PHYSICAL REVIEW VOLUME 109, NUMBER 4 FEBRUARY 15, 1958

Properties of s-Wave and p-Wave Neutron Resonances in Niobium*

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(Received October 24, 1957)

The neutron resonances of niobium were studied with the Argonne fast chopper. The transmission of a series of four samples was measured over the range of energy from 10 to 10 000 ev. Resonance parameters were deduced for thirteen levels, those below 700 ev. An examination of these parameters, calculations with the optical model, and a comparison of the cross sections at low and high energy all indicate that some of the resonances, even those below 100 ev, are due to a p-wave process. By using this evidence, average parameters are deduced. The radiation width is found to be 0.22 ev. The strength function $\bar{\Gamma}_n^0/\bar{D}$ for s-wave neutrons is 0.10×10^{-4}. The value of the corresponding quantity for p-wave neutrons is 4.6×10^{-4}. The resonance absorption integral deduced from the parameters is found to be in agreement with the result of a "pile oscillator" measurement.

I. INTRODUCTION

EARLY studies on the behavior of the neutron cross section of niobium seemed to give anomalous results. Measurements of the resonance-capture integral[1] gave relatively high values, indicating the presence of resonances; and yet no resonances,[2] or at most extremely small resonances,[3] were observed in direct measurements of the total cross section. The purpose of the present study was to find out whether the properties of the cross section of niobium were in any way qualitatively different from those of better understood nuclei. In our effort to fulfill this objective we have attempted to measure parameters for a large number of resonances so that the distribution of the widths could be studied and average values of parameters obtained.

II. APPARATUS

Some important components of the neutron time-of-flight spectrometer used in these measurements have not been described in the literature, although earlier systems[4] have been. The neutron chopper was a new instrument installed at the heavy-water reactor CP-5. The rotor of the chopper is roughly cylindrical in shape and rotates about a vertical axis, full speed operation being 15 000 rpm. Two sets of seven slits traverse the rotor perpendicular to the axis of rotation and to each other. Each slit is 2 in. high and 0.025 in. wide on the outer surface of the rotor, and widens to 0.055 in. at the center. This cigar-like geometry was chosen to enable slow neutrons to traverse the slit system more easily. The distribution in energy of the burst of neutrons transmitted by the rotor may be controlled to some extent by adjusting the lateral positions of entrance and exit collimators.

The neutrons in the beam transmitted by the chopper were detected by a boron-loaded liquid scintillator located 60 m from the chopper. The detector used in these measurements was detector No. 12 described by Bollinger and Thomas.[5]

Counts from the detection system were analyzed into their proper flight-time channels in a 1024-channel time analyzer which has been described by Schumann.[6] For neutron energies below 150 ev, a channel width of 1 μsec was used, whereas above this energy, $\frac{3}{4}$-μsec channels were used.

The energies involved in the measurements are deduced from the flight time of the neutrons, the zero time being determined by recording the gamma rays transmitted by the rotor when it is in the open position. It is interesting to note that for the long flight path used, the flight time of the gamma rays themselves is of a significant magnitude.

Under the operating conditions described above, the over-all resolution of the system was about 0.04 μsec per meter.

III. RESULTS

Transmission measurements were made for samples having thicknesses of 43.3, 21.7, 7.63, and 2.72 g/cm². The cross-section curve obtained is given in Fig. 1. For the thicker samples, the background was quite high, being approximately 30% of the effect being measured in the lower energy portion of the data. To insure that this background rate was accurately measured, it was determined by observing the minimum in the transmission dip which was produced at 27 ev by having a thick piece of selenium in the beam both for the open beam and sample runs. Earlier studies had shown that the background rate is independent of neutron energy over the energy range studied.

