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Weak-Field Magnetoresistance in *p*-Type Silicon

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Measurements of the three weak-field magnetoresistance coefficients and the Hall mobility have been made at a number of different temperatures between 77°K and 350°K on *p*-type silicon samples ranging in resistivity from 0.15 to 115 ohm-cm. The results indicate a marked temperature dependence of the anisotropies of the energy band structure and/or the scattering. The weak-field magnetoresistance coefficients happen to satisfy nearly the same symmetry relations above about 275°K as those satisfied by *n*-type germanium.

1. INTRODUCTION

A RECENT paper by one of us[1] (DL), which will hereafter be referred to as I, has presented the results of measurements of galvanomagnetic effects on 35 and 85 ohm-cm *p*-type silicon samples at temperatures between 77°K and 320°K. The magnetoresistance data reported in I were taken only at the two fixed temperatures of 77°K and 300°K. Because of the wide discrepancies found between the experimental galvanomagnetic effects in *p*-Si and the predictions of the best available theory of these effects,[2] it seemed important to continue the experimental investigation of this material in order to obtain sufficient data to form a basis for possibly constructing a more satisfactory theoretical model. The present paper gives the results of just such an extension of the galvanomagnetic experiments on *p*-Si. The emphasis here is on measurements of magnetoresistance effects over the entire temperature range of 77°K to 350°K in samples having room temperature resistivities ranging from 0.15 to 115 ohm-cm and in magnetic fields small enough to satisfy weak-field conditions.

The current density in a cubic crystal in the presence of weak electric and magnetic fields is given by[3]

$$\mathbf{j}=\sigma_0\mathbf{E}+\alpha\mathbf{E}\times\mathbf{H}+\beta\mathbf{E}H^2+\gamma\mathbf{H}(\mathbf{E}\cdot\mathbf{H})+\delta T\cdot\mathbf{E}, \qquad (1)$$

where T is a diagonal tensor with elements H_1^2, H_2^2, and H_3^2, and the subscripts refer to the three axes of cubic symmetry. We shall be interested in the three coefficients β, γ, and δ, since they give a complete characterization of the change of conductivity of a cubic crystal in a weak magnetic field. Nearly all electronic theories explicitly determine the current as a function of applied electric and magnetic fields, and so β, γ, and δ are generally the most convenient coefficients for quantitative comparison with theory; however, conventional magnetoresistance experiments are more closely related to the coefficients b, c, and d of the inverse equation,[4]

$$\mathbf{E}=\rho_0[\mathbf{j}+a(\mathbf{j}\times\mathbf{H})+b\mathbf{j}H^2+c\mathbf{H}(\mathbf{j}\cdot\mathbf{H})+dT\cdot\mathbf{j}], \qquad (2)$$

since in these experiments one holds j constant and measures the change in electric field due to an applied magnetic field. Simple relations exist between β, γ, and δ and b, c, and d, but these relations also involve the square of the Hall mobility μ_H. We have concentrated on measuring b, c, d, and μ_H in order to obtain complete information on the weak-field magnetoresistance in *p*-Si.

In Sec. 2 of this paper we describe the experimental methods and in Sec. 3 present the results obtained. Even though the primary purpose of this paper is simply to present the experimental results, we have included in Sec. 4 a short discussion of the results in which several interesting and possibly significant features of them are pointed out with respect to the valence band structure of silicon.

2. EXPERIMENTAL METHODS

The *p*-type silicon samples used in these experiments were cut from single crystals grown by the Czochralski method from duPont hyperpure material. The 115 ohm-cm samples were from a crystal which had been regrown and cropped several times in order to rid the silicon, by segregation, of as much of the impurities other than boron as possible.[5] Thus, we expect that the density of boron centers was probably much greater than that of any other impurity in these samples, although we have no direct evidence for this conclusion other than the fact that the resistivity profile was typical of that of a boron-doped crystal. All of the lower resistivity samples were from crystals which had been purposely doped with boron. The samples of a particular resistivity were taken from adjacent regions of the same crystal so that they would have similar properties.

The experiments to be described were done on samples cut in the conventional form of "bridges," with electrical contacts made by alloying aluminum into the silicon in the proper positions for resistivity and Hall effect measurements. Two kinds of crystallographic orientation were used. In one kind the direction of current flow was along the [100]-axis, and in the other it was along the [110]-axis. The Hall contacts in both cases were placed such that the magnetic field would be parallel to the [001]-axis in a Hall effect measurement. The room temperature resistivities and crystallographic

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[1] D. Long, Phys. Rev. **107**, 672 (1957).

