because of the relatively low concentration of ions; also, the strong resonance lines of the rare gas ions are well below 1250 A (e.g., 1100 A for xenon).

From the observed amplitude of the light pulses and assuming that the quantum yield of sodium salicylate is near unity, we obtain that the ratio between the number of metastable states formed to the total number of ions produced in the passage of the alpha particle is about 0.03. This does not appear to be inconsistent with the experiments of Hagstrum[3] who showed that one can detect metastable ions by the secondary electrons emitted on impact with metal surfaces. He has shown that the ratio of cross sections for the formation of metastable ions and normal ions by slow electrons is about 0.02. As a further support for this explanation of the scintillations, we may mention that in Ne^+ there are no metastable states similar to those occurring in Xe and Kr ions[3,4] and indeed, in the present experiments, neon was the only rare gas for which the presence of sodium salicylate did not produce a large increase in the total light output. (See Table I.)

It would seem that in addition to the importance of these phenomena for the understanding of gas scintillations, they indicate new methods of studying the properties and fate of metastable atoms.

A detailed description of these experiments is in preparation.

[3] H. D. Hagstrum, Phys. Rev. **104**, 309 (1956).

[4] C. E. Moore, *Atomic Energy Levels*, National Bureau of Standards Circular No. 467 (U. S. Government Printing Office, Washington, D. C., 1949), Vol. 1.

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Medium-Energy Deuteron Photodisintegration*

N. AUSTERN†
University of Pittsburgh, Pittsburgh, Pennsylvania
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It is shown that the distinctive features of the photoeffect angular distribution in the energy range 20–100 Mev probably result from a strong modification of the $^3P_0$ outgoing wave amplitude, to be understood as a result of the excitation of virtual mesons in a fashion which violates the Siegert theorem. Some evidence also is found which suggests the need for a repulsive-core modification of the ground-state wave function. The contributions from the transition $^3D_1 \rightarrow {}^3F_2$ are analyzed, and are found to be rather large.

I. INTRODUCTION

PHENOMENOLOGICAL analysis of the photodisintegration data can give one or another of two kinds of information about the deuteron system. If both the initial and final state wave functions are known, it might give information about the nature of the radiative interaction; if the interaction mechanism is known, it might give information about the wave functions. In the past it has seemed that the second of these two possibilities would apply in the medium-energy range, 20–100 Mev, and would provide useful information about the nuclear force in the $^3P_{0,1,2}$ states, the important final states of the process. In this paper it will be demonstrated to be unlikely that such information can be obtained, even though the energies are rather far below the meson threshold, for in the medium-energy region an unexpectedly strong modification seems to appear in the radiative-interaction mechanism.

Good data regarding the medium-energy photoeffect have been available for some time.[1] Nevertheless, the analysis only recently has become interesting, since nucleon scattering experiments with polarized beams have given information about the ^3P_J wave phase shifts.[2] Several authors[3] already have tested their nuclear-force ideas against the photoeffect data. The attitude which will be taken in this paper will be to attempt to extract the outgoing wave amplitudes from the data, and only after getting the amplitudes to attempt their interpretation. This approach is feasible because of the quite striking nature of the data.

The photodisintegration is well known to be reliably

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† On leave of absence during the 1957–58 academic year at the University of Sydney, Sydney, Australia.

[1] Lew Allen, Jr., Phys. Rev. **98**, 705 (1955); Whalin, Schriever, and Hanson, Phys. Rev. **101**, 377 (1956).

[2] For several analyses of the experiments, see H. Feshbach and E. A. Lomon, Phys. Rev. **102**, 891 (1956); A. M. Saperstein and L. Durand, III, Phys. Rev. **104**, 1102 (1956); J. L. Gammel and R. M. Thaler, Phys. Rev. **107**, 290 (1957). I am grateful to Gammel and Thaler for a prepublication copy of their paper, and to G. Breit for a prepublication copy of the Saperstein, Durand paper. Moreover, I am especially grateful to L. Wolfenstein for many discussions about the high-energy phase shifts. The present paper is a direct outgrowth of those discussions.

