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Transient Recombination of Excess Carriers in Semiconductors

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The recombination equations for a system containing an arbitrary number of Shockley-Read recombination centers are formulated. Transient solutions are obtained for the decay following the injection of a pulse of carriers into a system containing one or two centers. The specific cases considered include (1) the simple one-center case, which enables us to discuss (a) the situation for a large injection of carriers, (b) recombination through donors in n-type or acceptors in p-type material, and (c) recombination through centers in the presence of direct recombination; (2) the case of a recombination center with a temporary trap, and (3) the case of two recombination centers.

INTRODUCTION

Steady-state recombination through a single recombination center has been treated in detail by Shockley and Read[1] and Hall.[2] The transient case has been discussed[3] and a solution has been given by Sandiford,[4] a multicenter situation has been analyzed by Rose,[5] and the trapping process has been treated by Hornbeck and Haynes.[6]

The attempt is here made to examine certain elementary cases from a more rigorous point of view.

FORMULATION OF THE PROBLEM

The general recombination problem for a nondegenerate system may be formulated in terms of the capture rates U_n and U_p, for electrons and holes by impurity centers, given by Shockley and Read[1]:

$$\begin{aligned} U_{ni}&=\alpha_{ni}[N_i^0\delta n-(n_0+n_{1i}+\delta n)\delta N_i],\\ U_{pi}&=\alpha_{pi}[N_i^-\delta p+(p_0+p_{1i}+\delta_p)\delta N_i]. \end{aligned} \tag{1}$$

(The symbols are defined in Table I.) The rate of recombination by direct processes not requiring the action of a recombination center is

$$U_d=\alpha_d(n_0\delta p+p_0\delta n+\delta n\delta p). \tag{2}$$

If the direct process is radiative, then

$$\alpha_d=R_r/n_i^2, \tag{3}$$

where R_r has been given by van Roosbroeck and Shockley.[7]

The differential equations governing recombination may then be expressed in terms of these rates:

$$\begin{aligned} (d/dt)\delta n&=g-U_d-\textstyle\sum_i U_{ni},\\ (d/dt)\delta p&=g-U_d-\textstyle\sum_i U_{pi},\\ (d/dt)\delta N_i&=U_{ni}-U_{pi},\quad i=1,\cdots,k. \end{aligned} \tag{4}$$

The requirement of charge neutrality gives the additional relation

$$\textstyle\sum_i \delta N_i+\delta n=\delta p. \tag{5}$$

Equations (4) and (5) jointly constitute a system of $1+k$ independent coupled nonlinear differential equations, which determine the behavior of nonequilibrium carrier concentrations, provided transfer of charge between traps is neglected.

1. Single Recombination-Center Case

Eliminating δN_i from Eq. (1), making use of Eq. (5), and inserting the result into Eqs. (4), we obtain two differential equations of the form

$$\begin{aligned} (d/dt)\delta n&=g-\alpha\delta n+\beta\delta p,\\ (d/dt)\delta p&=g-\eta\delta p+\gamma\delta n, \end{aligned} \tag{6}$$

where

$$\begin{aligned} \alpha&=\alpha_n(N^0+n_0+n_1+\delta n)+\alpha_d(p_0+\delta p),\\ \beta&=\alpha_n(n_0+n_1+\delta n)-\alpha_d(n_0),\\ \eta&=\alpha_p(N^-+p_0+p_1+\delta p)+\alpha_d(n_0+\delta n),\\ \gamma&=\alpha_p(p_0+p_1+\delta p)-\alpha_d(p_0). \end{aligned} \tag{7}$$

Table I. List of symbols.[a]

Symbol	Definition
α_n	product of electron-capture cross section and thermal velocity of electrons
α_p	as above, but for holes
N^0	density of empty recombination centers (from the electron point of view)
N^-	density of filled recombination centers
δN	deviation from the thermal-equilibrium occupancy of a set of recombination centers
n_0, p_0	thermal-equilibrium carrier concentration
n_1, p_1	carrier concentration with the Fermi level at the energy level of the trap
$\delta n, \delta p$	deviations from the thermal-equilibrium carrier concentrations
g	rate of generation of carriers by external means
τ_{n0}	$(\alpha_n N)^{-1}$
τ_{p0}	$(\alpha_p N)^{-1}$

[a] The subscript i denotes properties of the ith group of recombination centers.