[2] B. Lax and J. G. Mavroides, Phys. Rev. **100**, 1650 (1955); J. G. Mavroides and B. Lax, Phys. Rev. **107**, 1530 (1957). (Referred to in text as LM.)

[3] F. Seitz, Phys. Rev. **79**, 372 (1950).

[4] G. L. Pearson and H. Suhl, Phys. Rev. **83**, 768 (1951).

[5] J. A. Burton, Physica **20**, 845 (1954).

directions of current flow for all the samples studied are listed in Table I.

The magnetic field for the experiments was provided by a Varian 12-inch electromagnet, and field strengths were measured to within ±1% by a calibrated Rawson rotating-coil gaussmeter. Temperatures constant at any value above 77°K were achieved simply by mounting the sample in an enclosed copper holder around which a heater wire was wound and by placing the holder above boiling nitrogen in a large Dewar flask. It was possible to compensate the cooling effect of the nitrogen with the heater in such a way that temperature drift was practically negligible during the course of a measurement. The sample was immersed in liquid nitrogen for the 77°K measurements.

3. EXPERIMENTAL RESULTS

Figures 1 and 2 show plots of the temperature dependences of the electrical resistivity and Hall mobility for samples *SH8A*, *SH7D*, and *SH6D*. These plots are intended to provide "background" information for the magnetoresistance results and in particular to illustrate the effects of different impurity concentrations on the electrical properties of p-Si. The resistivity curves in Fig. 1 are normalized to the resistivity values at 300°K, but the room temperature resistivity of each of the samples has already been given in Table I.

TABLE I. Resistivities and orientations of p-Si samples.

Sample number	Direction of current	Resistivity (ohm-cm)
SH8A	[110]	115
SH8C	[100]	115
SH9A	[110]	10
SH9C	[100]	10
SH7D	[110]	1.8
SH7C	[100]	1.8
SH10A	[110]	0.56
SH10C	[100]	0.56
SH6D	[110]	0.15
SH6A	[100]	0.15

The resistivity of sample *SH8A* follows a $T^{2.7}$ law above about 175°K, indicating that the lattice-scattering mobility varies as $T^{-2.7}$ in p-type silicon (at least above 175°K), since the free hole density is expected to be constant above this temperature in such a pure sample. This result is in agreement with previous investigations.[1,6] The Hall mobility of *SH8A* follows approximately a $T^{-2.9}$ law in the lattice-scattering range, because the ratio of Hall mobility to conductivity mobility exhibits a small negative temperature dependence in p-Si.[1,7] The importance of ionized-impurity scattering and impurity de-ionization at the lower temperatures is evident in the resistivity and Hall mobility curves for the samples of lower resistivity.

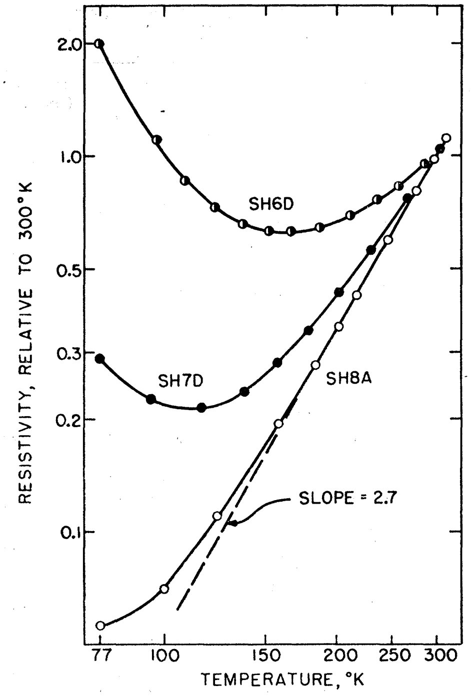

FIG. 1. Resistivity *vs* temperature in p-Si: samples *SH6D*, *SH7D*, and *SH8A*.