[3] S. H. Hsieh and M. Nakagawa, Progr. Theoret. Phys. (Kyoto) **15**, 79 (1956); S. H. Hsieh, Nuovo cimento **4**, 138 (1956); S. H. Hsieh, Progr. Theoret. Phys. (Kyoto) **16**, 68 (1956). Some work along related lines also was done by J. Bernstein and H. Feshbach (private communication).

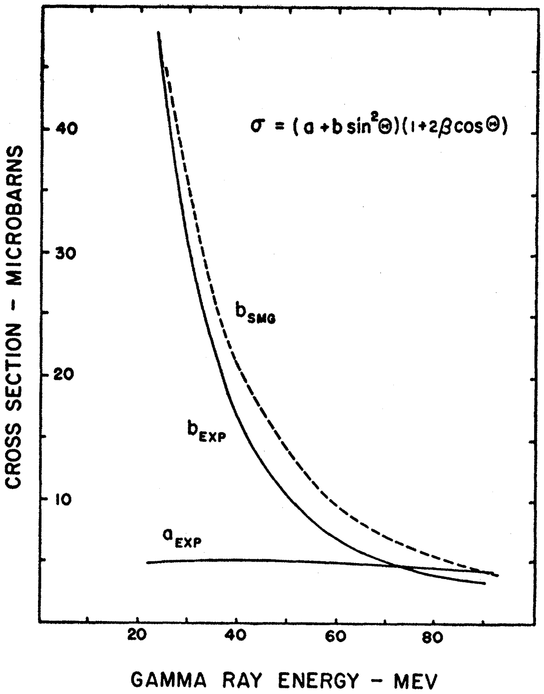

FIG 1. Experimental values for the cross section parameters, as given by Whalin, Schriever, and Hanson. The SMG prediction for b also is shown, the SMG prediction for a being zero.

a dipole process over the entire range thus far investigated, a consequence of the high relative momentum of the outgoing nucleons.[4] Electric quadrupole disintegration does yield an interesting interference in the medium-energy range, causing the protons to go preferentially forward,[5] but the actual $E2$ amplitude is quite small. It makes an entirely negligible contribution to the total cross section, and may be eliminated from the angular distribution by folding the experimental data about 90°. We then need only refer to the dipole form of the cross section,

$$\sigma = a + b\sin^2\theta. \quad (1)$$

Some further simplification is afforded by the fact that throughout the entire medium-energy range the photodisintegration is an *electric* dipole process,

$$\{{}^3S_1 + {}^3D_1\} \rightarrow \{{}^3P_{0,1,2} + {}^3F_2\}.$$

Magnetic dipole absorption is important at very low energy, as it leads to the advantageous ${}^1S_0$ final state,[6] and again at very high energy, being enhanced there by the pion-nucleon scattering resonance.[7,8] At medium energies, however, the $M1$ cross section only is a few percent of the $E1$ cross section, does not give any interference, and probably would be very difficult to detect.[9]

Figure 1 shows the cross-section data which will be analyzed in this paper, being a plot of the parameters a and b of Eq. (1), as a function of gamma-ray energy. We note immediately that the isotropic part of σ is extremely large. It will be argued in Sec. III that the only reasonable explanation of the large (a/b) ratio is that a large role is played by some new kind of $E1$ process, such as that suggested by Wilson,[10] and by Whalin, Schriever, and Hanson.[1] This is the two-step process, virtual production of an S-wave pion and then disintegration by reabsorption of the meson. Theory suggests that this mechanism is of no importance for the ${}^3P_{1,2}$ waves. The conclusion of this paper will be that it dominates in the ${}^3P_0$ wave, and determines the form of the cross section. It effectively masks the effects of final-state nuclear interactions.

The transition ${}^3D_1 \rightarrow {}^3F_2$ has not previously received much attention, most authors regarding the amplitude for this process as being small. Actually this amplitude is large,[11] and in the present paper will be carried on equal terms with the other amplitudes. A complete $E1$ photoeffect formula will be presented in Sec. II.

II. RIGID-NUCLEON THEORY

The photoeffect theory according to the "rigid-nucleon" assumption will be described in this section, and will be compared with experiment, so as to show in what respects the data are unexpected. The formulas introduced for this theory are the same ones, with a slight reinterpretation, which also will be used in Sec. III.

