physico-Technical Institute Report, 1956 (unpublished) [translation: Report 57 ICC Kl, Physics Department, University of Illinois (unpublished)]. The final report gives K-shell conversion coefficients for a finite nucleus including the effects of screening. The atomic potential differs from that used by Rose *et al.* by the inclusion of the electron-exchange correction.

[6] Sliv's results, as well as those of references 2 and 3, assume the nucleus to be a uniformly charged sphere of radius $R=1.20\times10^{-13}A^{\frac{1}{3}}$ cm, for the purpose of calculating the effect of the finite nuclear size on the electron wave functions. To obtain specific numerical values for the conversion coefficients, Sliv further assumed that the nuclear currents lie entirely on the nuclear surface. This assumption corresponds to the value $\lambda=+1$, where λ is defined by Eq. (10). To investigate the dependence of the conversion coefficients on these assumptions, Sliv *et al.* carried through calculations for $R=1.50\times10^{-13}A^{\frac{1}{3}}$ cm, and for a uniform distribution of nuclear currents. They found effects of only a few percent. Since a uniform distribution of nuclear currents corresponds to $\lambda=+\frac{2}{3}$, the latter effect is as expected.

electric-monopole, or $E0$ mode of nuclear de-excitation.[7] This process occurs by the transfer of the nuclear-excitation energy to an atomic electron, and its ejection as an internal-conversion electron carrying off zero units of angular momentum, with no parity change. There is no corresponding $E0$ gamma ray, and electric monopole transitions proceed solely by the effects of the penetration of the atomic electrons within the finite nuclear volume. The $E0$ mode of de-excitation has been well known for $0+ \rightarrow 0+$ transitions, for which all competing (single) gamma-ray modes of de-excitation are strictly forbidden. Recent work,[2] however, has emphasized that $E0$ de-excitation may also occur between any two nuclear levels of the same spin and parity—whether or not the spin is zero—and that in such cases the $E0$ de-excitation may compete successfully with the gamma-ray emission and internal-conversion of transitions of higher multipole order.

A similar effect of the finite nuclear size is also present in the internal conversion of $M1$ transitions.[3] However, in this case there is a corresponding $M1$ gamma-ray matrix element, and the principal part of the conversion probability is (usually) determined by this nuclear matrix element. Nevertheless, there is always a finite contribution to the $M1$ conversion-electron transition probability involving nuclear matrix elements distinctly different from the $M1$ gamma-ray matrix element, which obey different model-dependent selection rules.[3] Under certain circumstances the gamma-ray matrix element may be small, while the new conversion matrix elements, due to the finite nuclear size, may be uninhibited. In such cases, there would be a marked effect on the rate of $M1$ internal conversion and its directional distribution. Similar results hold for all higher electric and magnetic multipoles.[3,8]

In a preliminary study of the properties of the $2+ \rightarrow 2+$ transitions in some medium-weight "even-even" nuclei, an attempt was made to detect the $E0$ component by an analysis of conversion-coefficient and gamma-gamma correlation data. The results obtained indicated a negligible $E0$ component, and a significantly small upper limit for the $E0$ matrix element. It was then pointed out[2] that a more sensitive means for determining small $E0$ admixtures lies in the study of angular correlations involving the conversion electrons, since the correlation function contains a term due to the interference between the $E0$ and the (experimentally) dominant $E2$ conversion electrons. The magnitude of this interference term is proportional to the amplitude of the rate of $E0$ conversion, while the rate of conversion, and the conversion coefficient, involve the square of this amplitude. The present paper discusses the quantitative features of a proposed directional-correlation experiment between the conversion electrons of the $2+ \rightarrow 2+$ transition and the subsequent pure $E2$ gamma rays of the $2+ \rightarrow 0+$ ground-state transition. It is perhaps superfluous to mention that the interest in the monopole component in $2+ \rightarrow 2+$ transitions lies not with the $E0$ conversion *per se*, but rather with the information regarding the structure of these nuclei which may be derived from the study of their monopole matrix elements. A preliminary study of the expected properties of the monopole matrix element for various nuclear models has already been given elsewhere.[2] It is hoped that the preliminary results presented here will stimulate further interest in this field.

In general, the presence of the new nuclear matrix elements do not lead to dominant physical effects, except in the unique case of $0+ \rightarrow 0+$ transitions mentioned above. In addition, they do not appear singly, since the transitions may consist of a mixture of several multipoles, and each multipole order will, in general, involve a gamma-ray matrix element and a new conversion matrix element. In the case of the $2+ \rightarrow 2+$ transitions, for example, there are six principal nuclear matrix elements which determine the properties of the transition: two gamma-ray matrix elements ($M1$, $E2$), and four new conversion matrix elements ($E0$, $M1$, and two[8] $E2$) arising from the effect of the finite nuclear size. In principle, then, one would need at least six independent physical measurements in order to determine the values of the matrix elements involved.