Let us now consider the magnetoresistance measurements which have been made on p-Si. In performing the experiment one actually measures a parameter M defined by the relation,

$$M_j^H = \Delta\rho/\rho_0 H^2, \tag{3}$$

where the subscript and superscript specify the crystal-

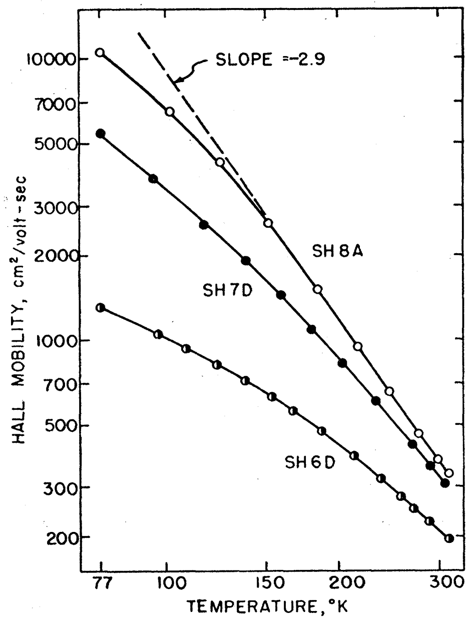

FIG. 2. Hall mobility *vs* temperature in p-Si: samples *SH6D*, *SH7D*, and *SH8A*.

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[6] G. W. Ludwig and R. L. Watters, Phys. Rev. 101, 1699 (1956).
[7] F. J. Morin and J. P. Maita, Phys. Rev. 96, 28 (1954).

TABLE II. Weak-field magnetoresistance M-parameters and Hall mobilities in p-Si samples.

Temperature (°K)	M-parameters (cm²/volt-sec)² M_{110}^{001}	M_{110}^{110}	$M_{110}^{1\bar{1}0}$	Hall mobility (cm²/volt-sec)
Sample *SH8A* (115 ohm-cm)				
77	14.6×10^7	3.75×10^7	14.9×10^7	10 400
123	4.46×10^7	1.22×10^7	4.91×10^7	4320
185	8.87×10^6	3.08×10^6	10.9×10^6	1520
217	4.04×10^6	1.56×10^6	5.33×10^6	950
246	21.1×10^5	8.45×10^5	28.8×10^5	660
275	11.3×10^5	4.80×10^5	15.9×10^5	470
299	7.36×10^5	3.25×10^5	10.62×10^5	370
310	6.06×10^5	2.74×10^5	8.82×10^5	335
324	4.80×10^5	2.21×10^5	6.97×10^5	300
334	4.12×10^5	1.90×10^5	6.07×10^5	275
349	3.31×10^5	1.58×10^5	4.88×10^5	245
Sample *SH9A* (10 ohm-cm)				
77	4.59×10^7	1.28×10^7	4.81×10^7	7450
193	3.66×10^6	1.41×10^6	4.81×10^6	1050
246	14.0×10^5	6.06×10^5	19.6×10^5	580
274	8.24×10^5	3.64×10^5	11.98×10^5	445
299	5.90×10^5	2.73×10^5	8.76×10^5	345
326	3.84×10^5	1.89×10^5	5.78×10^5	280
Sample *SH7D* (1.8 ohm-cm)				
77	17.3×10^6	5.20×10^6	18.6×10^6	5460
143	3.12×10^6	1.19×10^6	4.01×10^6	1800
223	9.81×10^5	4.29×10^5	14.15×10^5	670
275	4.90×10^5	2.18×10^5	7.31×10^5	400
299	3.63×10^5	1.74×10^5	5.54×10^5	320
324	2.65×10^5	1.24×10^5	4.09×10^5	270
341	2.22×10^5	1.06×10^5	3.44×10^5	210
Sample *SH10A* (0.56 ohm-cm)				
77	6.89×10^6	2.04×10^6	7.78×10^6	3730
197	6.24×10^5	2.73×10^5	8.73×10^5	680
275	2.55×10^5	1.27×10^5	3.90×10^5	355
298	2.12×10^5	1.06×10^5	3.28×10^5	305
Sample *SH6D* (0.15 ohm-cm)				
77	14.2×10^5	5.03×10^5	16.8×10^5	1310
246	12.1×10^4	6.13×10^4	18.1×10^4	300
298	7.62×10^4	3.95×10^4	11.50×10^4	215

lographic directions of the current and field respectively, $\Delta\rho$ is the increase of resistivity caused by the magnetic field H, and ρ_0 is the zero-field resistivity. Five different M's can be measured on the two kinds of oriented samples previously described, and the relations between the M's and the three magnetoresistance coefficients[4] are given below.

$$M_{110}{}^{001}=b, \tag{4a}$$

$$M_{110}{}^{110}=b+c+d/2, \tag{4b}$$

$$M_{110}{}^{1\bar{1}0}=b+d/2, \tag{4c}$$

$$M_{100}{}^{001}=b, \tag{4d}$$

$$M_{100}{}^{100}=b+c+d. \tag{4e}$$

We have measured the various M's on the samples listed in Table I at a number of different temperatures between 77°K and 350°K under weak-field conditions; i.e., the measurements were made at field strengths low enough that the effect satisfied the criterion, $\Delta\rho/\rho_0\propto H^2$. The weak-field Hall mobility was also measured at each of these temperatures. The values of the three M_{110}'s, which are really the only data needed to obtain b, c, and d, are given in Table II along with the Hall mobilities. It can be seen in Table II that more data have been taken on the highest resistivity sample (*SH8A*) than on any of the others, because we have been most interested in the properties of the purest silicon.