The chief experimental difficulty in the measurements

* Work performed under the auspices of the U. S. Atomic Energy Commission.
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[1] R. L. Macklin and H. S. Pomerance, *Proceedings of the International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1955* (United Nations, New York, 1956).
[2] Wu, Rainwater, and Havens, Phys. Rev. **71**, 174 (1947).
[3] Data of Bollinger, Dahlberg, and Palmer as given in *Neutron Cross Sections*, U. S. Atomic Energy Commission Report AECU-2040, Supplement 3 (Technical Information Division, Department of Commerce, Washington, D. C., 1954).
[4] Bollinger, Coté, Dahlberg, and Thomas, Phys. Rev. **105**, 661 (1957).
[5] L. M. Bollinger and G. E. Thomas, Rev. Sci. Instr. **28**, 489 (1957).
[6] R. W. Schumann, Rev. Sci. Instr. **27**, 686 (1956).

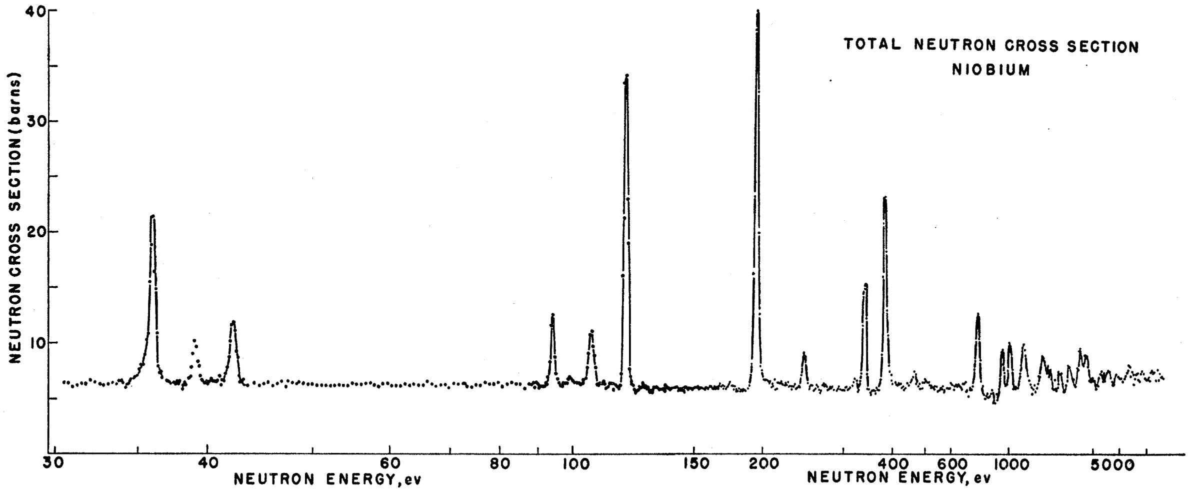

FIG. 1. The total neutron cross section of niobium. The well-defined resonance at 39 ev and the smaller peaks at 99 ev and elsewhere are due to a 0.09% impurity of tantalum in the sample.

was caused by a tantalum impurity in the samples, in spite of the fact that the niobium was of a very pure grade which contains only about 0.09% of tantalum. The small size of the resonances observed in niobium, however, caused even this small impurity to be important. A quantitative measure of the magnitude of the tantalum impurity was obtained from an area analysis of the transmission dip produced by its 39.3-ev resonance (see Fig. 1), using published values[7,8] for the parameters of the level. After a measure of the thickness of the tantalum impurity was obtained, the dip produced by the 35.9-ev level of niobium was corrected for the effects of the 35.4-ev and 36.1-ev resonances in tantalum.

Because of the small values of the peak cross sections σ_0 for the first few resonances in niobium, a second difficulty in the measurement was that of obtaining a thick sample for an area analysis. The thickest sample used (2 inches) was not enough for many of the resonances, yet it reduced the intensity in the off-resonance region to 20% of the value with an open beam. For most of the resonances, parameters were deduced by means of the standard method of area analysis.[9] For the resonances at 36.3 and 42.5 ev, however, greater accuracy was obtained by using a new method[10] which works directly with the partial area above a limited range of the transmission dip.