[1] W. Shockley and W. T. Read, Phys. Rev. **87**, 835 (1952).
[2] R. N. Hall, Phys. Rev. **87**, 387 (1952).
[3] E. S. Rittner, *Proceedings of the Conference on Photoconductivity, Atlantic City, November 4–6, 1954* (John Wiley and Sons, Inc., New York, 1956). A. Hoffman, *Halbleiter Problem II* (Friedrich Vieweg und Sohn, Braunschweig, 1955).
[4] D. J. Sandiford, Phys. Rev. **105**, 524 (1957).
[5] A. Rose, Phys. Rev. **97**, 322 (1955).
[6] J. A. Hornbeck and J. R. Haynes, Phys. Rev. **97**, 311 (1955).
[7] W. van Roosbroeck and W. Shockley, Phys. Rev. **94**, 1558 (1954).

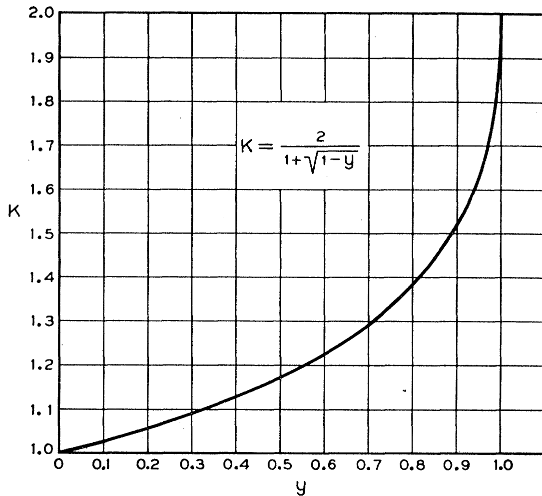

FIG. 1. Ratio of accurate to approximate solution as a function of the parameter y.

We note that

$$\alpha\geq\beta,\quad \eta\geq\gamma. \tag{8}$$

In special limiting cases, solutions to this set of equations can be obtained in closed form.

(a) *Simple Lifetime*

We assume that direct recombination is negligible and that injection is small:

$$\delta n\ll n_0+n_1,\quad \delta p\ll p_0+p_1. \tag{9}$$

The differential equations are now linear and have solutions of the form

$$\delta n=A_n+B_n\exp(-t/\tau_-)+C_n\exp(-t/\tau_+),$$

where

$$\tau_\pm=\frac{2}{(\alpha+\eta)[1\pm(1-y)^{\frac{1}{2}}]}, \tag{10}$$

and

$$y=4(\alpha\eta-\beta\gamma)(\alpha+\eta)^{-2}.$$

It follows from (8) that $y\leq 1$. This suffices to insure that the time constants are real and the solutions exponentials.

If $y\ll 1$, the square root of Eq. (10) may be expanded to obtain

$$\tau_-=(\alpha+\eta)(\alpha\eta-\beta\gamma)^{-1},\quad \tau_+=(\alpha+\eta)^{-1}. \tag{10a}$$

Expressions for the amplitudes in two simple cases are given in Appendix I. In general, interest focuses on the time constants of the decay. For a nondegenerate system two simple expressions are obtained:

$$\tau_-=[\tau_{p0}(N^0+n_0+n_1)+\tau_{n0}(N^-+p_0+p_1)]\times(n_0+p_0+N^-N^0/N)^{-1}, \tag{11a}$$

$$\tau_+=[\alpha_n(N^0+n_0+n_1)+\alpha_p(N^-+p_0+p_1)]^{-1}. \tag{11b}$$

These expressions have been given by Sandiford.[4] The first expression is the lifetime (mean life), the second the time for the adjustment of the change on the center to the condition where holes and electrons are captured at the same rate. In the present approximation $\tau_-/\tau_+=4/y\gg 1$. Measurements of lifetime yield τ_- since τ_+ is associated with a smaller amplitude and is generally too short to be resolved. For small recombination-center density, (11a) is identical with the steady-state solution.[1]