It is estimated that the error involved in the measurement of the field-induced change of resistivity was no greater than ±2% for most of the M-values in Table II. It might have been slightly larger in the lowest resistivity samples and at the lowest temperatures. This error was largely due to very slow temperature drifts during the course of a measurement. These error estimates do not include any of the small errors which may have affected each M_{110} at a given temperature in the same way, such as errors in field calibration, systematic errors, etc. The error in each Hall mobility value should not be greater than about ±5%.

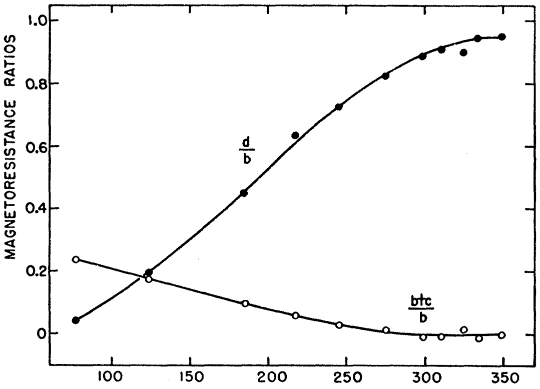

FIG. 3. Ratios of weak-field magnetoresistance coefficients *vs* temperature in 115 ohm-cm p-Si, sample *SH8A*.

According to Eqs. (4), M_{100} data should provide a check on the M_{110} results. We have measured the two M_{100} parameters on each of the [100]-oriented samples of Table I at a few of the temperatures at which corresponding M_{110} data had been obtained, and have in fact found the results to be entirely consistent with the values of b, c, and d deduced from the M_{110} measurements.

The results listed in Table II are plotted in Figs. 3 through 6 in the form of curves of the ratios $(b+c)/b$ and d/b *vs* temperature in order to illustrate more clearly the nature of the temperature dependence of the magnetoresistance in p-Si. It can be seen that the curves for all of the five sample resistivities studied exhibit the same general trends. The ratio d/b is positive and large around room temperature and above but decreases rapidly to become very small at 77°K, although still positive. The ratio $(b+c)/b$ is very nearly zero above about 275°K in each sample; however, it does seem to have a small negative value in the samples of intermediate resistivities, and this value is a little too large to lie within the expected experimental error if $(b+c)/b$ were really to be zero in these samples. In any case the value of $(b+c)/b$ appears to be exactly zero above around 275°K in the purest (115 ohm-cm) silicon, as seen in Fig. 3. In all the samples this ratio then slowly increases with decreasing temperature down to 77°K.

It should be mentioned here that the values of $M_{110}{}^{001}$, $M_{110}{}^{110}$, and $M_{110}{}^{1\bar{1}0}$ which were given for 85 ohm-cm p-Si at 77°K and 300°K in our earlier paper[1] are in good agreement with the values found in the present study for 115 ohm-cm p-Si at the same two temperatures. This agreement is to be expected, since the samples were of nearly the same resistivity.

4. DISCUSSION OF RESULTS

It is well known that magnetoresistance is generally quite sensitive to anisotropies in the surfaces of constant energy in k-space and in the scattering of free carriers and can therefore be used as a tool for the study of these properties.[8] The energy surface (effective mass) and scattering (relaxation time) parameters always appear in combination in the formulas, thus making it difficult when analyzing experimental results to separate clearly the anisotropies of the two; however, magnetoresistance is nonetheless probably the most fruitful type of effect available for the study of these anisotropies at temperatures at which cyclotron resonance experiments[9] cannot be done.