In general, the following data are available:

(1) a measure or estimate of the lifetime of the mixed transition or one of its components,

(2) a measure of the K-conversion coefficient of the mixed transition, and

(3) a measure of the directional correlation between the gamma-rays of the mixed transition and a coincident gamma ray. The results of such a measurement may be expressed in terms of the coefficients of the Legendre functions, $P_n(\cos\theta)$, required to express the observed directional distribution. In the present case there are two such numbers, the coefficients of P_2 and P_4. However, these are not independent, since they are in turn functions of a single nuclear parameter, namely, the ratio of the $M1$ gamma-ray matrix element to the $E2$ gamma-ray matrix element.

With the aid of the results of the present paper a fourth significant measurement may now be added to this list, namely:

[7] 2^L-pole electric (EL) and magnetic (ML) transitions carry off L units of angular momentum ($|\Delta \mathbf{I}| = L$), and obey the parity rules $\pi_i\pi_f = (-1)^L$ and $-(-1)^L$, respectively.

[8] T. Green and M. Rose, Bull. Am. Phys. Soc. Ser. II, **2**, 228 (1957). In the notation of this reference, there are two leading new matrix elements for $E2$ conversion:

$$\mu \equiv \int \mathbf{j}\cdot\mathbf{T}_{2,1}(r/R)^3 d\tau \Big/ \int \mathbf{j}\cdot\mathbf{T}_{2,1}(r/R)^1 d\tau$$

and

$$\nu \equiv \int \mathbf{j}\cdot\mathbf{T}_{2,3}(r/R)^3 d\tau \Big/ \int \mathbf{j}\cdot\mathbf{T}_{2,1}(r/R)^1 d\tau,$$

where the denominator is the $E2$ gamma-ray matrix element in the usual long-wavelength limit. In the limit of nuclear surface currents chosen by Sliv, these ratios have the values 1 and $(\frac{2}{3})^{\frac{1}{2}}$, respectively.

(4) a measure of the directional correlation between the K-conversion electrons of the mixed transition and a coincident gamma ray. The results of this measurement may also be expressed in terms of two empirical parameters, the coefficients of P_2 and P_4 which give rise to the observed directional distribution. Since these coefficients are functions of several nuclear conversion matrix elements, these two numbers constitute, in principle, two independent measurements. However, only the coefficient of P_2 is determinable with sufficient accuracy to be of use in the present context.

At the moment, additional measurements of the properties of the mixed transition are not generally available, although a number of experiments are possible in principle, the results of which would allow the six nuclear matrix elements to be completely determined. For example, a directional-correlation experiment involving the measurement of the transverse polarization of the conversion electrons of the mixed transition. However, in the absence of such data, we may still make a *preliminary* analysis of the existing data by introducing a reasonable physical assumption. In particular, we choose to neglect the new conversion matrix elements for $E2$ conversion. This neglect may be made plausible by the following arguments.

In general, the properties of the pure $E2$ component are rather insensitive to the effects of the finite nuclear size. This is evidenced by the slight difference between the values of the K-conversion coefficients computed by Rose for a point nucleus,[4] and those of Sliv for a finite nucleus.[5] For $Z=78$, for example, these coefficients differ by less than 7%. In addition, the deviations from Sliv's results due to the dependence on the new conversion matrix elements are expected to be small, especially since the $E2$ gamma-ray matrix elements are expected to be enhanced rather than retarded in these $2+\rightarrow 2+$ transitions.[2] In the following discussion, therefore, we base our calculations of the conversion properties of the $E2$ component on the conversion coefficients computed by Sliv *et al.*, and for simplicity, explicitly neglect the possible deviation from these. Although this is a convenient general assumption, it must eventually, of course, be justified for any particular transition considered.

If one makes the above assumption regarding the $E2$ conversion, the electron-gamma directional-correlation function may be written as a function of the $M1$ gamma-ray matrix element, and the new matrix elements for $E0$ and $M1$ conversion, all relative to the $E2$ gamma-ray matrix element. It is found that in the cases of interest here, the correlation is very insensitive to the new $M1$ conversion matrix element. This is due to several reasons. First, in these nuclei, the $M1$ gamma transitions amount to only a small fraction of the total transitions between the two $2+$ levels, and second, the $M1$ correlation itself is an extremely weak one. As is shown in detail in a later section, these factors conspire to make the correlation function almost completely independent of the effects of the finite nuclear size on the $M1$ component. In this case then, the correlation function depends almost exclusively on the properties of the $E0$ and $E2$ components and their interference.

The following sections present a discussion of the physical consequences of the interference between the $E0$ and $E2$ conversion electrons. The directional-correlation function expected between the K-conversion electrons of a mixed $E0+M1+E2$ transition and a subsequent gamma ray is computed explicitly, and numerical results given for the case of Pt ($Z=78$). The use of these results in the analysis of experimental data is then illustrated with reference to the particularly interesting case of Pt^{196}.