If $y\sim 1$, it may be shown that either $\alpha\gg\beta$, or $\eta\gg\gamma$ and $\alpha\sim\eta$. Under these conditions,

$$\begin{aligned}\tau_-&=\alpha_p^{-1}(N^-+p_0+p_1)^{-1},\\ \tau_+&=\alpha_n^{-1}(N^0+n_0+n_1)^{-1}.\end{aligned} \tag{12}$$

Since the values of y cannot be determined until a fit of the equation to the data has been made, it is significant to note that the error in the expansion for $y\ll 1$ is no greater than a factor of two. The ratio of the accurate solution to the approximate solution,

$$\frac{\tau_+}{\tau_{+y\ll 1}}=\frac{\tau_{-y\ll 1}}{\tau_-}=\frac{2}{1+(1-y)^{\frac{1}{2}}}=K, \tag{13}$$

is given in Fig. 1.

In electron-bombarded n-type silicon,[8] the behavior of the lifetime as a function of bombardment and temperature is adequately represented by Eq. (11a). Bombardment produces an energy level 0.27 ev above the edge of the valence band. The imperfections associated with this level act as recombination centers which can be introduced in controlled density. We specialize the equation for this case, and obtain

$$\tau_-=(\alpha_pN)^{-1}+(\alpha_nn_0)^{-1}(1+p_1/N).$$

This equation indicates that a lower limit in lifetime is reached as bombardment, and therefore N is in-

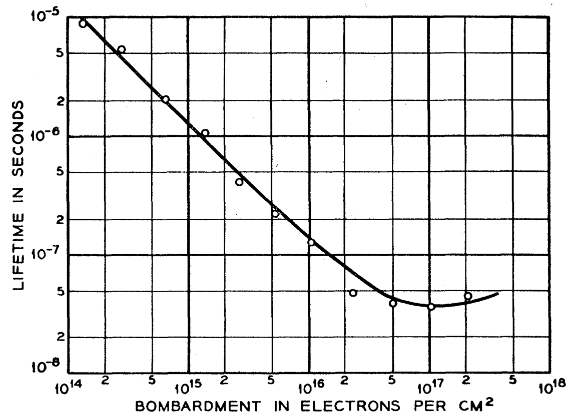

FIG. 2. Lifetime (mean life) at 60°C in 7 ohm-cm, n-type silicon as a function of bombardment, n_e. Curves computed with $\alpha_n=9.5\times10^{-8}$, $\alpha_p=1.6\times10^{-6}$, $p_1=\frac{1}{2}N_v\exp(-0.27/kT)$, $n_0=6.2\times10^{14}$, $N=5\times10^{-3}n_e$.

[8] G. K. Wertheim, Phys. Rev. **105**, 1730 (1957).

creased. In practice a minimum is observed because n_0 decreases as bombardment progresses. This is observed (Fig. 2) at 10^{17} electrons/cm^2. It may be noted that it is here possible to obtain the capture cross section without explicit knowledge of the density of recombination centers. The equation also correctly describes the behavior of the lifetime as a function of temperature for various values of bombardment (Fig. 3).

An example of a case where Eq. (11a) is not adequate because y passes through unity is found in silicon containing a large concentration of indium. This case is discussed in Sec. (c).

