

in the calculation. Finally we remark that the extension of the basic results (5) and (6) to an actual solid is trivial; as Hubbard has remarked, one need only incorporate the appropriate solid dielectric constant in these expressions.

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¹ D. Bohm and D. Pines, [Phys. Rev. **92**, 605 (1953)], as revised along the lines of P. Nozières and D. Pines, II, [Phys. Rev. **109**, 762 (1958) this issue].

² M. Gell-Mann and K. A. Brueckner, Phys. Rev. **106**, 364 (1957); Sawada, Brueckner, Fukuda, and Brout, Phys. Rev. **108**, 507 (1957); K. Sawada, Phys. Rev. **106**, 369 (1957).

³ J. Hubbard, I, Proc. Roy. Soc. (London) **A240**, 539 (1957); II, Proc. Roy. Soc. (London) **A243**, 336 (1959).

⁴ P. Nozières and D. Pines, II, reference 1; and IV (to be published).

⁵ See, for instance, J. Hubbard, Proc. Phys. Soc. (London) **A68**, 976 (1955).

⁶ Usually the dielectric constant is extended to the range of negative frequencies by the requirement that $\epsilon(-\Omega) = \epsilon^*(\Omega)$, while Hubbard prefers to use the relation $\epsilon(-\Omega) = \epsilon(\Omega)$. This does not affect our result, since we integrate only from 0 to ∞ . We further remark that to get the correct result in the high-density limit, we should add to (5) the second order exchange energy.

⁷ The integration in (8) is understood to run along a line slightly above the real axis. It does not involve any principal part at the plasma pole.

⁸ For actual metallic densities the higher order exchange terms (which are negligible in the high-density limit) actually cancel one-half of the direct terms for the short-wavelength momentum transfers of interest in this domain. For a detailed discussion of this point we refer the reader to a forthcoming paper by us.

Energy Levels of the Two-Electron Atom to Order α^3 ry; Ionization Energy of Helium*

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THE α^3 ry corrections to the energy levels of the two-electron atom have been studied and techniques have been developed for calculating at least some of the corrections of order α^4 ry. The starting point has been a "mixed gauge" Bethe-Salpeter equation, extended to the case where an external field is present. The term "mixed gauge" refers to the fact that interaction between the electrons is described in Coulomb gauge while self-interaction is treated in covariant gauge. A general proof has been obtained that such a mixing yields correct results in the calculation of energy levels.¹ The correctness of previous calculations for hydrogen² and for positronium,³ in which some terms were calculated by comparison with the corresponding, non-hole-theoretic Breit equation,⁴ has been incidentally confirmed.

The increase ΔW of the nonrelativistic energy W due to relativistic and quantum-electrodynamic effects is expressible as the expectation value of certain operators with φ , the Schrödinger-Pauli wave function which

satisfies

$$W\varphi = H_0\varphi = \left(\frac{\mathbf{P}_1^2}{2m} + \frac{\mathbf{P}_2^2}{2m} - \frac{Ze^2}{r_1} - \frac{Ze^2}{r_2} + \frac{e^2}{r} \right) \varphi. \quad (1)$$

One finds

$$\Delta W = (\Delta W)_{\alpha^2 \text{ ry}} + (\Delta W)_{\alpha^3 \text{ ry}} + O(\alpha^4 \ln \alpha \text{ ry}), \quad (2)$$

where $(\Delta W)_{\alpha^2 \text{ ry}}$ is obtained in agreement with previous work on the α^2 ry corrections.⁵ The general formula for $(\Delta W)_{\alpha^3 \text{ ry}}$ is lengthy.⁶ For a singlet S state, with wave function of the Hylleraas-Kinoshita type,⁷ i.e. (in atomic units)

