assume $\cos\theta=1$ and $\sin\theta=\theta$. Also we neglect the effect of leakage of singly scattered neutrons out the side of the specimen. Consider the first scattering to occur at x_1 and the second at x_2. The rate at which neutrons are scattered twice into $d\omega_2$ and emerge in the direction of the detector is as follows, where the brackets indicate the individual processes mentioned above:

$$\int_{\omega_1}\int_{x_2=0}^{t}\int_{x_1=0}^{x_2}\left[Q\pi\left(\frac{d}{L}\right)^2 e^{-\sigma_t n x_1}\right][n dx_1\sigma_e(\theta)d\omega_1]\times[e^{-\sigma_t n(x_2-x_1)}][n dx_2\sigma_e(\theta)d\omega_2][e^{-\sigma_t n(t-x_2)}].$$

Integration over x_1 and x_2 is straightforward. Since $\sigma_e(\theta)$ is large only for small values of θ, the integral over ω_1 can be evaluated approximately by setting $\sigma_e(\theta)=\sigma_e(0)$ and integrating up to some θ_m which includes the forward lobe of the scattering angular distribution. An appropriate value of θ_m is $1/kR$ according to Feld *et al.*[20] The contribution to the observed transmission due to double scattering is given by

$$T_2/T_1=\tfrac{1}{2}\pi n t\sigma_e(0)\theta_m^2.$$

For 14.1-Mev neutrons on a lead specimen of 50% transmission, this ratio is 0.06.

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Nucleon Exchange Effects in the $B^{10}(d,p)$ Stripping Reaction

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The consequences of an exchange contribution to a specific stripping reaction are worked out in some detail. The particular transition considered is that which leads to the first excited state of B^{11} and which is forbidden by the angular momentum restrictions of ordinary stripping theory. The exchange calculation provides a fair measure of agreement with experiment on the angular distribution, yield, and energy dependence of the reaction. The relation of the present analysis to treatments of heavy-particle and spin-flip stripping is briefly discussed.

I. INTRODUCTION

In 1954 Evans and Parkinson[1] reported a study of the $B^{10}(d,p)$ reaction at deuteron energies between 6 and 8 Mev. The angular distributions of the various groups of outgoing protons could be best fitted, using stripping theory, by assuming that the B^{10} nucleus captures a neutron with $l=1$ in order to form the ground state or any of the first four excited states of B^{11}. This conforms to shell model theory, according to which B^{11} will have the properties of the polyad p^7 and will, with increasing excitation, pass successively through the total angular momentum states $\frac{3}{2}$ (ground state), $\frac{1}{2}$, $\frac{5}{2}$, $\frac{7}{2}$, $\frac{3}{2}$, $\frac{5}{2}$ under reasonable conditions of intermediate coupling.[2,3] There is the special difficulty, however, that conservation of angular momentum does not permit formation of a state $J=\frac{1}{2}$ by simple addition of a p-neutron to B^{10} ($J=3$). Thus it would appear that the first excited state of B^{11} could not be produced in a stripping reaction by this means if its spin is indeed $\frac{1}{2}$.

Experimental evidence supports the theoretical expectation that $\frac{1}{2}(-)$ is the correct spin/parity assignment to the 2.14-Mev level of B^{11}. The spin value is implied by the relative gamma-ray transition probabilities in $Li^7(\alpha,\gamma)$,[4] and by the isotropy of the p-γ angular correlation in the reactions $B^{10}(d,p\gamma)$ [5] and $B^{11}(p,p'\gamma)$.[6] More recently Wilkinson[7] has shown that the gamma-ray transition from the first excited state to the ground state of B^{11} is fast; from this and other evidence he concludes that it is $M1$ and hence that the excited state has odd parity. It is the concern of the present paper to fit the results of the deuteron stripping experiments into this scheme of things.

II. POSSIBILITY OF NUCLEON EXCHANGE

It was recognized at the time of the original stripping measurements on $B^{10}(d,p)$ that the angular distribution of the proton group Q_1 (leading to the first excited state of B^{11}) is distinctly anomalous and can only with difficulty be reconciled with $l_n=1$. To illustrate the anomalous character, we show in Fig. 1 an angular distribution of the ground-state protons Q_0 (extended

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over a wider range of angles than in the original work), and in Fig. 2 the angular distribution of Q_1 at two energies.[8] For both groups the contribution of a quasi-isotropic "background" is abnormally high; but whereas in Fig. 1 the superimposed forward peak can be quite well fitted by a theoretical stripping curve, in Fig. 2 the agreement is far from convincing, especially as the character of the experimental curve varies markedly with deuteron energy.