(b) *Moderately Large Deviation from Equilibrium*

The preceding analysis may be extended to include the case of moderately large injection, defined by

$$\delta n \lesssim N^0+n_0+n_1, \quad \delta p \lesssim N^-+p_0+p_1. \tag{14}$$

For $y \ll 1$ we obtain

$$\tau_-^* = \frac{\tau_- + (\tau_{p0}\delta n + \tau_{n0}\delta p)(n_0+p_0+N^-N^0/N)^{-1}}{1+[n_0\delta n/(n_0+n_1)+p_0\delta p/(p_0+p_1)](n_0+p_0+N^-N^0/N)^{-1}}, \tag{15a}$$

$$1/\tau_+^* = 1/\tau_+ + \alpha_n \delta n + \alpha_p \delta p. \tag{15b}$$

The expression for τ_-^* represents the "instantaneous lifetime" for a given excess carrier concentration; that is, although the decay will not be exponential it will at any instant approximate an exponential with a time constant given by Eq. (15a).

For a density of recombination centers small compared to the majority-carrier concentration, and for $\delta N < \delta n + \delta p$, i.e., for a recombination rather than a trapping process, the equation may be specialized to obtain

$$\tau_-^* = [\tau_- + (\tau_{n0}+\tau_{p0})\delta n/(n_0+p_0)] \times [1+\delta n/(n_0+p_0)]^{-1}. \tag{16}$$

This is identical with the steady-state, large-injection lifetime given by Shockley and Read.[1] It must be remembered, however, that in the present case, the range of validity is restricted by Eq. (14). For larger injection the differential equations are nonlinear and other solutions must be obtained.

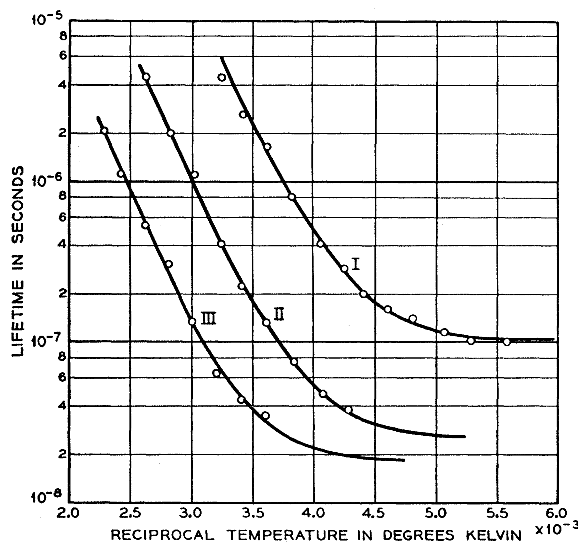

Fig. 3. Lifetime (mean life) in 7 ohm-cm, n-type silicon as a function of temperature for three amounts of bombardment; I $n_e = 1.4\times 10^{14}$ electrons/cm^2, II $n_e = 1.4\times 10^{15}$, III $n_e = 1.1\times 10^{16}$.

(c) *Donor- or Acceptor-Limited Lifetime*

The lifetime in p-type material may be limited by recombination through the chemical acceptor which determines the carrier concentration. In n-type material the donor may play the corresponding role. We shall analyze the p-type case for which some experimental evidence is available. The usual approximations must be examined because the density of centers may be much larger than the majority carrier concentration.

If we make the assumption $N^0 \gg n_0+n_1$, valid in heavily doped p-type material, Eq. (11a) may be transformed to obtain

$$\tau_- = (\alpha_n N^0)^{-1} + \alpha_p^{-1}(N^- + p_0 + p_1)^{-1}. \tag{17}$$

If the sample contains N_D donors/cm^3, and N_A acceptors/cm^3 with larger ionization energy and in greater concentration, so that the latter will have a dominant effect on lifetime,

$$N^0 = N_A - N_D - p_0, \quad N^- = p_0 + N_D. \tag{18}$$

The lifetime then becomes

$$\tau_- = \alpha_n^{-1}(N_A - p_0 - N_D)^{-1} + \alpha_p^{-1}(2p_0 + p_1 + N_D)^{-1}. \tag{19}$$