$$\varphi = e^{-ks} \sum c_{lmns} s^l u^m v^n, \quad (3)$$

we find

$$\begin{aligned} (\Delta W)_{\alpha^3 \text{ ry}} = & \left[\frac{8}{3} \frac{19}{30} + \ln \left(\frac{m}{2k_0} \right) \right] \frac{\alpha}{\pi} E_1'' - \frac{14}{3} (\ln \alpha) \frac{\alpha}{\pi} E_1''' \\ & + \frac{5}{2} \frac{\alpha}{\pi} E_3'' - \left(\frac{14}{3} \ln k + \frac{302}{45} \right) \frac{\alpha}{\pi} E_1'''' \\ & - \frac{7}{3} k^3 \left(\frac{F}{G} \right) \frac{\alpha^3}{\pi} \text{ ry}, \quad (4) \end{aligned}$$

where E_1'' , E_1''' , and E_3'' are quantities defined in reference 5, k_0 is an average excitation energy defined by Kabir and Salpeter⁸ and evaluated by them for the ground state of helium, and

$$F = \sum_{\mu+\nu \neq 0} c_{lmn} c_{l'm'n'} S_{\lambda, \mu-3, \nu} + 2 \sum_{l,l'} c_{l00} c_{l'00} \sum_{p=1}^{l+l'+2} \frac{1}{p}, \quad (5a)$$

$$G = \sum c_{lmn} c_{l'm'n'} S_{\lambda, \mu, \nu}, \quad (5b)$$

with $\lambda = l+l'$, $\mu = m+m'$, $\nu = n+n'$, and

$$\begin{aligned} S_{\lambda, \mu, \nu} = & [1 + (-1)^\nu] (\lambda + \mu + \nu + 5)! \\ & \times [(\nu+1)^{-1} (\mu+\nu+3)^{-1} - (\nu+3)^{-1} (\mu+\nu+5)^{-1}]. \quad (6) \end{aligned}$$

The terms in the first line of Eq. (4) have already been obtained and evaluated by Kabir and Salpeter⁸ for the ground state of helium. Their results combined with the results of Kinoshita⁷ for W and $(\Delta W)_{\alpha^2 \text{ ry}}$ have previously yielded a value of 198 310.38 cm^{-1} for the ionization energy of helium.⁷ We have evaluated the remaining terms in Eq. (4) by using Kinoshita's values⁷ for k and E_1'''' in the second line of Eq. (4) and the 6-parameter wave function of Hylleraas⁹ in the last line of Eq. (4). We find an additional contribution to $(\Delta W)_{\alpha^3 \text{ ry}}$ of amount 0.072 cm^{-1} . The theoretical value for the ionization energy thus becomes

$$J_{\text{theor}} = 198\,310.31 \text{ cm}^{-1}, \quad (7)$$

which may be compared with the experimental value¹⁰

$$J_{\text{exp}} = 198\,310.5 \pm 1 \text{ cm}^{-1}. \quad (8)$$

The uncertainty in J_{theor} comes mainly from uncertainty in the values of W and $(\Delta W)_{\alpha^2 \text{ ry}}$, as has been discussed by Kinoshita. The agreement between (7)

and (8) is of course very satisfactory but an increase in the knowledge of the precision of the present values of W and $(\Delta W)_{\alpha^{2,ry}}$ and in the precision of Eq. (8) would be very desirable, if the combined results of Kabir and Salpeter⁸ and of the present calculation are to be put to a more severe test.¹¹

A detailed account will be published. Professor H. M. Foley suggested this work and his constant advice and encouragement are gratefully acknowledged. Thanks are due Professor N. M. Kroll for many profitable discussions.

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¹ See J. Sucher, [Phys. Rev. **107**, 1448 (1957)], for a sketch of the proof.

² E. E. Salpeter, Phys. Rev. **84**, 328 (1952).

³ T. Fulton and P. Martin, Phys. Rev. **95**, 811 (1954).

⁴ See reference 2, Sec. 3, and reference 3, Sec. 3.

⁵ J. Sucher and H. M. Foley, Phys. Rev. **95**, 966 (1954).

⁶ See J. Sucher, Ph.D. thesis, Columbia University, 1957 (unpublished) and reference 11.

⁷ T. Kinoshita, Phys. Rev. **105**, 1490 (1957).

⁸ P. Kabir and E. E. Salpeter, Bull. Am. Phys. Soc. Ser. II, **1**, 46 (1956).

⁹ E. Hylleraas, Z. Physik **54**, 347 (1929).