By the "rigid-nucleon" assumption it is meant that the Siegert theorem[4,12] is used, so that the matrix elements for the $E1$ transition are just the matrix elements of $\mathbf{r}$, the neutron-proton separation. The nucleons are rigid in the sense that the proof of the

[4] J. G. Brennan and R. G. Sachs, Phys. Rev. **88**, 824 (1952).

[5] A quantitative evaluation of this effect is given in the papers of Schiff, and of Marshall and Guth, references 15 and 16. The physical basis of the effect is discussed very clearly by A. Sommerfeld, *Atombau und Spektrallinien, Wellenmechanischer Ergänzungsband* (Friedrich Vieweg und Sohn, Braunschweig, 1929), pp. 211–215, as part of his analysis of hydrogenic atoms. Essentially, for gamma rays of a given energy it may be seen that protons which are emitted forward come from lower momentum parts of the ground-state wave function than those which are emitted backward, and so are more likely. This explanation relies upon the rigid-nucleon assumption (see Section II).

[6] For example, see the analysis by H. A. Bethe and C. Longmire, Phys. Rev. **77**, 647 (1950).

[7] B. T. Feld, Suppl. Nuovo cimento **2**, 145 (1955), N. Austern, Phys. Rev. **100**, 1522 (1955).

[8] F. Zachariasen, Phys. Rev. **101**, 371 (1956); R. Suzuki, Progr. Theoret. Phys. (Kyoto) **15**, 536 (1956); D. Ito *et al*, Progr. Theoret. Phys. (Kyoto) **15**, 74 (1956).

[9] It is possible that measurement of the polarization of the outgoing protons will give information about the $M1$ process at medium energies. See reference 23.

[10] R. R. Wilson, Phys. Rev. **104**, 218 (1956).

[11] I first learned of the large value of the ${}^3D_1 \rightarrow {}^3F_2$ matrix element through a sum-rule discussion in the Ph.D. thesis of M. L. Rustgi [Louisiana State University, 1957 (unpublished)]. I am grateful to J. S. Levinger for the opportunity to read this thesis.

[12] A. J. F. Siegert, Phys. Rev. **52**, 787 (1937); R. G. Sachs and N. Austern, Phys. Rev. **81**, 705 (1951); L. L. Foldy, Phys. Rev. **92**, 178 (1953).

Siegert theorem assumes that the motions of the virtual mesons in the deuteron system always follow the motions of the nucleons, their relative motion not being affected by the gamma-ray interaction, that, therefore, the mesons need not be taken into account explicitly in computing the process. Of course, at sufficiently high energies the nucleons no longer should be expected to be rigid, and the Siegert theorem should not hold. It is a low-energy theorem. In Sec. III we will see how the Siegert theorem fails at medium energies for transitions to the $^3P_0$ state.

The parameters a and b of the electric-dipole photodisintegration are given by the formulas[13]:

$$a=[B(k)/36]\{4L_0^2-8L_0L_2\cos(\delta_0-\delta_2)-12L_0L_f\cos(\delta_0-\delta_f)+9L_1^2+13L_2^2-18L_1L_2\cos(\delta_1-\delta_2)+18L_f^2+18L_1L_f\cos(\delta_1-\delta_f)-6L_2L_f\cos(\delta_2-\delta_f)\},\quad(2)$$

$$b=[B(k)/24]\{8L_0L_2\cos(\delta_0-\delta_2)+12L_0L_f\cos(\delta_0-\delta_f)+3L_1^2+7L_2^2+18L_1L_2\cos(\delta_1-\delta_2)+12L_f^2-18L_1L_f\cos(\delta_1-\delta_f)+6L_2L_f\cos(\delta_2-\delta_f)\}.\quad(3)$$