DIRECTIONAL-CORRELATION FUNCTION FOR AN $M1+E2$ TRANSITION

We consider the nuclear decay scheme characterized by the spin sequence $J_i\rightarrow J_i\rightarrow J_f$, where the two levels of spin J_i are understood to have the same parity, while the spin of the final state, J_f, and its parity are arbitrary. If $J_i=1$, the first transition can proceed via the usual $M1$ and $E2$ gamma-ray modes of de-excitation (i.e., gamma-ray emission and internal conversion), in addition to $E0$ conversion. For higher values of J_i higher multipoles are also possible, but for low-energy transitions such higher multipoles are not expected to be of practical importance. We therefore restrict our attention to the particular case,

$$J_i^{(1)} \xrightarrow{E0+M1+E2} J_i^{(2)} \xrightarrow{L} J_f.$$

For the moment we consider the second transition, from $J_i^{(2)}$ to J_f, to proceed by a transition of a single multipole order, namely L.

A simple correlation experiment is considered: the measurement of the directional-correlation between the K-conversion electrons of the $J_i^{(1)}\rightarrow J_i^{(2)}$ transition, and the gamma rays of the $J_i^{(2)}\rightarrow J_f$ transition. If no spins or polarizations are measured, the directional-correlation function, $W(\theta)$, can be written in the general form,

$$W(\theta)=P_0(\cos\theta)+A_2P_2(\cos\theta)+A_4P_4(\cos\theta), \qquad (1)$$

where $P_0=1$, P_2, and P_4 are Legendre functions. The coefficients A_2 and A_4 may each be written as the product of two factors, one depending only on the $J_i^{(1)}\rightarrow J_i^{(2)}$ transition, and the second depending only on the $J_i^{(2)}\rightarrow J_f$ transition. In the proposed experiment, the coefficient A_2 is found to depend almost exclusively on a single nuclear parameter, namely, the ratio of the $E0$ matrix element to the $E2$ gamma-ray matrix element. An experimental value of A_2, therefore, plus an independent measure or estimate of the $E2$ gamma-ray matrix element, leads to the desired value of the $E0$ matrix element. The expected electron-gamma correlation coefficient A_2 is discussed below.

The coefficient A_4, which arises solely from the $E2$ component, is independent of nuclear properties, except for a normalization factor.

The directional-correlation function between the conversion electrons of a mixed $E0+M1+E2$ transition and a subsequent gamma ray may be expressed in part in terms of the correlation function when only the $M1$ and $E2$ components are present. The $M1+E2$ electron-gamma correlation function is, in turn, related to the corresponding gamma-gamma correlation. The relation between the electron-gamma correlation and the gamma-gamma correlation is discussed below. For a mixed $M1+E2$ transition, we have the following explicit form for the *gamma-gamma* correlation function[9]:

$$W(\gamma\gamma:M1+E2)=P_0+\frac{1}{1+\delta^2}[A_2{}^e+2\delta A_2+\delta^2 A_2{}^m]P_2+\frac{1}{1+\delta^2}[A_4{}^e]P_4, \quad (2)$$

where $A_2{}^e$, A_2, $A_2{}^m$, and $A_4{}^e$ are coefficients tabulated by Biedenharn and Rose, and δ^2 is the ratio of the $M1$ gamma-ray transition probability to the $E2$ gamma-ray transition probability. For a mixed $M1+E2$ transition, the corresponding *electron-gamma* correlation is[9]

$$W(e\gamma:M1+E2)=P_0+\frac{1}{1+p^2}[b_2{}^eA_2{}^e+2pb_2A_2+p^2b_2{}^mA_2{}^m]P_2+\frac{1}{1+p^2}[b_4{}^eA_4{}^e]P_4, \quad (3)$$

where p^2 is the ratio of the rate of $M1$ conversion-electron ejection to the rate of $E2$ conversion-electron ejection, i.e.,

$$p^2\equiv\frac{\beta_1{}^K}{\alpha_2{}^K}\delta^2=\frac{\text{No. of } M1\ K\text{-shell conversion electrons/sec}}{\text{No. of } E2\ K\text{-shell conversion electrons/sec}}, \quad (4)$$

where $\beta_1{}^K$ is the $M1$ K-shell internal-conversion coefficient, and $\alpha_2{}^K$ is the $E2$ K-shell internal-conversion coefficient. The sign of p is the sign of δ, which is in turn empirically determined from the gamma-gamma correlation (2). Values of the particle parameters, $b_2{}^e$, $b_4{}^e$, b, and $b_2{}^m$, have been tabulated by Biedenharn and Rose[9] *for a point nucleus*. Values appropriate for a finite nucleus with $Z=78$ are discussed later.

[9] Rose, Biedenharn, and Arfken, Phys. Rev. **85**, 5 (1952); L. Biedenharn and M. Rose, Revs. Modern Phys. **25**, 729 (1953). A factor of $-(i)^{2l(\kappa)}$ has been omitted from Eq. (82) of the latter paper. The notation used in the present paper differs somewhat from that used in the above references, but in most cases the differences are obvious. However, it should be noted that the definition, $\delta^2\equiv\mathcal{W}_\gamma(M1)/\mathcal{W}_\gamma(E2)$, used in the present paper is the *reciprocal* of that used in these papers.