¹⁰ G. Hertzberg and R. Zbinden (unpublished), quoted in reference 7.

¹¹ As this work was being completed, there appeared a paper by H. Araki [Progr. Theoret. Phys. Japan **17**, 619 (1957)] in which the same problem is considered. Although the approach of Araki differs in many details from ours, as does the appearance of his final formula, the results are in fact the same. This tends to confirm the correctness of both calculations.

Effect of Neutron Irradiation on Infrared Absorption in Silicon*

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IT was observed by Becker, Fan, and Lark-Horovitz¹ that irradiation by fast neutrons or high-energy deuterons produces in silicon an absorption band with a peak at about 1.8 microns. A second effect observed was an apparent shift of the intrinsic absorption edge toward longer wavelengths. Both effects were found to be roughly proportional to the irradiation. The absorption band which was attributed to the excitation of defect centers in the lattice has been observed also in electron-irradiated samples.² Three additional effects of fast-neutron irradiation have been found. (1) An absorption band is produced at 20.5 microns with weaker bands at longer wavelength. (2) An absorption band is observed at 3.9 microns. It becomes more pronounced after the irradiated sample has been annealed at some elevated temperature. (3) Annealing may cause a general decrease of infrared transmission. This effect was first observed by Longo.³

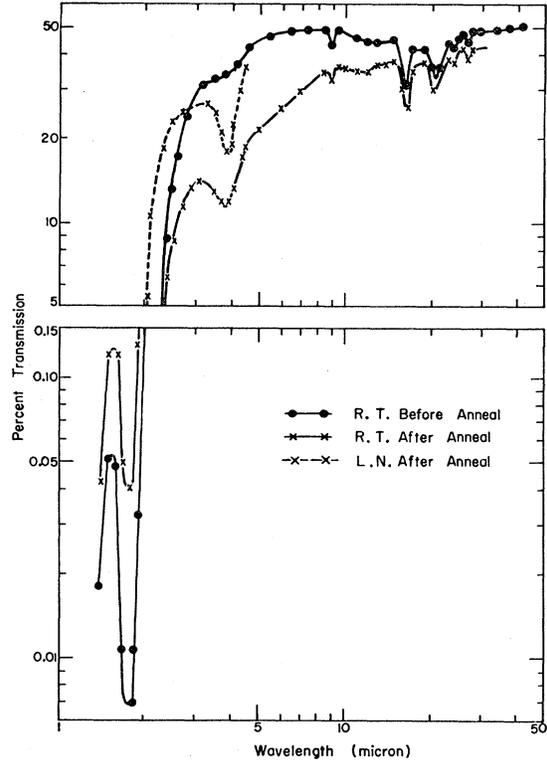


FIG. 1. Percent transmission as a function of wavelength for a p -type silicon sample irradiated with $8.5 \times 10^{18} \text{ cm}^{-2}$ fast neutrons. The sample had originally a resistivity of 0.007 ohm cm at room temperature. The anneal consisted of heating at 145°C for 80 minutes in vacuum.

Figure 1 shows the transmission curves for a neutron-irradiated sample (1.2×10^8 ohm cm after irradiation), before and after an anneal at 145°C in vacuum for 80 minutes. Two of the absorption bands in normal silicon, the most prominent lattice band at 16.5 microns, and the 9-micron band due to the presence of oxygen, are seen in these curves. The strong 1.8-micron band, produced by the irradiation, is seen at the short-wavelength end of the curves. The newly observed band at 3.9 microns is seen to be more pronounced after the anneal, especially in the low-temperature curve. The additional band found at 20.5 microns with two weaker bands at longer wavelengths are also shown by the curves.

The strongest of the additional bands at long wavelengths is close to the normal 16.5-micron lattice band. This band at 20.5 microns is insensitive to temperature as shown clearly by the curves in Fig. 2. The apparent decrease of this band at the lower temperature is entirely accounted for by the diminished background of normal lattice absorption⁴ in this wavelength region. The 16.5-micron band, which decreases with decreasing temperature,⁴ is seen to be stronger than the 20.5-micron band at room temperature but comparatively smaller at the lower temperature. The normal modes of silicon are optically inactive due to the crystal sym-