Here the subscripts 0, 1, 2 refer to the $^3P_{0,1,2}$ waves, and f refers to the $^3F_2$ wave. The above formulas are complete, except insofar as the $^3P_2-{}^3F_2$ mixing by the tensor force is neglected, a reasonable approximation for the energies under discussion. The quantities δ_J and δ_f are the scattering phase shifts, with the approximation $\delta_f=0$ being employed henceforth. The quantities L_J and L_f are the amplitudes of the outgoing waves, with the rigid-nucleon assumption giving

$$L_0=\int_0^\infty(\gamma r)[U-\sqrt{2}W]v_0(kr)dr,\quad(4)$$

$$L_1=\int_0^\infty(\gamma r)[U+W/\sqrt{2}]v_1(kr)dr,\quad(5)$$

$$L_2=\int_0^\infty(\gamma r)[U-W/5\sqrt{2}]v_2(kr)dr,\quad(6)$$

$$L_f=(3\sqrt{2}/5)\int_0^\infty(\gamma r)W[krj_3(kr)]dr.\quad(7)$$

For these equations the deuteron wave function has been taken as

$$\psi_{\text{deuteron}}=[N/r(4\pi)^{\frac{1}{2}}]\{U+8^{-\frac{1}{2}}S_{12}W\}\chi_1^m,\quad(8)$$

with N being chosen to give the standard effective-range normalization

$$\lim_{r\to\infty}U(r)=e^{-\gamma r},\quad(9)$$

so

$$N=2.77\times10^6\ \text{cm}^{-\frac{1}{2}}.\quad(10)$$

Also v_J are the P-wave radial functions, normalized so that

$$\lim_{r\to\infty}v_J(kr)=kr\{\cos\delta_J j_1(kr)-\sin\delta_J n_1(kr)\}.\quad(11)$$

The $j_1(kr)$ and $n_1(kr)$ are the spherical Bessel and Neumann functions.[14] Finally,

$$B(k)=\frac{1}{12}\frac{e^2}{\hbar c}\frac{M\omega}{\hbar k}\frac{N^2}{\gamma^2},\quad(12)$$

where $\hbar\omega$ is the gamma-ray energy. A graph of the dimensionless function $B(k)$ is given as Fig. 2.

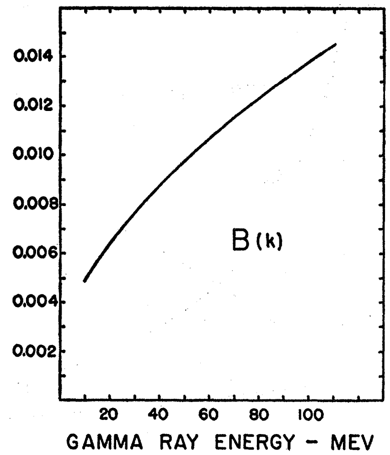

FIG. 2. The dimensionless coefficient $B(k)$, of (12).

It should be noted that the amplitudes L_J are related to the Rarita-Schwinger amplitudes I_J as

$$L_J=(\gamma/N)I_J.$$

This modification is convenient. The L_J are radial integrals over dimensionless functions, and are scaled so as to be lengths of the order of those which actually are effective in this problem.

(a) SMG (Schiff, Marshall, Guth) Theory

If all noncentral force effects are neglected, the equations of this section reduce to the case treated by Schiff,[15] and Marshall and Guth.[16] In this case

$$L_0=L_1=L_2=L_{\text{SMG}},\quad\delta_0=\delta_1=\delta_2,\quad L_f=0.$$

We find that a vanishes, the SMG theory predicting a purely $\sin^2\theta$ form for the cross section (exclusive of $M1$ and $E2$ effects). For b the result is

$$b_{\text{SMG}}=\tfrac{3}{2}B(k)(L_{\text{SMG}})^2.\quad(13)$$

[13] The F-wave terms of these formulas are presented here for the first time. Otherwise the formulas are the usual ones of W. S. Rarita and J. Schwinger, [Phys. Rev. **59**, 556 (1941)]. The corresponding Eq. (9) of the paper by N. Austern, [Phys. Rev. **85**, 283 (1952)] is missing a factor ($\frac{1}{2}$) in front of the $\sin^2\theta$ term.

[14] L. I. Schiff, *Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1949), pp. 77, 78.

[15] L. I. Schiff, Phys. Rev. **78**, 733 (1950).

[16] J. F. Marshall and E. Guth, Phys. Rev. **78**, 738 (1950).