With a view to resolving some of the difficulties, it was proposed[9] that a significant amount of nucleon exchange might occur, so that the outgoing proton in a (d,p) stripping reaction (for example) might come from the target nucleus, and not solely from the deuteron as conventional stripping theory would require. As applied to the proton group Q_1 in $B^{10}(d,p)$ the exchange mechanism would, through the increased latitude in coupling angular momenta, allow the capture of the neutron with $l=1$ and yet lead to a final B^{11} state of spin $\frac{1}{2}$. As applied to other proton groups, the coherence (at least partial) of normal and exchange stripping where both are allowed would lead to interference effects that are often suggested by the form of the experimental angular distributions.[1]

The detailed features of an exchange contribution to stripping were subsequently worked out[10,11] with the help of some simplifying assumptions. If the total wave function describing the initial state of deuteron+target nucleus is made antisymmetric with respect to interchange of similar nucleons, the reaction amplitude in Born approximation is a sum of two terms, F and G. F corresponds to direct stripping in which the outgoing particle comes from the deuteron, and G to exchange stripping, in which the outgoing particle comes from the target nucleus. F and G can each be resolved into two parts, one arising from the neutron-proton interaction in the deuteron, and the other arising from the interaction of the outgoing particle with the target nucleus. The special interest of the proton group Q_1 in $B^{10}(d,p)$ is that the F terms cannot contribute,[12] so our attention is confined to the exchange terms G.

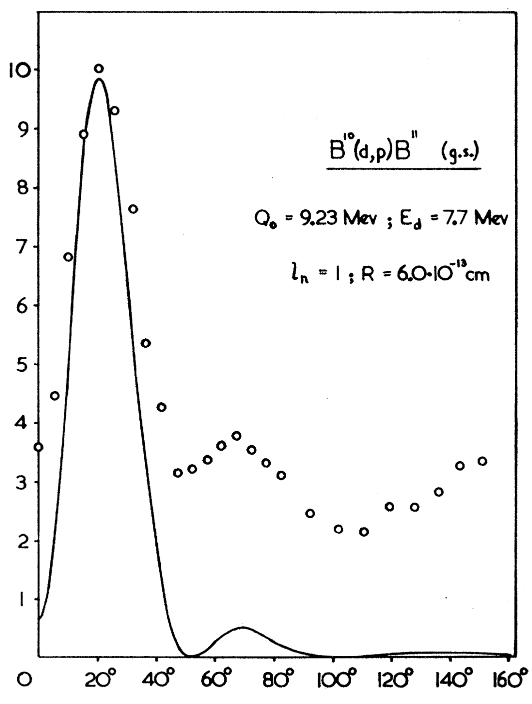

FIG. 1. Stripping angular distribution for ground-state transitions in $B^{10}(d,p)$ at 7.7 Mev. The full line is the theoretical (Butler) curve for $l_n=1$.

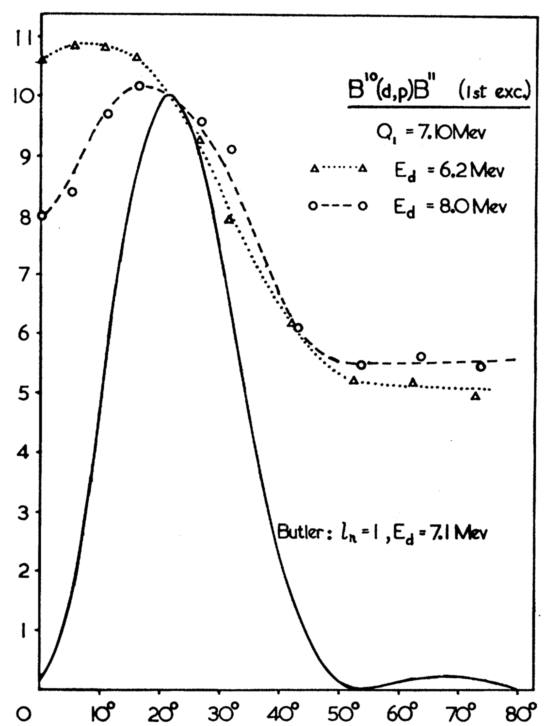

FIG. 2. Stripping angular distributions for first excited state transitions in $B^{10}(d,p)$.