The resonance parameters which were obtained are listed in Table I. For the first two resonances, the radiation width Γ_γ was obtained directly from the analysis. Note that these two values are consistent with the postulate that all values of Γ_γ are approximately equal for a given nuclide. The weighted average value of 0.224 ± 0.045 ev for Γ_γ is found to be in extremely good agreement with that of neighboring nuclei.[11] It is in poor agreement, however, with the value of 0.34 ± 0.06 ev obtained by Rae[12] for the level at 194 ev.

The range in sample thicknesses used was not great enough to allow a complete analysis of the resonances above the one at 45 ev. It was therefore necessary to use the average value of Γ_γ obtained for the first two resonances in deducing the other parameters for the resonances at higher energy, assuming that they are all due to *s*-wave neutrons. However, an examination

TABLE I. Parameters for the neutron resonances of niobium. For all except the first two levels, it is assumed that $\Gamma_\gamma=0.22\pm0.045$ ev. The uncertainties listed are standard statistical errors. For *s*-wave neutrons g may be either 9/20 or 11/20 so that $2g\Gamma_n$ is almost equal to Γ_n. For *p*-wave neutrons, however, g may be 7/20, 9/20, 11/20, or 13/20 and $2g\Gamma_n$ may not be a very good approximation for Γ_n. The asymmetric shapes of the resonances at 194 and 381 ev indicate that they are due to *s*-wave neutrons.

E_0 (ev)	σ_0 (barns)	Γ (10^{-3} ev)	Γ_γ (10^{-3} ev)	$2g\Gamma_n$ (10^{-3} ev)	$2g\Gamma_n^0\times10^6$ (ev)
35.9	25.9 ± 7	206 ± 51	205 ± 51	$0.15+0.01$	25
42.2	10.7 ± 3	256 ± 84	256 ± 84	0.13 ± 0.01	20
94.3	21.8 ± 6	220 ± 46	220	0.35 ± 0.03	36
106	21.5 ± 6	220 ± 46	220	0.39 ± 0.04	37
119	137 ± 50	223 ± 46	220	2.8 ± 0.5	257
194	810 ± 300	250 ± 48	220	30.2 ± 6	2160
244	39 ± 9	222 ± 46	220	1.64 ± 0.3	105
320	34 ± 10	222 ± 46	220	1.85 ± 0.4	103
336	350 ± 100	242 ± 41	220	21.9 ± 3.7	1190
381	1115 ± 215	327 ± 38	220	107 ± 14	5500
462	97 ± 37	228 ± 46	220	7.8 ± 2.3	365
503	85 ± 31	227 ± 46	220	7.5 ± 2.2	333
744	925 ± 128	460 ± 38	220	240 ± 27	8820

[7] Harvey, Hughes, Carter, and Pilcher, Phys. Rev. **99**, 10 (1955).
[8] Fluharty, Simpson, and Simpson, Phys. Rev. **103**, 1778 (1956).
[9] Melkonian, Havens, and Rainwater, Phys. Rev. **92**, 702 (1953).
[10] Lowell M. Bollinger and Jeanne P. Marion (to be published).
[11] J. S. Levin and D. J. Hughes, Phys. Rev. **95**, 645 (1954).
[12] E. R. Rae, *Proceedings of the International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1955* (United Nations, New York, 1956).