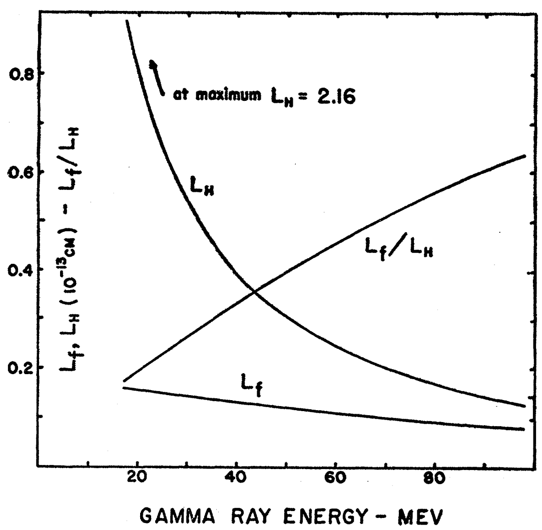

FIG. 3. Predictions of the rigid-nucleon theory for the $^3F_2$ amplitude, L_f, and for the SMG value of the amplitudes L_J, computed under the circumstance that the ground-state wave function has the Hulthén form. Their ratio also is shown.

The special case of the SMG theory which is most widely employed is obtained if all final-state interactions are set equal to zero, including the central interaction, and a Hulthén form is used for U. Then

$$U_H=(e^{-\gamma r}-e^{-\zeta r}), \tag{14}$$

$$L_{\text{SMG}}=L_H=\int_0^\infty k\gamma r^2 U_H j_1(kr)dr$$
$$=2k^2\gamma\{(\gamma^2+k^2)^{-2}-(\zeta^2+k^2)^{-2}\}. \tag{15}$$

The introduction of a repulsive core would tend to increase L_{SMG} by several percent in the energy range of interest.[17]

The SMG prediction for b was recalculated for this paper, using the above equations, and is shown in Fig. 1. The values[18] used for γ and ζ are 0.2315 and 1.340, respectively. The latter value is somewhat smaller than the value used by Schiff, the result of a simple correction of the effective range so as to take into account the ground-state D wave. This correction leads our SMG values to be slightly smaller than the values actually given by those authors.

The experimental data are seen to differ from the SMG prediction in two ways. Not only is a very large and comparable to b over most of the energy range, rather than zero as SMG predicted, but b consistently lies below the SMG prediction. Interestingly, the two sorts of deviation from the SMG prediction tend to cancel when the total cross section is computed, suggesting that they might both be produced by way of the interference terms, these not appearing in the total cross section.

(b) Evaluation of L_f

A useful estimate of L_f may be obtained by approximations similar to those of the SMG theory. Thus in Eq. (7) the final wave already is assumed to be free. For the initial wave, W, the form to be used now will be $W_{\text{asymptotic}}$ multiplied by a factor to cut off the function near $r=0$,

$$W\approx(M/N)(1-e^{-\beta r})^3e^{-\gamma r}[1+(3/\gamma r)+(3/\gamma^2r^2)]. \tag{16}$$

The parameters (M/N) and β are so chosen as to give a good fit to the suitable one of the numerically calculated functions of Feshbach and Schwinger[19]:

$$(M/N)=0.0359,\quad \beta=0.463. \tag{17}$$

By using these expressions for W, the integral of Eq. (7) is performed, and the result for L_f is plotted as Fig. 3. Also shown in Fig. 3 are L_H and the interesting ratio (L_f/L_H).

It should be noted that the values for L_f computed here probably are rather reliable. The integral which determines L_f involves the function $r^2Wj_3(kr)$, and is not sensitive to the shape of W at small r. This is exhibited in the numerical work by a weak dependence of L_f upon the detailed form of the cutoff factor in W. Also the normalization factor, M, while coming from a calculation which gives the particular value of 2.7% D state in the deuteron,[19,20] is by no means as freely variable as is the percentage of D state. It is chosen to give the best normalization for W at large r, and is determined to about ten percent accuracy from the deuteron quadrupole moment.[21]

The ratio (L_f/L_H) is as large as 0.23 even at an energy of 25 Mev and even though L_f expresses the transitions from the 2.7% deuteron D wave. In part, this should be understood to arise because it is the D-wave amplitude which determines L_f/L_H, rather than the D-wave probability, and in part because the off-diagonal matrix element of S_{12} which links $^3P_2$ and $^3F_2$ has the large value of $6^{\frac{3}{2}}/5$.