III. METHOD FOR EVALUATING EXCHANGE TERMS

We designate the outgoing proton in a (d,p) reaction by 1. Then the amplitude G can be written as $G_{1n}+G_{1c}$, where G_{1n} arises from the neutron-proton interaction V_{1n}, and G_{1c} from the interaction of proton 1 with the core of the target nucleus. These terms are in essence the exchange analogs of a pure stripping amplitude (F_{1n}) and of the extra contribution due to scattering of the outgoing particle by the target nucleus. Explicit formulas for G_{1n} and G_{1c} are given in Eqs. (31) and (48) of reference 11. In order to make them more tractable,

[8] Note the difference of horizontal scales in Figs. 1 and 2.
[9] A. P. French, as quoted in reference 1.
[10] N. T. S. Evans, Ph.D. thesis, Cambridge, 1955 (unpublished).
[11] A. P. French, Phys. Rev. 107, 1655 (1957).

[12] This limitation may be relaxed if one chooses to invoke special mechanisms (e.g., as in reference 7).

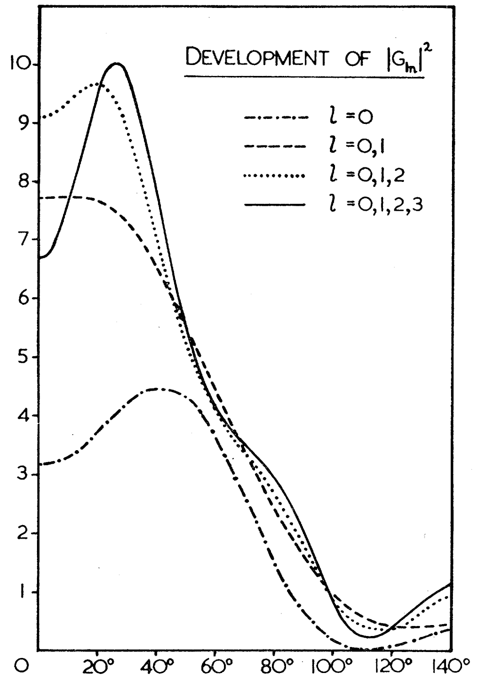

FIG. 3. The effect on the calculated exchange intensity of using various numbers of terms in the deuteron internal wave function expansion. The relative intensities are normalized to a peak value of 10 in the curve for $l=0, 1, 2, 3$.

we assume that the reaction takes place on the surface of a sphere of interaction of radius R. The radial integrals appearing in G_{1n} and G_{1c} are thus replaced by the integrands evaluated at R. The important simplification that ensues is that G can then be written in terms of a series development of the internal wave function of the deuteron, and the intensities due to G_{1n} and G_{1c} can be expressed, when reduced to their bare essentials, in the following way:

$$|G_{1n}(\theta)|^2 = A_1(E_d,R) \sum_{L'=a,a'} j_a(KR) j_{a'}(KR) \sum_{L=b,b'} \sum_{l=l_1,l_2} w_1(L'lL; E_d,R) g(L'lL; \cos\varphi), \quad (1)$$

$$|G_{1c}(\theta)|^2 = A_2(E_d,R) \sum_{L'=a,a'} \sum_{m,m'} w_2(lL'; E_d,R) \times P_a{}^m(\cos\theta) P_{a'}{}^{m'}(\cos\theta), \quad (2)$$

where θ=angle of emission of proton; A=a parameter of the dimensions of area, depending on the deuteron energy E_d and on the radius R; $\mathbf{K}=\frac{1}{2}\mathbf{k}_d-\mathbf{k}_1$ (k_d=wave number of deuteron, k_1=wave number of emitted proton); w=a numerical coefficient depending on the quantities indicated; $g(L'lL; \cos\varphi)$=an explicit function of the angle φ between k_d and K; and $P_a{}^m(\cos\theta)$ =a Legendre polynomial. The number l identifies a given term of the deuteron wave function expansion; the numbers L' and L depend on l and on the participating orbital angular momenta.