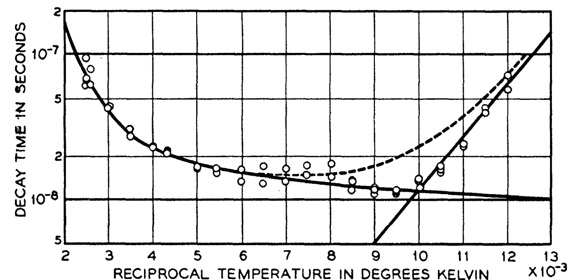

Fig. 4. Lifetime (mean life) in indium-doped silicon. Curves computed with $\alpha_n = 1.1\times 10^{-14} T^{-1} v_n$, $\alpha_p = 1.5\times 10^{-9} T^{-2} v_p$; v_n, v_p are average thermal velocities of electrons and holes; $p_1 = \frac{1}{2} N_v \times \exp(-0.16/kT)$.

At low temperature, $\alpha_n(N_A-N_D)/\alpha_p(2p_0+N_D)$ may approach unity provided N_D is sufficiently small. Under this condition, $y\sim 1$ and the two time constants become

$$\tau_-=\alpha_p^{-1}(2p_0+p_1+N_D)^{-1}, \tag{20a}$$

$$\tau_+=\alpha_n^{-1}(N_A-p_0-N_D)^{-1}. \tag{20b}$$

Behavior of this type has been observed in heavily indium-doped silicon[9] with $N_A=1.4\times 10^{17}$ and $N_D<10^{12}$, Fig. 4. The dotted line represents τ_- computed from Eq. (19), assuming that $y\ll 1$. The solid line consists of Eq. (19) at high temperature and continues as Eq. (20b) at low temperature. The rising curve at low temperature is Eq. (20a). It is found that $y=1$ at 100°K. The temperature dependence of the capture cross sections of indium were derived from a comparison of the data with these equations.

In the case of an acceptor, the cross section for hole capture, σ_p, is expected to be larger than the cross section for electron capture, σ_n, because the former is aided by Coulomb attraction, while the latter is not. At room temperature the lifetime is consequently dominated by the first term of Eq. (19). Using the relation

$$p_0^2+p_0(N_D+p_1)-p_1(N_A-N_D)=0,$$

one obtains

$$\tau_-=p_1\alpha_n^{-1}p_0^{-2}(1+N_D/p_0)^{-1},$$

which indicates that the lifetime will depend on the inverse square of the carrier concentration when the donor concentration is small.

The case where a second acceptor N_A' with smaller ionization energy is also present may be treated by substituting N_D-N_A' for N_D in these equations.

(d) *Direct Recombination in Conjunction with Recombination through Centers*

Direct and indirect recombination may compete in certain semiconductors such as InSb. We consider only the case of small injection defined by

$$\delta n\ll n_0+n_1,\quad \delta p\ll p_0+p_1.$$

For $y\ll 1$, Eqs. (10a) yield

$$\tau_-^*=\frac{\alpha_n(N^0+n_0+n_1)+\alpha_p(N^-+p_0+p_1)+\alpha_d(n_0+p_0)}{\alpha_n\alpha_p(n_0N+p_0N+N^0N^-)+\alpha_d(n_0+p_0)[\alpha_n(n_0+n_1)+\alpha_p(p_0+p_1)]+\alpha_d(\alpha_n n_0N^0+\alpha_p p_0N^-)},$$

$$\tau_+^*=[\alpha_n(N^0+n_0+n_1)+\alpha_p(N^-+p_0+p_1)+\alpha_d(n_0+p_0)]^{-1}. \tag{21}$$