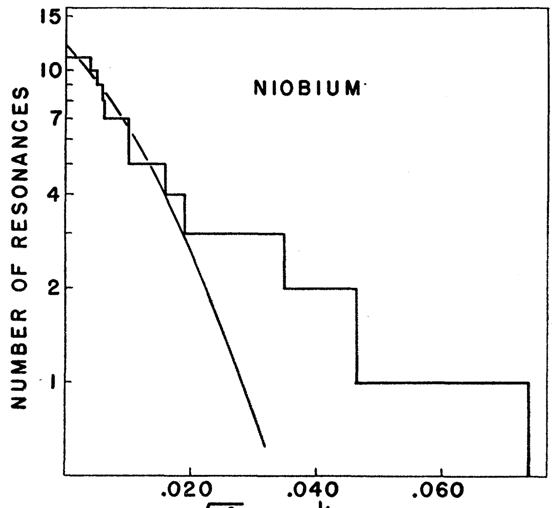

FIG. 2. Distribution in reduced neutron widths for niobium, plotting the number of resonances having reduced neutron widths $>\Gamma_n^0$ *vs* $\sqrt{\Gamma_n^0}$. The smooth curve is a Porter-Thomas distribution drawn to fit the data for the 8 narrowest resonances.

of the results obtained in this way, as given in Table I, throws some doubt on this last assumption. The distribution of reduced neutron widths Γ_n^0, which is plotted in Fig. 2, does not seem to have the exponential-like form that is usually obtained. Instead, it appears to consist of two components for which the mean widths differ by about a factor of 40. Let us consider this possibility more quantitatively under the assumption that the widths are governed by the Porter-Thomas distribution,[13] which has the form $x^{-\frac{1}{2}}e^{-x/2}$, where $x=\Gamma_n^0/\bar{\Gamma}_n^0$. Let us consider only the 11 levels below 480 ev, since the narrowest resonances could not be observed above this energy. If we fit the 8 smallest reduced widths to a Porter-Thomas distribution, as is done in Fig. 2, we find that the two largest widths are inconsistent with the distribution; the probability of finding two reduced widths greater than 2.1×10^{-3} ev is about 0.002 and the probability of finding one value greater than 5.4×10^{-3} ev is about 10^{-4}. An argument of this kind cannot be conclusive, of course, but it is suggestive that in Fig. 2 we have the superposition of two independent distributions having quite different mean widths.

One possible explanation of the abnormal distribution in Γ_n^0 for our data is that some or most of the narrow resonances are caused by *p*-wave neutrons.[14] The measurements of the individual levels give little information on this point. We only observe that the larger resonances exhibit the asymmetric shape which would be expected for *s*-wave scattering. The smaller resonances would be expected to appear symmetrical in any case because they are almost entirely due to the radiative capture process.

Some information about the likelihood that the smaller resonances are due to a *p*-wave process may be obtained by making a theoretical estimate of the expected neutron widths for such levels. The optical model of nuclear reactions,[15] with a square-well potential, has been shown to be capable of predicting the average cross section for compound-nucleus formation $\bar{\sigma}_c^0$ for *s*-wave neutrons to within a factor of 2 or 3 when appropriate parameters are used. Moreover, as will be shown in a later paragraph, this model gives results for *s*-wave neutrons that are in good agreement with our experimental data for niobium. Thus we may have some expectation that $\bar{\sigma}_c^1$, the average cross section for compound-nucleus formation for *p*-wave neutrons, may be estimated to within an order of magnitude. Using Eq. (3.10) of Feshbach *et al.*,[15] we find the ratio $\bar{\sigma}_c^0/\bar{\sigma}_c^1$ to be about 51 at an energy of 100 ev for niobium. In this calculation the nuclear well depth V_0 was 44 Mev, the absorption factor ζ was 0.03, and the nuclear radius was obtained from $R=1.45A^{\frac{1}{3}}\times10^{-13}$ cm.

In comparing the calculated ratio $\bar{\sigma}_c^0/\bar{\sigma}_c^1$ with the experimental data, let us assume that a *p*-wave process is responsible for the 7 narrowest of the first 11 resonances. This division is the most probable since the level density is expected to be proportional to the sum of all allowed values of $(2J+1)$, where J is the total angular momentum of the state involved. Taking into account the nuclear barrier penetration factor for the assumed *p*-wave resonances, we find that the experimental data give a value of 24 for $\bar{\sigma}_c^0/\bar{\sigma}_c^1$ at 100 ev. The agreement between this measured value of 24 and the calculated value of 51 is as close as could be expected.