[17] N. Austern, Phys. Rev. **88**, 1207 (1952). Some related calculations have been made by M. L. Rustgi and J. S. Levinger (private communication).

[18] These values have been taken over from the paper N. Austern, [Phys. Rev. **92**, 670 (1953)]. Note that all lengths and reciprocal lengths will be given in units of 10^{-13} cm or 10^{13} cm^{-1}, these dimensions generally not being indicated explicitly.

[19] H. Feshbach and J. Schwinger, Phys. Rev. **84**, 194 (1951). The same Feshbach, Schwinger case was used for this analysis as for the author's 1952 paper (see reference 13). However Eq. (16) is a more accurate representation of the D wave than was considered necessary for the earlier paper.

[20] The connection between the percentage of D wave and the deuteron magnetic moment has been clarified by the discovery that the meson field is pseudoscalar. A recent analysis which makes use of this property is given by M. Sugawara, [Phys. Rev. **99**, 1601 (1955); Arkiv. Fysik. **10**, 113 (1955); Progr. Theoet. Phys. Japan **14**, 535 (1955)]. Sugawara finds the percentage of D wave to lie in the range (3 ± 1)%.

[21] See J. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952), pp. 105–108.

III. DETERMINATION OF THE AMPLITUDES

The amplitudes L_J in Eqs. (2) and (3) need not be interpreted as the integrals of Eqs. (4), (5), and (6). In general, they are the amplitudes with which the outgoing waves are generated by the photoeffect process, whatever it may be, with the expressions (4), (5), and (6) only being the consequences of the rigid-nucleon assumption, the reaction mechanism at low energy. At sufficiently high energies new sorts of virtual meson effects can be expected, the Siegert theorem no longer holding, and the L_J can differ rather significantly from the values of (4), (5), and (6). Nevertheless (2) and (3) correctly relate the cross section to the outgoing-wave amplitudes, whatever may be the reaction mechanism.

In the present section a phenomenological analysis of (2) and (3) will be conducted, so as to see from experiment what are the values of the L_J and how these do compare with (4), (5), and (6). On the whole the analysis procedure simply will be to conduct a numerical exploration of the possible sets of solutions of (2) and (3), given the measured values of the parameters a and b. Equations (2) and (3) have too many solutions, though, for such an exploration to be possible without some judgment in advance as to which solutions might be reasonable. Not only are the numerical values of the "known" quantities more or less unreliable, but the equations have multiple solutions, and there are only two equations for three unknowns. It, however, is fortunate that reasonable bases for selection among the solutions may be established. The following ones will be used: (1) theory suggests that only for the $^3P_0$ wave is the Siegert theorem likely to break down to any important extent. For this reason the rigid nucleon predictions will be retained for L_1 and L_2. (2) An analysis to be given below suggests that the most general rigid nucleon predictions for L_1 and L_2 cannot be very different from the SMG predictions. Therefore, while L_1 and L_2 will be permitted large variations, these always will be within a factor of two about L_{SMG}. On the basis of (1) and (2) the analysis can be carried through, and a value found for L_0.

As important as the *a priori* judgments (1) and (2) are for the analysis, it also may be remarked here that it really is the large observed value for (a/b) which restricts the range of variation for L_0 in this analysis, giving for L_0 a surprisingly definite numerical value. In other circumstances (1) and (2) would not be sufficient to lead to any useful result.

The argument that virtual-meson production only affects L_0 has been given before.[1,10] Through $E1$ absorption an S-wave charged pion is produced, neutral pions being very unlikely. Then in the intermediate state we have two identical nucleons in a state of relative motion having even parity, and $J=0$ or 1, or 2. The only two possibilities are $^1S_0$ and $^1D_2$, with angular momentum considerations favoring the former over the latter. Reabsorption of the meson leads from $^1S_0$ to the $^3P_0$ final state.