DIRECTIONAL CORRELATION FUNCTION FOR AN $E0+M1+E2$ TRANSITION

We now inquire into the effects on the directional correlation due to the presence of an $E0$ component in the mixed transition. The gamma-gamma correlation, of course, would be unaffected, since there are no (single) $E0$ gamma rays. Pure electric-monopole conversion can lead only to an isotropic correlation function. However, interference between $E0$ and $E2$ conversion results in a characteristic P_2 (cosθ) interference term. Interference between the $E0$ and $M1$ conversion could only appear in a P_1 (cosθ) term. In the absence of a longitudinal-spin or circular-polarization measurement, however, the fact that parity is a good quantum number implies that only even terms appear in the directional-correlation function. In the directional correlation then, there is no observable interference between the $E0$ and $M1$ conversion electrons, but only between the $E0$ and $E2$ conversion electrons.

The effect of the presence of an $E0$ component on the conversion-electron directional-correlation function is twofold. First, a simple change in the normalization, and second, the addition of the $E0-E2$ interference term. As a measure of the monopole contribution we define a quantity q^2, in analogy with (4), as

$$q^2\equiv\frac{1}{\alpha_2{}^K}\frac{\mathcal{W}_K(E0)}{\mathcal{W}_\gamma(E2)}=\frac{\text{No. of } E0\ K\text{-shell conversion electrons/sec}}{\text{No. of } E2\ K\text{-shell conversion electrons/sec}}, \quad (5)$$

where $\mathcal{W}_K(E0)$ is the rate of $E0$ K-conversion-electron ejection, and $\mathcal{W}_\gamma(E2)$ is the rate of $E2$ gamma-ray emission. The dependence of $\mathcal{W}_K(E0)$ on the monopole matrix element, ρ, is given to lowest order by

$$\mathcal{W}_K(E0)=\Omega_K(Z,k)\rho^2, \quad \rho=\int\phi_f{}^*(r_p/R)^2\phi_i d\tau, \quad (6)$$

where Ω_K is a known function of atomic number, Z, and nuclear transition energy, k. Values of Ω appropriate for a finite nucleus have been given elsewhere.[2]

The directional-correlation function between $E0+M1+E2$ conversion electrons and a subsequent gamma ray may then be written as

$$W(e\gamma:E0+M1+E2)=\frac{1+p^2}{1+p^2+q^2}W(e\gamma:M1+E2)+\frac{q^2}{1+p^2+q^2}P_0+\frac{q}{1+p^2+q^2}b_0P_2. \quad (7)$$

The numerical $E0-E2$ interference parameter, b_0, appearing in (7) can be calculated in a straightforward manner, as is discussed in the next section. This coefficient depends only on electron wave functions, and is a calculable function of atomic number and transition energy. Quantitative results are given in Fig. 1 for the particularly interesting case of K conversion in Pt ($Z=78$), over a range of nuclear transition energies.

CALCULATION OF THE INTERFERENCE PARAMETER b_0

The actual manipulations and calculations involved in the evaluation of directional-correlation functions are well known, and only the briefest description suffices here.[9] In short, the interaction between the nuclear and electron currents and charges can be written in terms of the multipole expansion. This formulation, in terms of irreducible tensors, permits the application of the Wigner-Eckart theorem. The angular parts of the matrix elements are then written as a sum of products of Clebsch-Gordan coefficients, which are particularly amenable to the Racah methods.

$E0$ conversion consists of the transfer of the nuclear transition energy and zero units of angular momentum to an atomic electron. The final state electrons resulting from $E0$ conversion in the K shell are, then, $s_{\frac{1}{2}}$ electrons, or in relativistic notation, those for which $\kappa=-1$. $E2$ conversion results in the ejection of a K electron either into a $d_{\frac{3}{2}}$ or a $d_{\frac{5}{2}}$ continuum state, for which $\kappa=+2$ and $\kappa=-3$, respectively. These latter possibilities appear in the calculation of the interference coefficient, b_0, as terms weighted as their fractional amplitude contributions, η_κ, appearing in the $E2$ internal conversion probability, and by appropriate phase factors.

The results of the calculation give the following general expression for the interference parameter b_0:

$$b_0=-2\,\mathrm{Re}\Big[(\tfrac{3}{5})^{\frac{1}{2}}|\eta_{-3}|e^{i(\Delta_{-1}-\Delta_{-3}-\theta_{-3})} - (\tfrac{2}{5})^{\frac{1}{2}}|\eta_{+2}|e^{i(\Delta_{-1}-\Delta_{2}-\theta_{2})}\Big]F_2(L,J_f,J_i), \quad (8)$$

where Re stands for "the real part of." The square bracket is concerned only with the $J_i^{(1)}\rightarrow J_i^{(2)}$ transition, while the factor F_2 is concerned only with the succeeding $J_i^{(2)}\rightarrow J_f$ gamma ray. The second factor, F_2, is tabulated by Biedenharn and Rose for various spin sequences. The quantities Δ_{-1}, Δ_2, Δ_{-3} appearing in (8) are the Coulomb phase factors corresponding to $s_{\frac{1}{2}}$, $d_{\frac{3}{2}}$, $d_{\frac{5}{2}}$ electrons of the appropriate energy. The quantities θ_2, θ_{-3} are the phases of the conversion matrix elements for $d_{\frac{3}{2}}$ and $d_{\frac{5}{2}}$ electrons, respectively.[10]