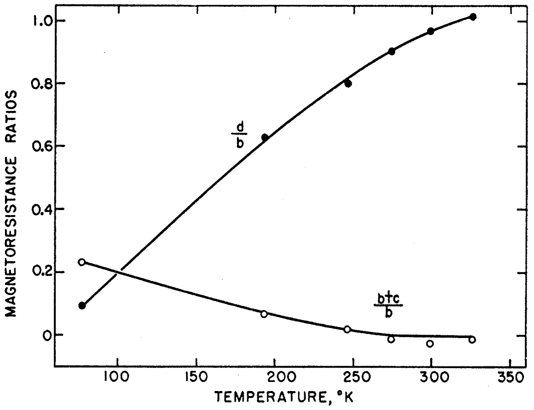

FIG. 4. Ratios of weak-field magnetoresistance coefficients *vs* temperature in 10 ohm-cm p-Si, sample *SH9A*.

In trying to relate the results of magnetoresistance measurements to the energy surface and scattering anisotropies it is most informative to examine the symmetry relations among the three weak-field coefficients b, c, and d of Eq. (2). The relative values and signs of these three coefficients are determined by the model assumed for the forms of the energy surfaces and of the scattering, and so one can gain information about these basic properties of a material simply by comparing the observed relations among b, c, and d with those predicted by various models. The remainder of this section consists principally of a discussion of our results in terms of the symmetry relations among the three magnetoresistance coefficients.

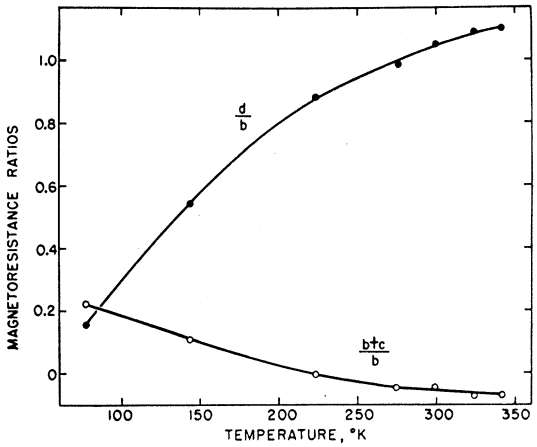

FIG. 5. Ratios of weak-field magnetoresistance coefficients *vs* temperature in 1.8 ohm-cm p-Si, sample *SH7D*.

It was demonstrated in I that there are serious discrepancies between the observed weak-field magnetoresistance in p-Si at 77°K and at 300°K and the predictions of a model, due to Lax and Mavroides,[2] which is based on the assumptions of the warped-sphere energy surface structure of the valence band edge that

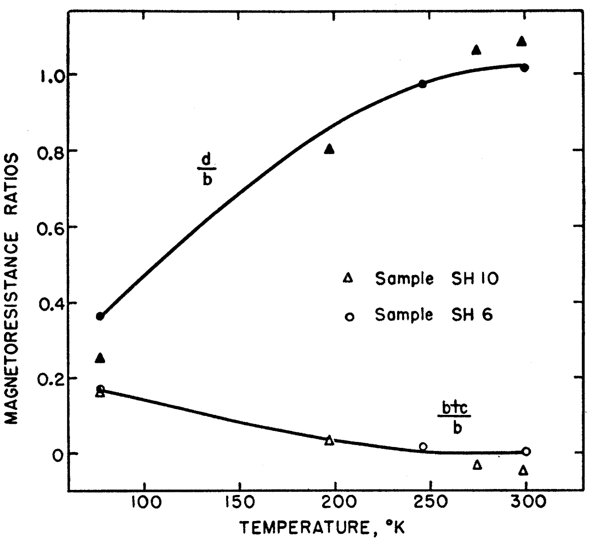

FIG. 6. Ratios of weak-field magnetoresistance coefficients *vs* temperature in 0.56 and 0.15 ohm-cm p-Si, samples *SH10A* and *SH6D*.

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[8] See, for example, A. H. Wilson, *The Theory of Metals* (Cambridge University Press, Cambridge, 1953), second edition, chapter VIII.

[9] Dresselhaus, Kip, and Kittel, Phys. Rev. **98**, 368 (1955); Dexter, Zeiger, and Lax, Phys. Rev. **104**, 637 (1957).

has been deduced from cyclotron resonance experiments[9] done at 4°K and of a scalar (isotropic) energy-dependent relaxation time. Interband scattering between the two sets of warped spheres is neglected except insofar as it can be described by a suitable energy dependence of the relaxation time. This model is of course also based on the fundamental assumptions that the Boltzmann transport equation applies and that a relaxation time does exist.[8]

One can show from the calculations of Lax and Mavroides[2] that their model predicts the following symmetry relations among the weak-field magnetoresistance coefficients:

$$b+c>0, \tag{5a}$$

$$d<0. \tag{5b}$$

The experimental results which were presented in Sec. 3 obey the relations given below in the indicated ranges of temperature.