Further information concerning the expected widths for *p*-wave resonances is obtained by comparing the off-resonance cross section of 6.2 barns, as measured between 10 and 100 ev, with the average cross section of 9.2 barns[16] at 100 kev. Again using the relationships given by Feshbach *et al.*,[15] it is found that the off-resonance cross section of niobium decreases by 0.1 barn between 100 ev and 100 kev. Thus the measured cross section for compound-nucleus formation $\bar{\sigma}_c$ at 100 kev is 3.1 barns. Assuming all the levels observed in our measurements to be *s*-wave in nature, we find that the *s*-wave cross section $\bar{\sigma}_c^0$ at 100 kev is only 0.2 barn. Clearly the major part of $\bar{\sigma}_c$ must be due to *p*-waves. Now if we again assume that the 7 narrowest resonances of Table I give a measure of $\bar{\sigma}_c$ at low energies for *p*-waves, we would expect $\bar{\sigma}_c$ at 100 kev to be 3.8 barns. This value is in surprisingly good agreement with the measured value of 3.1 barns.

All the evidence presented above, namely the

[13] C. E. Porter and R. G. Thomas, Phys. Rev. **104**, 483 (1956).

[14] The importance of *p*-wave resonances at a somewhat greater energy has already been shown for niobium. See H. H. Barschall and S. E. Darden, Phys. Rev. **100**, 1242 (1955).

[15] Feshbach, Porter, and Weisskopf, Phys. Rev. **96**, 448 (1954).

[16] Miller, Adair, Bockelman, and Darden, Phys. Rev. **88**, 83 (1952).

distribution in reduced widths, the results of calculations with the optical model, and the value of $\bar{\sigma}_c$ at 100 kev, are inconsistent with the assumption that only s-wave resonances were observed in our measurements. Moreover, the smaller resonances were found to be of the expected width for p-wave excitation. We conclude, therefore, that some of the narrow resonances are p-wave in nature and, for the purpose of deriving average parameters, will somewhat arbitrarily assume those at 35.9, 42.2, 94.3, 106, 244, 320, and 462 ev to be p-wave.

The probability that the resonances at 35.9 and 42.2 ev are excited by p-wave neutrons introduces an unexpected uncertainty into our analysis of the other levels, since the radiation width Γ_γ is determined for these two levels and this width is used to determine the neutron widths for the other levels. Experimental evidence on the ratio of the radiation widths for s-wave and p-wave neutrons is essentially nonexistent. Niobium is one of the most favorable nuclides for which to attempt to measure the ratio but the existing data are apparently not precise enough to be useful. Comparing the radiation width of 0.34 ± 0.06 ev measured by Rae[12] for a resonance that is surely caused by s-wave neutrons and the width of 0.22 ± 0.05 ev measured in the present experiment for resonances that are thought to be due to p-wave neutrons, we see that, although there is a suggestion of a real difference between the two values, the errors are too large for us to arrive at a certain conclusion. The theory of radiation widths is also somewhat uncertain at the present time. Perhaps the most refined treatment has been given by Cameron.[17] He finds,[18] using experimental information concerning neighboring nuclei, that the s-wave radiation width for niobium should be 0.304 ev. Again, in view of the uncertainties involved, we have no convincing evidence that the radiation widths differ for s-wave and p-wave neutrons. The results of Table I will, therefore, be allowed to stand as they are. In any case, the value that is obtained for the neutron width is not very dependent on the assumed value of Γ_γ.