If the second transition, $J_i^{(2)}\rightarrow J_f$, is mixed rather than pure, F_2 is to be replaced by the mixture coefficient,

$$\frac{1}{1+\Delta^2}\{F_2(L,J_f,J_i)+\Delta^2F_2(L',J_f,J_i) -2\Delta(-1)^{J_i-J_f}[(2J_i+1)(2L+1)(2L'+1)]^{\frac{1}{2}} \times G_2(L,L',J_f,J_i)\}, \quad (9)$$

where the notation is that of Biedenharn and Rose,[9] and $\Delta^2=\mathcal{W}_\gamma(L')/\mathcal{W}_\gamma(L)$ is the gamma-ray mixing ratio of the second transition.

To illustrate the above results, values of b_0 have been computed for K conversion in Pt ($Z=78$), and are given graphically in Fig. 1. As given, these results are appropriate for the particular interesting case of the electron-gamma directional correlation in a $2+\rightarrow 2+ \rightarrow 0+$ cascade. It should be noted, however, that the dependence on the spins of the level sequence J_i and J_f enter only through the factor F_2 in (8). The results of Fig. 1 can then be easily modified for some other spin sequence—however, only for $Z=78$.

A more complete discussion of the methods and approximations involved in the calculation of the values of b_0 shown in Fig. 1 is given in the Appendix.

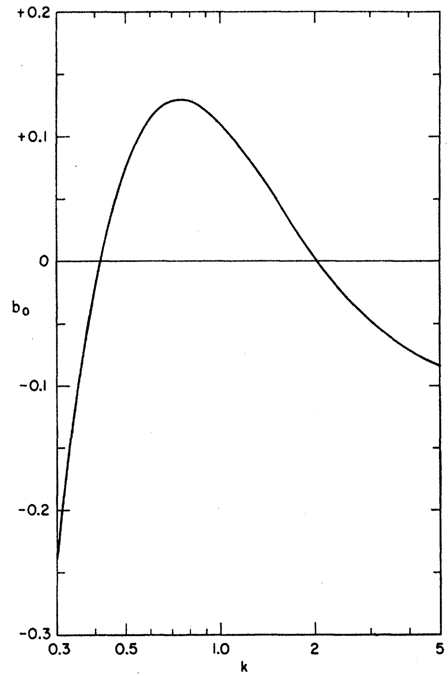

FIG. 1. The $E0$-$E2$ interference, b_0, appearing in Eq. (7). Results are given for $Z=78$ as a function of the nuclear transition energy, k, in units of the electron rest mass. As given, the results are appropriate for the electron-gamma directional correlation is a 2–2–0 cascade. However, as indicated in the text, the results for any other spin sequence are easily derivable from these. Critical parts of this curve were drawn with the aid of a polynomial interpolation formula based on the six computed points for which conversion calculations were available.

[10] Using the definitions in reference 12, $|\eta_2|^2=\alpha_2^K(d_{\frac{3}{2}})/\alpha_2^K$ and $|\eta_{-3}|^2=\alpha_2^K(d_{\frac{5}{2}})/\alpha_2^K$. The phase factors Δ_κ and θ_κ are defined as in reference 9.

EFFECT OF THE FINITE SIZE ON THE $M1$ COMPONENT

To lowest order, the new matrix element for $M1$ internal conversion is[3]

$$\int \phi_f{}^*\mathbf{j}\cdot\mathbf{A}_{1M}{}^{m*}\left(\frac{r}{R}\right)^2\phi_i d\tau \equiv \lambda \int \phi_f{}^*\mathbf{j}\cdot\mathbf{A}_{1M}{}^{m*}\phi_i d\tau, \quad (10)$$

where the integral on the right is the usual $M1$ gamma-ray matrix element in the long-wavelength limit. The real dimensionless parameter, λ, then characterizes the effects of the finite nuclear size on the $M1$ component. In Sliv's calculations of the $M1$ K-shell conversion coefficient, $\beta_1{}^K$, his assumption of nuclear surface currents is equivalent to setting $\lambda = +1$. The explicit dependence of $\beta_1{}^K$ on λ is given approximately by

$$\beta_1{}^K(\lambda) \sim [1-(\lambda-1)C(Z,k)]^2\beta_1{}^K(1), \quad (11)$$

where the numerical coefficient $C(Z,k)$ has been tabulated elsewhere.[3,11] The values of this coefficient used in the present calculation are given in Table I. It should be emphasized at this point that because of the many simplifying physical assumptions underlying the evaluation of the coefficient $C(Z,k)$, these numerical values are only illustrative. These include the assumption of a spherical nucleus of uniform charge with a sharp surface, the assumption that the transitions involving the new matrix element (10) take place entirely within the uniform charge distribution, and the neglect of numerous higher order effects. The point is simply that this coefficient is somewhat model-dependent, and a generally applicable number cannot be given. However, because of the insensitivity of the correlation function to the finite-size effects on the $M1$ component, as shown later, such uncertainties have negligible effect on the directional-correlation experiments proposed here.