If the density of recombination centers is small,

$$N^0<n_0+n_1,\quad N^-<p_0+p_1. \tag{22}$$

Equations (21) may be simplified to obtain

$$1/\tau_-^*=(1/\tau_-+1/\tau_d)(1+\tau_+/\tau_d)^{-1},\quad 1/\tau_+^*=1/\tau_++1/\tau_d, \tag{23}$$

where τ_- and τ_+ are those given by Eqs. (11a) and (11b) and

$$\tau_d^{-1}=\alpha_d(n_0+p_0). \tag{24}$$

Here again τ_+ represents the time for the adjustment of the charge on the recombination centers to the condition where holes and electrons are captured at the same rate. It is always smaller than τ_-. Consequently, when τ_- is smaller than τ_d, the addition of reciprocal time constants customarily employed is valid. It must be remembered, however, that this result is predicated on a density of recombination centers sufficiently small to satisfy Eqs. (22).

2. Recombination in Crystals Containing Two Centers

Recombination in crystals containing two centers is complicated by the fact that the quasi-Fermi levels of the centers are independent. (The case where a single quasi-Fermi level applied to a spectrum of centers has been treated by Rose.[5]) Here we consider the filling of the two centers to be uncoupled, and ignore the possibility of direct exchange between them. Two simple cases will be analyzed, (a) the temporary trap, and (b) the two recombination-center case.

(a) *Temporary Trap*

A center will act as a temporary trap for minority carriers if its majority-carrier cross section is so small that the rate of capture of majority carriers is negligible compared to the net rate of emission of minority carriers from the trap.

We consider the case of a hole trap in n-type material in conjunction with an ordinary recombination center. If the electron-capture cross section of the trap is zero, we obtain

$$(d/dt)\delta n=-U_n,\quad (d/dt)\delta p=-U_p-U_{pt},\quad \delta n+\delta N+\delta N_t=\delta p,\quad (d/dt)\delta N_t=-U_{pt}, \tag{25}$$

where the subscript t stands for trap. This is a system of three coupled nonlinear differential equations. The case where the filling of the traps is small yields a set of linear equations, which have solutions in terms of three time constants. These can be obtained in useful form when one of the three is much longer than either of the other two. This condition is usually met in trapping processes.

[9] G. K. Wertheim, Bull. Am. Phys. Soc. Ser. II, 2, 314 (1957).

The differential equations may be written as a set of operator equations in the form

$$(D+a_{11})\delta n+a_{12}\delta p+a_{13}\delta N_t=0,$$
$$a_{21}\delta n+(D+a_{22})\delta p+a_{23}\delta N_t=0,$$
$$a_{31}\delta n+a_{32}\delta p+(D+a_{33})\delta N_t=0.$$

These have solutions only when the determinant of the coefficients is zero. This requirement produces a cubic equation of the form

$$D^3+pD^2+qD+r=0,$$

whose smallest root (corresponding to the largest time constant) is approximately

$$\omega=r/q=\tau^{-1}.$$

The result for n-type material with a small density of recombination centers and small fractional filling of the trap is

$$\tau=\frac{[1+\alpha_t(N_t^-+p_t)/\alpha_pN^-](\alpha_tp_t)^{-1}+[1+p_1(N_t^-+p_t)/p_tN^-][\alpha_n(n_0+n_1)]^{-1}}{1+p_1N^0N_t^-/p_tN^-(n_0+n_1)}. \quad (26)$$

The denominator will approximate unity, provided the recombination centers lie below the Fermi level and the trapping centers below the recombination centers.

At low temperature, where $p_t<N_t^-$, the equation may be written

$$\tau=(\alpha_tp_t)^{-1}(\alpha_pN^-+\alpha_tN_t^-)/\alpha_pN^- + (\alpha_nN)^{-1}(N^-+p_1)/n_0+(\alpha_nN)^{-1}p_1N_t^-/p_tn_0. \quad (27)$$