$$\left.\begin{array}{l} b+c>0 \\ d>0 \end{array}\right\} \text{ from } 77°\text{K to } \sim 275°\text{K}. \quad \begin{array}{r}(6\text{a})\\(6\text{b})\end{array}$$

$$\left.\begin{array}{l} b+c=0 \\ d>0 \end{array}\right\} \text{ from } \sim 275°\text{K to } 350°\text{K}. \quad \begin{array}{r}(7\text{a})\\(7\text{b})\end{array}$$

Actually, relation (7a) was not exactly satisfied in the samples of intermediate resistivities, as was pointed out previously, but the observed *negative* deviation of $(b+c)/b$ from zero was never greater than 7%.

Thus, neither of the relations predicted by the LM model is satisfied by *p*-Si above about 275°K, and only (5a) is satisfied between this temperature and 77°K; however, d is very small at 77°K, and the trend of d with decreasing temperature indicates that it may reverse sign to become negative somewhere below 77°K in which case both (5a) and (5b) would then be satisfied. The valence band edge is known to have the structure at 4°K on which the LM model is based so that one might expect the model to apply at a low enough temperature, provided of course that the scattering mechanisms are not too complex to be consistent with the theoretical assumptions. On the other hand, one would not really expect the model to apply very well at temperatures of 77°K and above. As was pointed out in I, the model is based on the structure of the very edge of the valence band, at $k=0$; whereas, the band shapes at energies only slightly below the $k=0$ edge ($\sim kT$ at 77°K and above) is quite different because of the relatively small spin-orbit splitting in silicon.[10] Nevertheless, it does not seem probable that this situation would lead of itself to such a marked change of the symmetry relations among b, c, and d, since at all energies the surfaces of constant energy still have the form of warped spheres.[10] Of course, sufficiently complex scattering or other effects could invalidate this intuition, and one would have to calculate the three coefficients for this situation anyway to be sure of the effect on b, c, and d.

It is interesting and perhaps important to note that the symmetry relations (7a) and (7b), which are satisfied (or almost satisfied) by *p*-Si at the higher temperatures, *happen* to be the same as those predicted by the 4-spheroid (or 8-spheroid) band structure model which Abeles and Meiboom[11] and Shibuya[12] have employed successfully to explain magnetoresistance effects in *n*-type germanium. This model assumes four or eight equivalent band extrema located along [111] axes in *k*-space with the surfaces of constant energy at each extremum being spheroids. The relaxation time can either be a scalar or have the same type of anisotropy as the band structure.[13] This model is the only one of those which have so far been examined in attempting to explain magnetoresistance effects in cubic semiconductors[2,3,11–13] which predicts *both* conditions (7a) and (7b). Applicability of such a model to *p*-Si would, however, require that a change occur as the temperature increases of the point (or points) in *k*-space which determines the valence band edge, but such a change seems unlikely and there is no other evidence for it at present. On the other hand, to the writers' knowledge no other experimental result has yet been obtained on *p*-Si in the vicinity of room temperature which is definitely inconsistent with an explanation based on a 4-spheroid (or 8-spheroid) model.[14]

In summary, the experimental results indicate a marked temperature dependence of the anisotropies of the energy band structure and/or the scattering in *p*-Si, but it is uncertain just what mechanisms are contributing to these temperature dependences. It is hoped that the results presented in this paper will stimulate further investigation of band structure and scattering in *p*-type silicon.

5. ACKNOWLEDGMENTS

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[10] E. O. Kane, J. Phys. Chem. Solids **1**, 83 (1956).

[11] B. Abeles and S. Meiboom, Phys. Rev. **95**, 31 (1954).

[12] M. Shibuya, Phys. Rev. **95**, 1385 (1954).

[13] C. Herring and E. Vogt, Phys. Rev. **101**, 944 (1956).

[14] See, for example, C. S. Smith, Phys. Rev. **94**, 42 (1954); Morin, Geballe, and Herring, Phys. Rev. **105**, 525 (1957); G. G. Macfarlane and V. Roberts, Phys. Rev. **98**, 1865 (1955); W. C. Dash and R. Newman, Phys. Rev. **99**, 1151 (1955); W. G. Spitzer and H. Y. Fan, Phys. Rev. **106**, 882 (1957); T. H. Geballe and G. W. Hull, Phys. Rev. **98**, 940 (1955).