The dependence on finite-size effects of the various particle parameters, b, appearing in the electron-gamma directional-correlation function (3), is more subtle, and has been obtained in the manner discussed below. We have derived this dependence from the point-nucleus results of Rose *et al.*[4] and the finite-nucleus results of Sliv *et al.*[5] on the basis of three assumptions. First, we have assumed that of the two final electron states in $M1$ conversion, $s_{\frac{1}{2}}$ and $d_{\frac{3}{2}}$, the finite nuclear size affects only the $s_{\frac{1}{2}}$ electron matrix element, and that the value of the $d_{\frac{3}{2}}$ electron matrix element is that computed for a point nucleus. Second, it is assumed that the effect of the finite nuclear size on the $s_{\frac{1}{2}}$ component can be represented by the addition of a pure imaginary increment to the $s_{\frac{1}{2}}$ radial electron matrix element. This assumption is based on the observation that all the contributions to the radial electron matrix elements from the region about the origin are pure imaginary, and is supported by the observation that the $s_{\frac{1}{2}}$ matrix elements computed for a point nucleus are themselves almost pure imaginary. The third and final assumption is concerned with the small effect of the finite nuclear size on $E2$ conversion. Again there are two final electron states, $d_{\frac{3}{2}}$ and $d_{\frac{5}{2}}$. We have assumed that the finite-size effects on these matrix elements can be satisfactorily taken into account by altering the values computed by Rose *et al.* by a common (real) factor. As a consequence of this last assumption, the parameters $b_2{}^e$ and $b_4{}^e$ are identical with those given by Biedenharn and Rose. When the individual values of the $E2$ matrix elements become available, the detailed dependence of these parameters on the $E2$ finite-size effects may be computed. However, it is not anticipated that these effects will significantly alter the values of the particle parameters considered here.

The values of $b_2{}^m$ and b_2 depend on the ratios of the radial electron matrix elements for $M1$ conversion into the $s_{\frac{1}{2}}$ and $d_{\frac{3}{2}}$ final electron states.[12] The exact formulas are given by Biedenharn and Rose.[9] It is assumed for the purpose of the present calculation that the $d_{\frac{3}{2}}$ matrix element is that computed by Rose *et al.*[4] for a point nucleus, while the $s_{\frac{1}{2}}$ matrix element is that of Rose *et al.* plus two correction terms. The magnitude of the first of these is that necessary to give Sliv's value of $\beta_1{}^K$, and its phase is taken as identically $+\pi/2$. This correction includes the effect of the finite nucleus on the electron wave functions. To this must be added the

TABLE I. Values of $C(Z,k)$ used in the present calculations.

k	0.3	0.5	1.0	1.8	3.0	5.0
$C(78,k)$	0.02_{15}	0.02_{23}	0.02_{45}	0.02_{74}	0.03_{10}	0.03_{66}

TABLE II. Values of the particle parameter b_2 appearing in Eq. (3), appropriate for a finite nucleus of $Z=78$. Results have been derived as described in the text as a function of the nuclear transition energy, k, in units of the electron rest mass. The parameter λ, characterizing the effects of the new nuclear matrix element for $M1$ conversion, is defined by Eq. (10). Values for $\lambda=+1$ correspond to the K-shell conversion-coefficient calculations of Sliv *et al.*

k	$\lambda=-3$	$\lambda=+1$	$\lambda=+5$
0.3	+0.089	+0.087	+0.086
0.5	−0.128	−0.131	−0.135
1.0	−0.385	−0.392	−0.401
1.8	−0.541	−0.554	−0.569
3.0	−0.633	−0.654	−0.679
5.0	−0.706	−0.739	−0.781

[11] The values of $C(Z,k)$ tabulated in reference 3 are in error, and must be corrected by a factor of $(\frac{2}{3})^{\frac{1}{2}}$.

[12] In the notation of reference 9, the total $M1$ K-shell conversion coefficient may be written as $\beta_1{}^K=\beta_1{}^K(s_{\frac{1}{2}})+\beta_1{}^K(d_{\frac{3}{2}})$ where $\beta_1{}^K(s_{\frac{1}{2}}) = (4\pi^2\alpha k/3)|Q(-1,1,m)|^2$, $\beta_1{}^K(d_{\frac{3}{2}}) = (8\pi^2\alpha k/3)|Q(2,1,m)|^2$. Similarly, the total $E2$ K-shell conversion coefficient may be written $\alpha_2{}^K=\alpha_2{}^K(d_{\frac{3}{2}})+\alpha_2{}^K(d_{\frac{5}{2}})$, where $\alpha_2{}^K(d_{\frac{3}{2}})=(8\pi^2\alpha k/15)|Q(2,2,e)|^2$, $\alpha_2{}^K(d_{\frac{5}{2}})=(4\pi^2\alpha k/5)|Q(-3,2,e)|^2$. The particle parameters $b_2{}^m$, $b_2{}^e$, b_2, and $b_4{}^e$ depend only on the ratios $T_m \propto Q(2,1,m)/Q(-1,1,m)$ and $T_e \propto Q(2,2,e)/Q(-3,2,e)$, with appropriate phase factors.

second correction to the $s_{\frac{1}{2}}$ matrix element,

$$-i(\lambda-1)C(Z,k)\frac{\beta_1{}^K(1)}{[\beta_1{}^K(1)-\beta_1{}^K(d_{\frac{3}{2}})]^{\frac{1}{2}}},\tag{12}$$

which takes the effect of the new $M1$-conversion matrix element into account. Values of $b_2{}^m$ and b_2 computed in this way are given in Tables II and III.