The first term is the reciprocal of the net rate at which holes are captured by the recombination center. This follows from the facts that α_tp_t is the rate of regeneration from the traps and

$$\alpha_pN^-/(\alpha_pN^-+\alpha_tN_t^-)$$

is the fraction which are captured by the recombination center. The rest are retrapped. The second term is the electron-capture time of the recombination centers

$$\tau_n=(\alpha_nN)^{-1}(N^-+p_1)N_0^{-1}. \quad (28)$$

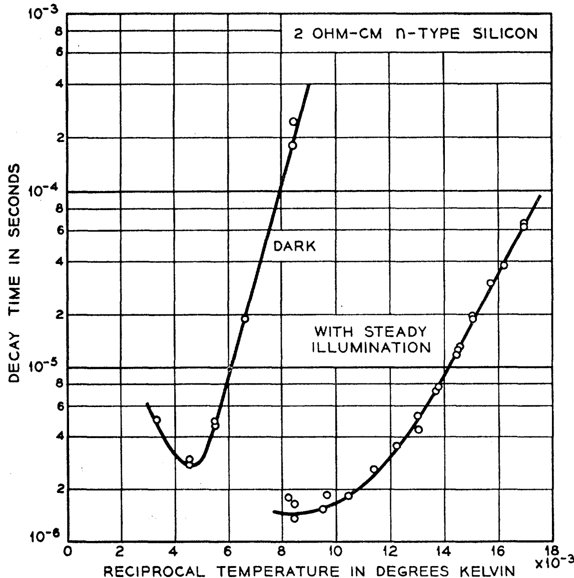

FIG. 5. Low-temperature trapping in 2 ohm cm n-type silicon. The trapping levels are 0.11 ev and 0.059 ev above the valence band.

We now define the regeneration time $\tau_g=(\alpha_tp_t)^{-1}$, the hole-trapping time $\tau_t=(\alpha_tN_t^-)^{-1}$, and the hole-capture time of the recombination center $\tau_p=(\alpha_pN^-)^{-1}$ and rewrite Eq. (27) in the form

$$\tau=\tau_g\tau_p\tau_t^{-1}+\tau_g+\tau_n+p_1N_t^-(\alpha_nn_0p_tN)^{-1}. \quad (29)$$

This is similar to the result obtained by Hornbeck and Haynes,[6] who did not consider the individual processes of the recombination center.

Behavior of the type suggested by Eq. (27) is often found in silicon at low temperatures. Figure 5 shows the lifetime in a sample of 2 ohm-cm n-type silicon containing two sets of such trapping centers. The dark behavior of this crystal is well represented by Eq. (27). Below 125°K the trap release-time has become sufficiently long so that weak, steady illumination keeps this trap filled. A second shallower trap is now observed. The behavior here cannot be interpreted in terms of Eq. (27) which applies only for small deviations from thermal equilibrium.

(b) *Two Recombination Centers*

We now consider the general, small-signal case for two recombination centers, under the assumption that the density of recombination centers is smaller than the majority-carrier concentration

$$N_1+N_2<n_0+p_0.$$

It may be shown that under these conditions

$$1/\tau=[\tau_1^{-1}(1+\mu_1)+\tau_2^{-1}(1+\mu_2)] \times[1+\mu_1(1+\nu_1)+\mu_2(1+\nu_2)]^{-1}, \quad (30)$$

where, in n-type material,

$$\begin{aligned}\mu_1&=N_1^-/[p_0+p_{11}+(n_0+n_{11})\alpha_{n1}/\alpha_{p1}],\\ \mu_2&=N_2^-/[p_0+p_{12}+(n_0+n_{12})\alpha_{n2}/\alpha_{p2}],\\ \nu_1&=\alpha_{n1}(n_0+n_{11})/[\alpha_{n2}(n_0+n_{12})+\alpha_{p2}(p_0+p_{12})],\\ \nu_2&=\alpha_{n2}(n_0+n_{12})/[\alpha_{n1}(n_0+n_{11})+\alpha_{p1}(p_0+p_{11})].\end{aligned} \quad (31)$$

It may be seen from Eqs. (30) and (31) that the customary addition of reciprocal time constants is justified only when $\mu_i \ll 1$ and $\mu_i \nu_i < 1$. Both conditions are met when the recombination-center density is sufficiently small. For recombination centers near the middle of the gap, this requires that

$$\alpha_{pi} N_i^- / \alpha_{ni} n_0 < 1,$$

which suggests that deviations may be found when one or both of the centers are negatively charged, so that $\alpha_p > \alpha_n$.