(a) Limits of Variation of L_1 and L_2

The ^3P_J scattering phase shifts furnish a very helpful guide to the values of the amplitudes L_1 and L_2. The photodisintegration cross section is not very sensitive to the δ_J as they explicitly appear in (2) and (3), inasmuch as they appear only in cosines and certainly do not become very big over the energy range of interest. The cosines then do not differ much from unity. This point will be considered again. Meanwhile, the more interesting question of the magnitudes of the L_J integrals may be related to the δ_J. The δ_J play a role in these integrals insofar as they fix the asymptotic forms of the functions v_J, according to Eq. (11), so providing knowledge from experiment concerning the functions v_J. Everywhere outside the region of strong nuclear force the v_J have the form (11); certainly this form holds for $r\gtrsim 2\times 10^{-13}$ cm, and very likely it is not yet grossly wrong for $r=1\times 10^{-13}$ cm. For still smaller values of r we only know that the nuclear force must be of such a nature that v_J departs from the form (11) in such a way as to pass through zero at $r=0$, and that v_J must be a reasonably smooth function.

A pictorial and numerical study of the L_J may be based upon these ideas. Numerical values of the δ_J may be found in the papers of Feshbach and Lomon[2] and Gammel and Thaler.[2] Both analyses agree that δ_1 and δ_2 are small, of the order of or less than about 20° (in the paper of Gammel and Thaler) for center-of-mass energies less than 100 Mev. However Feshbach and Lomon find $\delta_0=-40°$, at a center-of-mass energy of 65 Mev. The possible consequences of such a large value for δ_0 will be considered again later in this paper, and then will be seen not to be very important. Meanwhile, the δ_1 and δ_2 phase-shift values may be summarized as, probably small, less than or equal to 20°.

In the following discussion the D-wave contributions to the L_J, at first, will be ignored.

Figure 4(a) presents the information required for a graphical estimate of the influence of the forces on the integrands of the L_J. This figure shows the functions v_J, plotted from the asymptotic formula (11), for a variety of possible phase shifts. Somewhere at small r Eq. (11) becomes invalid, the v_J becoming affected by the nuclear force. Presumably this happens for $r\lesssim 1\times 10^{-13}$ cm. Also shown in Fig. 4(a) is the function (γrU), plotted on the assumption that U has Hulthén form. Visual estimates of the contribution to L_J from a certain range of r may be made by estimating the average of the product of v_J by (γrU), over the range considered, and multiplying this by the value of the range of r. In making such estimates, it is necessary to bear in mind that however strange the nuclear force may be, the wave functions must vary smoothly and satisfy $v_J(0)=0$. Then it is clear, for example, that the

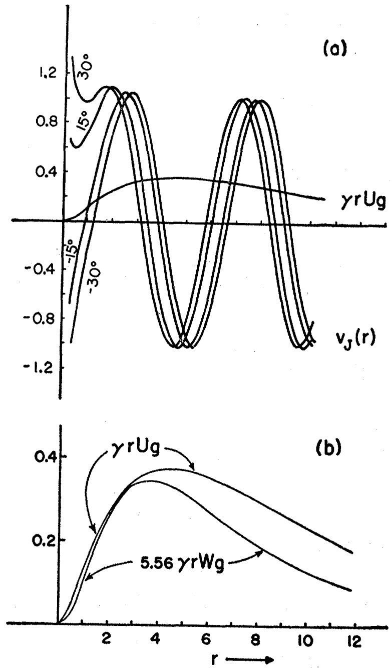

FIG. 4. (a) Graphs of the two factors of the integrands of (5) and (6). The asymptotic form of the factor $v_J(r)$ is shown for a variety of possible phase shifts. The factor $(\gamma r U_g)$ is shown for the circumstance that U_g has the Hulthén form. (b) Comparison of the shapes of the functions $(\gamma r U_g)$ and $(\gamma r W_g)$. The functions U_g and $5.56W_g$ reach their maxima at about the same value of r, and have the same value there. These particular wave functions are taken from the paper of Feshbach and Schwinger.

contribution to L_J from the region $r<1$ tends to have the same sign as δ_J. Only with difficulty could the magnitude of the contribution be as large as 0.1×10^{-13} cm, and its actual order of magnitude probably lies in the range $(0.01-0.03)\times10^{-13}$ cm.