APPLICATION TO Pt^{196}

Pt^{196} is an even-even nucleus possessing a 0+ ground state and two 2+ excited states, located 354 and 685 kev above the ground state, respectively.[13] Gamma-gamma directional-correlation experiments indicate $\delta^2=4.71\%$ for the $2+\rightarrow2+$ transition, with δ positive. The measured K-conversion coefficient of this transition is $\beta^K=(5.9\pm0.4)\times10^{-2}$.

If, as usual, we neglect possible deviations from Sliv's values of the $E2$ K-conversion coefficient, $\alpha_2{}^K$, we have the simple expression for the net K-conversion coefficient, β^K,

$$\beta^K=\frac{1}{1+\delta^2}[(1+q^2)\alpha_2{}^K+\delta^2\beta_1{}^K(\lambda)].\tag{13}$$

Combining the experimental values of β^K and δ with (11) and (13), we obtain a relation between λ and q which is presented in Fig. 2. This figure illustrates the region in the $\lambda-q$ plane nearest the origin which is consistent with the quoted experimental values of β^K and δ. Values of the conversion parameters used in the calculation are $\beta_1{}^K(1)=0.195$, $\alpha_2{}^K=0.0492$, and $C(78,0.65)=0.02_3$. To illustrate the use of correlation data, we have also included curves corresponding to an *assumed* value of $A_2=-0.149\pm10\%$, where it must be emphasized that no such measurement has yet been made. The mean value of A_2 has been arbitrarily chosen to pass through the point $\lambda=+1$, $q=0$. The value of the interference parameter used was $b_0=+0.124$. It is seen that in the present case a measurement of β^K determines λ only to within rather broad limits, while measurement of A_2 determines q, and hence ρ, almost independently of λ.

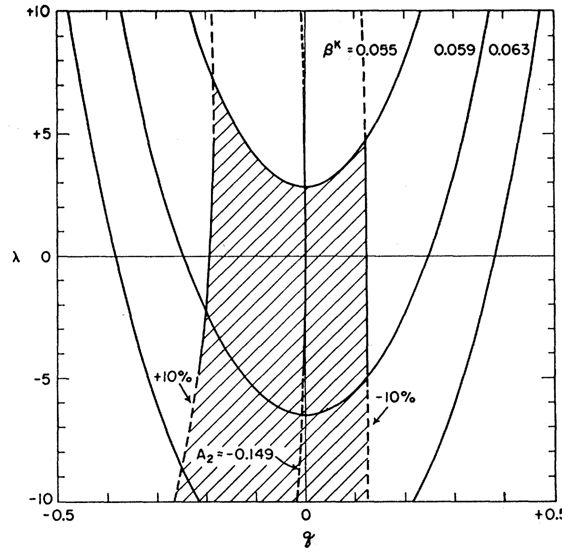

FIG. 2. Plot of the section of the $\lambda-q$ plane appropriate for the analysis of electron-gamma correlation and conversion-coefficient data for the 331-kev $2+\rightarrow2+$ transition in Pt^{196}. λ is the ratio of the new matrix element for $M1$ conversion to the $M1$ gamma-ray matrix element, and q is the ratio of the $E0$ K-conversion intensity to $E2$ K-conversion intensity. The indicated curves correspond to the measured K-conversion coefficient, $\beta^K=0.059\pm0.004$. The near-vertical curves correspond to the *assumed* values $A_2=-0.149\pm10\%$ for the coefficient of P_2 in the K-electron-gamma directional correlation. The mean value of A_2 has been arbitrarily chosen to pass through the point $\lambda=+1$, $q=0$. The shaded area represents the region consistent with the quoted values of β^K and A_2.

The insensitivity of the correlation function to the properties of the $M1$ component illustrated in the figure justifies, in a sense, the approximations made in their calculation, and the neglect of such effects in the analysis of the proposed directional-correlation experiment, in the case of $Z=78$.