We see that Eq. (2) describes an angular distribution similar to that resulting from compound nucleus formation; but Eq. (1), containing products of spherical Bessel functions, will exhibit the kind of peaking towards small θ that one associates with a surface reaction. One will also expect to have an interference term between G_{1n} and G_{1c}, the properties of which can be visualized in general terms from the equations given.

IV. EXCHANGE STRIPPING CURVES FOR $B^{10}(d,p)B^{11*}$ (2.14 Mev)

Equations (1) and (2) were applied to the Q_1 group in $B^{10}(d,p)$, assuming that (a) the final state spin is $\frac{1}{2}$, (b) the departing proton emerges from a $p_{\frac{3}{2}}$ state, (c) the captured neutron enters a $p_{\frac{3}{2}}$ orbit, combining with the core of the target nucleus to give spin zero, and (d) the captured proton enters a $p_{\frac{1}{2}}$ state. We are not concerned to defend this scheme in detail, but it is not altogether implausible, and it is relatively amenable to calculation.

We shall not dwell on the properties of $|G_{1c}|^2$ and of the interference term. A rough evaluation of $|G_{1c}|^2$ indicated approximate isotropy, as one might expect from its form. The cross terms between G_{1n} and G_{1c} are very complicated summations over large numbers of subsidiary indices, and it appeared from inspection (without detailed calculation) that the coherence between G_{1n} and G_{1c} is largely lost as a result. Our efforts were therefore directed to a detailed evaluation of

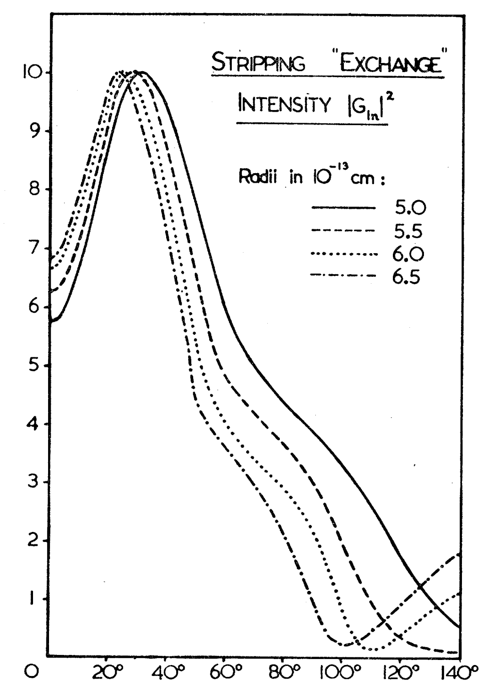

FIG. 4. The calculated exchange intensity as a function of the interaction radius R.

$|G_{1n}|^2$, to which an isotropic intensity could be added to approximate the effects of the other terms.

Most of our calculations were carried out for an assumed deuteron energy of 7.7 Mev, with an energy release of 7.10 Mev for the Q_1 proton group. The value of the interaction radius R was the only free parameter. Figure 3 shows (for $R=6.0\times10^{-13}$ cm) the effect of using various numbers of terms, through $l=3$, in the deuteron wave function expansion. The shape of the angular distribution appears to be fairly well established for $\theta\gtrsim20°$; there is some uncertainty for the region of smaller angles, but it did not seem to us profitable or reasonable (especially in view of the numerous other approximations) to go beyond $l=3$. Figure 4 then shows the angular distribution of $|G_{1n}|^2$ for various R as calculated with the inclusion of all terms through $l=3$. We see that the main features of a stripping type curve are reproduced. In Fig. 5 we show the measure of agreement with experiment that could be obtained by superimposing a conventional stripping curve (for $l_n=1$) or our curve of $|G_{1n}|^2$ (for $R=6.5\times10^{-13}$ cm) on an isotropic background of appropriate size. It may be seen that the exchange calculation leads to an appreciably better fit in the region of small angles. (It so happens that the fit is almost perfect if one terminates the deuteron wave function expansion at $l=2$ rather than at $l=3$—see Fig. 3.) On the other hand, the falloff at larger angles is not sufficiently rapid in the exchange curve, despite the choice of a quite large value of R.