APPENDIX I

Following injection of a short pulse of carriers in equal number $\delta n(0) = \delta p(0) = \Delta$ we have

$$A_n = 0, \qquad A_p = 0,$$

$$B_n = \frac{\alpha - \beta - \omega_+}{\omega_- - \omega_+}\Delta, \qquad B_p = \frac{\eta - \gamma - \omega_+}{\omega_- - \omega_+}\Delta,$$

$$C_n = \Delta - B_n, \qquad C_p = \Delta - B_p,$$

where $\omega = \tau^{-1}$.

If injection is suddenly begun at a rate g, then

$$A_n = \left(\frac{\beta + \eta}{\alpha\eta - \beta\gamma}\right) g, \qquad A_p = \left(\frac{\gamma + \alpha}{\alpha\eta - \beta\gamma}\right) g,$$

$$B_n = \frac{(\alpha - \omega_+) A_n - \beta A_p}{\omega_+ - \omega_-}, \qquad B_p = \frac{(\eta - \omega_+) A_p - \gamma A_n}{\omega_+ - \omega_-},$$

$$C_n = -(A_n + B_n), \qquad C_p = -(A_p + B_p).$$

The same time constants are obtained in both cases.

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Dielectric Properties of Single Domain Crystals of $BaTiO_3$ at Microwave Frequencies

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Small-signal dielectric constant measurements were made on single domain crystals of $BaTiO_3$ from 25°C to 170°C at 24 kMc/sec. A typical Curie behavior was observed with a dielectric constant ϵ_{11} of about half the dc value below the upper Curie point. Measurements of ϵ_{11} were also made with a dc field (E_3) applied along the c axis in order to obtain information concerning the distortion of the potential well perpendicular to the c axis when the ion is displaced along the c axis. The change $\Delta\epsilon_{11}$ observed was about 50 for an applied field of 10^4 volts/cm.

INTRODUCTION

THE dielectric properties of ceramic and polycrystalline samples of $BaTiO_3$ have been studied by other authors[1] at microwave frequencies. These results show a relaxation effect in the neighborhood of 10^{10} cps which has been attributed to inertia of the domain boundaries[2] and the piezoelectric resonance of the crystallites.[3] These experiments also show a variation of the dielectric constant with an applied dc field. The sections which follow describe experiments done on single domain crystals. The experiments were of two different kinds: (*a*) measurement of the small-signal dielectric constant at 24 kMc/sec as a function of temperature, and (*b*) measurement of the effect of a dc field on the dielectric constant at that frequency.

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[2] C. Kittel, Phys. Rev. **83**, 458 (1951).
[3] A. F. Devonshire, Phil. Mag. **42**, 1065 (1951).

DIELECTRIC CONSTANT *VS* TEMPERATURE

Single crystals of area ~1 cm² and thickness ~0.040 cm were grown by the method described by Remeika.[4] The crystals were inspected under a polarizing microscope and those in which the c axis was aligned predominantly perpendicular to the plane of the crystal plate were chosen for investigation. These crystals were then poled at 60 cps until microscopic observation showed them to be polarized in the c direction (c domain) over an area large enough to fill the cross section of a standard K-band wave guide (0.420 in.×0.170 in.). The crystals were shaped to fit the guide by sandblasting away the unwanted portion. It was found that sandblasting was preferred to shaping with a diamond saw because fewer strains were produced near the edges of the crystals during the cutting operation. The crystals were then mounted in brass slugs with silver paste providing the contact between the crystal and the brass. The plate was mounted perpendicular to the

[4] J. P. Remeika, J. Am. Chem. Soc. **76**, 940 (1954).