To illustrate the sensitivity of the method of analysis proposed here, we note that the analysis of the K-conversion-coefficient and gamma-gamma-directional-correlation data for Pt^{196} indicates $|\rho|\leq1/26$.[2] If now we arbitrarily take $|\rho|=1/150$, and assume the reduced transition probability for the $E2$ component of the $2+\rightarrow2+$ transition to be twice that measured for the ground state $2+\rightarrow0+$ transition,[2] we find $|q|\sim0.08$. A value of q of this magnitude would lead to only an $\sim0.6\%$ increase in the K-conversion coefficient of the $2+\rightarrow2+$ transition. The detection of an effect of this magnitude is well beyond the present limits of experimental accuracy. However, it is easily shown with the aid of the previously derived expressions, that the effect of a q of this magnitude would change the coefficient of P_2 in the electron-gamma directional correlation by $\sim\pm6\%$. The direction of the change depends on the sign of q—in other words, on the relative phases of the

TABLE III. Values of the particle parameter $b_2{}^m$ appearing in Eq. (3) appropriate for a finite nucleus of $Z=78$. Results have been derived as described in the text as a function of the nuclear transition energy, k, in units of the electron rest mass. The parameter λ characterizing the effects of the new nuclear matrix element for $M1$ conversion is defined by Eq. (10). Values for $\lambda=+1$ correspond to the K-shell conversion-coefficient calculations of Sliv *et al.*

k	$\lambda=-3$	$\lambda=+1$	$\lambda=+5$
0.3	−0.020	−0.022	−0.024
0.5	+0.061	+0.066	+0.071
1.0	+0.187	+0.204	+0.223
1.8	+0.295	+0.324	+0.359
3.0	+0.381	+0.424	+0.476
5.0	+0.461	+0.525	+0.603

[13] References to the papers giving the experimental values are to be found in reference 2.

$E0$ and $E2$ gamma-ray matrix elements.[14] Detection of effects of this magnitude does not appear to be beyond the present limits of experimental accuracy.[15] Their detection would yield important information not presently available by other means.

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APPENDIX

In this appendix we discuss further details of the calculation of the $E0-E2$ interference parameter, b_0.

In computing the values of b_0 shown in Fig. 1, the values of Δ_κ and θ_κ used were taken from the point-nucleus K-shell internal-conversion calculations of Rose *et al.* These calculations involve two approximations in the calculation of the electron wave functions. First, the atomic potential is assumed to be an unscreened Coulomb potential, and second, the atomic nucleus is assumed to be a point. For the reasons discussed below, these approximations are expected to be satisfactory for the calculation of the $E0+E2$ interference parameter, b_0.

The effects of screening on the $E2$ internal-conversion coefficient can be seen by comparing values of $\alpha_2{}^K$ computed with and without screening. For energies above $\sim$150 kev the effects are quite negligible. Although the calculations with screening are not in a form to allow a separation of the $d_{\frac{3}{2}}$ and $d_{\frac{5}{2}}$ contributions, it is reasonable to assume that the effects of screening are negligible for both the phases, θ_κ, and amplitudes, η_κ, of the components of the final states taken separately. This is supported by a nonrelativistic WKB estimate of the effects of screening.

Screening also affects the continuum phase factors, Δ_κ. While Coulomb phases have an infinite logarithmic term, the interference coefficient, b_0, depends only on differences in phase shifts, and hence no logarithmic term enters, even in the limit of no screening. Screened phases differ from Coulomb phases by the absence of the logarithmic term. It might be thought, then, that screening, in addition to removing the infinite logarithmic term, might significantly change the finite phase differences. We are aware of no suitable relativistic computations which can be used to compare screened and Coulomb phase differences. We have, however, carried through a nonrelativistic WKB calculation to estimate the order of magnitude of the effect expected. In this approximation the effects of screening on the phase factors, Δ_κ, are completely negligible at the energies of interest here.

Sliv has found that the effects of the finite size on the magnitude of the $E2$ conversion are generally small. As mentioned previously, we have assumed that the fractional amplitudes, η_κ, and phases, θ_κ, of the $E2$ conversion matrix elements are those computed by Rose *et al.* The effect of the finite nuclear size on the continuum phase shifts, Δ_κ, may be shown to be negligible.[16] We have, therefore, neglected all such finite-size effects in the calculation of the interference parameter, b_0. This neglect is equivalent to the assumptions regarding $E2$ conversion already mentioned in the text. Although, in principle, the interference parameter, b_0, also depends on the finite-size effects on the $E2$ component, it is not expected that these dependences will significantly change the results of the present paper.

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[14] The sign of q is the sign of $(J_f\|E0\|J_i)/(J_f\|E2\|J_i)$, where

$$(J_fM_f|E0|J_iM_i)=(J_f\|E0\|J_i),$$
$$(J_fM_f|\mathbf{j}\cdot\mathbf{A}_{LM}{}^e|J_iM_i)=(J_f\|EL\|J_i)C(J_iLJ_f;\,M_iMM_f),$$
$$\mathbf{A}_{LM}{}^e=-i^{L+1}[\tfrac{1}{2}\pi L(L+1)]^{-\frac{1}{2}}\nabla\times(\mathbf{r}\times\nabla)j_LY_L{}^M,$$

and where the phases of the $Y_L{}^M$ and Clebsch-Gordan coefficients are those of Condon and Shortley.

[15] J. Kane and S. Frankel, Bull. Am. Phys. Soc. Ser. II, **2**, 25 (1957). This paper reports preliminary measurements on the K-electron-gamma directional correlation in Pt^{196}. However, because of the large experimental error, the results quoted do not place significant restrictions on q.

[16] L. Sliv and B. Volchok, Leningrad Physico-Technical Institute Report, 1956 (unpublished) [translation: Atomic Energy Commission Report AEC-2875 (unpublished)].