Figure 6 shows the calculated variation of the exchange angular distribution with deuteron energy. The main effect appears to be an accentuation of the rise between 0° and the position of the peak as the energy is raised. This is in qualitative accord with the trend found experimentally (Fig. 2).

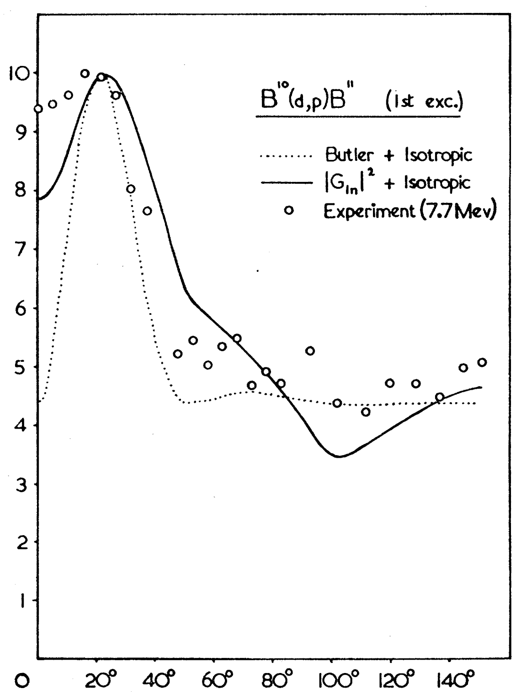

FIG. 5. Comparison of theory and experiment for $B^{10}(d,p)$ (first excited state transition) for $E_d=7.7$ Mev.

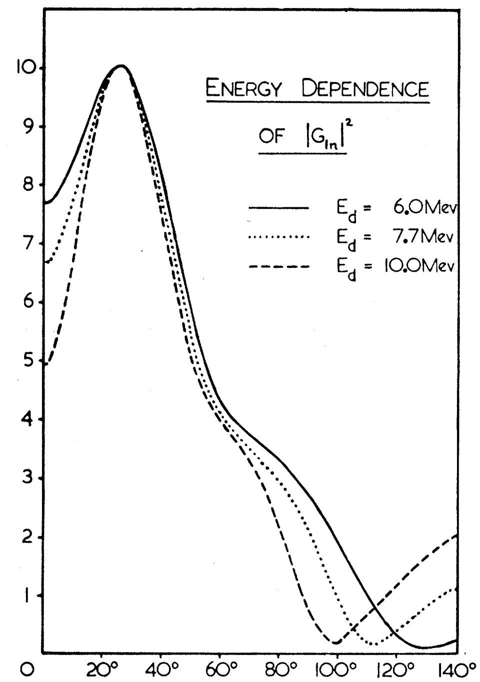

FIG. 6. Calculated energy dependence of exchange angular distribution.

It should be mentioned that, although the exchange theory[11] was worked out for an infinitely heavy target nucleus, we have sought to make rough allowance for center-of-mass motion in the present calculations by using the reduced-mass values of k_d and k_1.

V. RELATIVE CROSS SECTIONS FOR DIRECT AND EXCHANGE PROCESSES

In Eq. (62) of reference 11 we give a formula for the direct stripping intensity $|F_{1n}|^2$ evaluated analogously to $|G_{1n}|^2$. If in $|F_{1n}|^2$ we again replace the radial integrals by the values of the integrands at radius R, we find that the following expression may be used to provide an estimate for the ratio of $|F_{1n}|^2$ to $|G_{1n}|^2$ at the same angle:

$$\frac{|F_{1n}|^2}{|G_{1n}|^2}\approx\frac{1}{8\pi}\cdot\frac{(2j_d+1)(2j_r+1)}{(2l_1+1)(2l_2+1)(2l_n+1)}\times\left(\frac{R}{r_0}\right)^2\left(\frac{R}{\Delta R}\right)^2\frac{\gamma^2[u_d(r_0)]^2j_n^2(kR)}{[f_1^2R^3][f_2^2R^3]\{X\}}, \quad (3)$$