Having settled the question of the nature of the levels which we have observed, let us attempt to derive average parameters from the data of Table I. If the narrower resonances are p-wave, their average width increases rapidly with increasing energy, as is observed, and the probability of missing levels is decreased. For this reason it is probable that most of the levels below 480 ev were detected. It is estimated that approximately one narrow resonance was missed, however, so we will assume that there are 12 levels below 480 ev which can be excited by s-wave and p-wave neutrons. Under the assumption that the level density is proportional to the sum of all allowed values of $(2J+1)$, it is expected that only 4 of the 12 resonances are excited by s-waves. Thus the average level spacing $\bar{D}$ per spin state is 240 ± 55 ev, where the error listed is a probable error for an exponential distribution in spacings. Note that, under the assumptions made, the error in $\bar{D}$ depends on the total number of levels observed and not on the number that are assigned as being s-wave scattering.

One of the more important quantities that can be obtained from the data of Table I is a value for the strength function $\bar{\Gamma}_n^0/\bar{D}_0$ for s-wave scattering. Fortunately, the ambiguity as to which levels are excited by p-waves is unimportant here because of their small contribution at low energies. Moreover, we may use a somewhat wider range of energy than was used in obtaining $\bar{D}$ because, as has been pointed out elsewhere, no significant error is made if small resonances are missed. For the levels below 850 ev, then, we obtain the value of 0.10×10^{-4} for $\bar{\Gamma}_n^0/\bar{D}_0$, with probable errors of $+55\%$ and -36%. In this case the quoted errors depend entirely on the number of s-wave resonances included in the average and on the assumption that both Γ_n^0 and D are distributed exponentially.

Although the strength function of niobium is one of the smallest that has yet been measured, it is not in disagreement with the general trend for other nuclides in the same range of A, particularly for the nuclides heavier than niobium.[7] Moreover, it is in excellent agreement with the prediction of the optical model for a square-well potential of the form $V=-V_0(1+i\zeta)$. For $V_0=44$ Mev and $\zeta=0.03$, the parameters which give the best fit to the data for lighter nuclides, the optical model gives a value of 0.136×10^{-4} for $\bar{\Gamma}_n^0/\bar{D}$. The experimental value is in very much poorer agreement with the result obtained for a potential having a diffuse edge. Weisskopf[19] gives a curve for such a calculation from which we read a value of about 1.0×10^{-4}.

The strength function S_1 for p-wave neutrons may also be determined from the data. Because of the scarcity of experimental results this quantity seems not to have been defined in the literature. We will therefore use an approach that is analogous to the one commonly used for s-wave neutrons at low energies. Let S_1 be defined by requiring that it play the same role for p-wave neutrons as Γ_n^0/D does for s-wave neutrons in determining the average cross section for compound-nucleus formation, namely let it be required that

$$\bar{\sigma}_c^1=\left(\frac{2\pi^2}{k^2}E^{\frac{1}{2}}\frac{x^2}{x^2+1}\right)(2l+1)S_1 \qquad (1)$$

at low energy. Here E is the incident neutron energy and $x=kR$, where k is the wave number of the incident neutron and R is the nuclear radius. The quantum number for the angular momentum of the incident neutron is denoted by l and is, of course, unity for

[17] A. G. W. Cameron, Chalk River Report UK/C 6/123, 1957 (unpublished).

[18] A. G. W. Cameron (private communication).

[19] V. Weisskopf, Revs. Modern Phys. 29, 174 (1957).

p-wave neutrons. A comparison of Eq. (1) with Eq. (3.10) of Feshbach *et al.*[15] shows that at low energy S_1 is almost independent of the neutron energy and that it may be expected to vary smoothly with the nuclear radius.

Accepting Eq. (1) as a definition of the strength function for p-wave neutrons, it is easily shown that it is related to the parameters of individual resonances by

$$S_1=\frac{1}{\Delta E}(2l+1)^{-1}\sum_r (g\Gamma_n{}^1)_r, \qquad (2)$$

where $\Gamma_n{}^1$ is defined as being $\Gamma_n E_0{}^{-\frac{1}{2}}(x_0{}^2+1)x_0{}^{-2}$, with E and x evaluated at the resonance energy. The sum of $g\Gamma_n{}^1$ is over all the p-wave resonances r in an energy range ΔE. The statistical weight factor g is given by

$$g=\tfrac{1}{2}(2J+1)/(2I+1),$$

where I is the spin of the target nucleus and J of the compound nucleus.