where j_d, j_r=spins of deuteron and residual nucleus; l_1, l_2, l_n=orbital momenta of initial bound proton, final bound proton, and neutron; r_0=nuclear force range ($\approx2\times10^{-13}$ cm); ΔR=effective thickness of surface reaction layer; γ^2 = "smearing factor" for deuteron in direct stripping [see Eq. (61) of reference 11]; $u_d(r_0) = \exp(-\alpha r_0)[1-\exp(-\beta r_0)]$, expressing the deuteron wave function amplitude (Hulthén) at $r=r_0$; $j_n(kR)$ = spherical Bessel function of order $l_n+\frac{1}{2}$ ($\mathbf{k}=\mathbf{k}_d-\mathbf{k}_1$) corresponding to direct stripping; $[f^2R^3]$ = numerical factor proportional to reduced width for initial (1) or final (2) proton[13]; and $\{X\}$ = sum of all contributions to the angular distribution of $|G_{1n}|^2$ after removal of extractable common factors [see Eq. (31) of reference 11] when radial integrals are placed by integrands.

Substitution of numerical values into Eq. (3) (taking into account the presence of three equivalent p-protons that could contribute to the particular exchange reaction considered) gave a ratio of peak cross sections close to 10 in favor of a direct stripping process. The value of j_r was taken as $\frac{1}{2}$ (with $l_1=l_2=l_n=1$) so as to make for a comparison of direct and exchange stripping in the same transition, and the value of ΔR was set equal to $2r_0$ (≈ 1 nucleon diameter). Putting $j_r=\frac{3}{2}$, we would have a calculated ratio of about 20 between the direct and exchange peak cross sections for the proton groups Q_0 and Q_1 in $B^{10}(d,p)$ at 7.7 Mev; the observed ratio[1] at this energy is rather less than 10.

Our calculations also suggest that as the deuteron energy rises (over the region 6–10 Mev) the ratio of $|G_{1n}|^2$ to $|F_{1n}|^2$ should fall by about 25%; this trend is in the same direction (though only about half as rapid) as that found experimentally.

VI. DISCUSSION

It should be emphasized that we have been concerned with a very specific (and certainly rare) example of a pure exchange stripping reaction. The type of problem treated here should be viewed within the broad context of the study of direct surface reactions in general (as discussed, for example, in a recent paper by Butler[14]). From our results it does, however, appear possible that an exchange contribution of the order of 10% might occur in normal stripping processes and could, through coherence of direct and exchange amplitudes, exert a significant influence on the angular distribution of such reactions. It will be noticed that our treatment leads naturally to a forward peak in the angular distribution of outgoing particles with respect to the incident deuteron direction. The approximations involved in our analysis (particularly the use of a limited series of terms in the internal wave function expansion) begin to fail for the backward directions to which the method devised by Madansky and Owen[15] is particularly adapted. It would thus appear that an exchange contribution to a stripping reaction can be expected to give rise to both forward and backward peaks; the latter will be the more noticeable effect[15] when both normal and exchange transitions are permitted by the selection rules.

One final comment can be made on the question of the polarization of the outgoing particles. Recent measurements by Hensel and Parkinson[16] indicate that the favored spin direction for the outgoing protons of the group Q_1 in $B^{10}(d,p)$ is opposite to that for other groups so far studied through deuteron stripping reactions. Such a reversal would be a natural consequence of an exchange stripping process, regardless of its exact details, because of the operation of the Pauli principle under the physically necessary condition that the exchanging protons must come close together.[17] The experimental result need not, however, be construed as particularly favoring the detailed mechanism discussed in the present paper. Reduced to its simplest terms, the formation of the first excited state of B^{11} in $B^{10}(d,p)$ is a problem in conserving angular momenta. This cannot be done by an orthodox stripping process, but becomes possible if a nucleon exchange reaction is assumed, or, alternatively, if the outgoing proton, regarded as coming from the deuteron (as in normal stripping), is deemed to undergo a spin reversal during the reaction.[7] The anomalous polarization is thus explicit in this latter treatment, and implicit in our own. Since both descriptions involve the close interaction of the whole deuteron with the target nucleus, the difference between them is perhaps more apparent than real.

ACKNOWLEDGMENT

One of us (N.T.S.E.) wishes to acknowledge the financial support of the Department of Scientific and Industrial Research (U. K.) during the performance of this work.

[13] See, for example, C. R. Lubitz, "Numerical Table of Butler-Born Approximation Stripping Cross Sections," University of Michigan, 1957 (unpublished).

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[17] We are grateful to Dr. G. R. Satchler for discussions bearing on this point.