Note that Eq. (2) is just the relationship commonly used for s-wave resonances if we replace $\Gamma_n{}^1$ by $\Gamma_n{}^0$. It is also important to notice that, whereas for s-wave neutrons the strength function is defined as being $\bar{\Gamma}_n{}^0/\bar{D}$, the quantity S_1 is not necessarily equal to $\bar{\Gamma}_n{}^1/\bar{D}$. The reason for this inequality is that for a p-wave resonance having a given J the width Γ_n may consist of two components due to two spin states. For example, in niobium, for which $I=9/2$, if the vector sum of I and the intrinsic spin of the neutron is 4, J may be 3, 4, or 5, whereas for a vector sum of 5, J may be 4, 5, or 6. Thus there are two spin states for which J is either 4 or 5. If the reduced neutron width $\Gamma_n{}^1$ had been defined in such a way that $(\Gamma_n{}^1)_i$ referred to a particular spin state i, then it would be true that S_1 would be equal to $(\bar{\Gamma}_n{}^1)_i/\bar{D}_i$. In the above discussion it has been tacitly assumed that $(\bar{\Gamma}_n{}^1)_i/\bar{D}_i$ is the same for all of the possible spin states.

Returning now to the experimental results, if we again assume that the 7 narrowest resonances at an energy less than 480 ev are due to p waves, we obtain a value of 5.5×10^{-4} for S_1. This result is uncertain by about a factor of two. A more reliable value is that which gives the experimental result for the difference in the average cross section at 100 kev and the off-resonance cross section at low energy, namely $S_1=4.6\times10^{-4}$. It is estimated that this latter value is accurate to within about 25%.

Since it was what appeared to be the anomalously high resonance-absorption integral of niobium that partially prompted the present study, it is now of interest to deduce this quantity from our results. The absorption integral is defined as

$$\Sigma_\gamma=\int_{E_1}^{E_2}\sigma_\gamma(dE/E),$$

where σ_γ is the radiative-capture cross section. The lower limit E_1 is the energy of cadmium cutoff, about 0.4 ev, and the upper limit is an equivalent upper limit for the pile spectrum, say 2 Mev; in any case, the exact values of the limits are unimportant. Let us separate the integral into four components; the contributions from observed resonances below 750 ev, from a $1/v$ term in the cross section, from s-wave resonances above 750 ev, and from p-wave resonances above 750 ev. The contribution of the observed levels, calculated directly from the parameters given in Table I, is 4.35 ± 0.6 barns. The $1/v$ component of the cross section which is determined from the absorption cross section at thermal energy, gives a contribution of 0.47 ± 0.1 barn. The effects of s-wave and p-wave levels above 750 ev were calculated numerically and found to be 1.67 ± 0.7 and 1.79 ± 0.5 barns, respectively. The whole resonance abosrption integral is, therefore, 8.3 ± 1.1 barns. This result is in perfect agreement with the value of 8.3 barns obtained by Langsdorf by the "pile oscillator" method (quoted by Macklin *et al.*[1]) but is much higher than the value of 3.87 barns that was obtained in an activation measurement.[1]

In summary, it seems clear that the neutron cross section of niobium is unusual but entirely understandable. The original indication that its behavior was anomalous, namely that a high-resonance integral was combined with an observation of only negligibly small resonances, is, in fact, real. This contradiction has now been shown to be due to the chance occurrence of atypically narrow neutron widths, probably caused by a p-wave process, for the first few levels. It might well be expected that the cross sections of neighboring nuclides will exhibit similar features when more complete measurements are made.