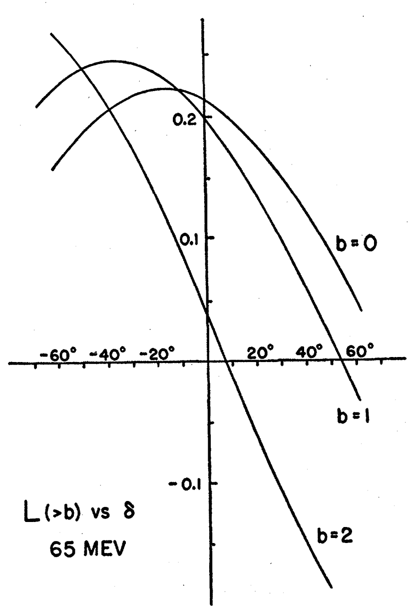

FIG. 5. The integral $L(>b)$, of (18), as a function of δ, for several values of b.

In conjunction with Fig. 4(a), which aids the discussion at small r, it is necessary to evaluate analytically the contributions to the L_J from large r. These contributions are the functions

$$L_J(>b)\equiv\int_b^\infty(\gamma rU)v_Jdr, \tag{18}$$

with U the Hulthén function, and v_J having its asymptotic form. Figure 5 is a graph of $L_J(>b)$ *vs* δ_J, for several values of b. Only the values $b=1$ need be considered. It will be observed that the values of $L_J(>1)$ are of the order of 0.1 or 0.2×10^{-13} cm, certainly much larger than the contribution from the interval $r<1$. Furthermore, the contributions from $r<1$ tend to have the same sign as δ_J, so augment $L_J(>1)$ where it is small, and reduce it where it is large.

Apparently it follows as a fairly reliable consequence of the rigid-nucleon assumption that the following qualitative observations are true, that L_1 and L_2 certainly are positive and roughly equal to L_H, and that $L_1\approx L_2$. In the further work the latter equality will be taken to hold within a factor of two.

The D-wave contributions to the L_J are not sufficiently large to change these qualitative conclusions. Equations (4), (5), and (6) show that the D-wave influence is largest in L_0, which will be determined from experiment, and only appears in L_1 and L_2 with the quite small coefficients 0.7 and 0.14. If W and U are presumed to have the same shape, then $(W/U)\approx0.2$, and the D-wave *relative* contributions to L_1 and L_2 are 0.14 and 0.03, well within our range of uncertainty. Actually W is slightly more sharply peaked than U, as may be seen in Fig. 4(b), so its contribution is a little bigger than these estimates. Detailed calculation shows that the change is not significant. In support of this conclusion it may be noted that the D-wave contribution to the L_J was found to be small even in a calculation in which its effects were deliberately exaggerated.[22]

(b) Solution of Equations

Now the knowledge that L_1 and L_2 lie close to L_{SMG}, and within a factor of two of each other, will be used to solve (2) and (3).

It is convenient to replace the experimental quantities a and b in (2) and (3) by the equivalent quantities

$$\alpha\equiv9a/B(k),\quad \beta\equiv6b/B(k). \tag{19}$$

Table I gives values of α and β at several interesting energies. Then the total cross section is proportional to $(\alpha+\beta)$, and

$$\alpha+\beta=L_0^2+3L_1^2+5L_2^2+(15/2)L_f^2, \tag{20}$$

[22] Y. Yamaguchi and Y. Yamaguchi, Phys. Rev. 98, 69 (1955).

TABLE I. Summary of experimental data. The parameters α and β of Eq. (19).

$\hbar\omega$ (in Mev)	α	β
25	0.636	3.607
40	0.529	1.125
65	0.404	0.307
90	0.297	0.152

no interference terms appearing in the total cross section. Equation (20) will be solved simultaneously with (2), of which the working form is

$$\alpha = L_0^2 - 2L_0L_2\cos(\delta_0-\delta_2) - 3L_0L_f\cos\delta_0 + (9/4)L_1^2 + (13/4)L_2^2 - (9/2)L_1L_2\cos(\delta_1-\delta_2) + (9/2)L_f^2 + (9/2)L_1L_f\cos\delta_1 - (3/2)L_2L_f\cos\delta_2. \quad (21)$$

It clearly is impossible for L_0 to dominate in $(\alpha+\beta)$, for if L_1 and L_2 are not much smaller than L_{SMG} they must in themselves already contribute a large part of $(\alpha+\beta)$. The L_f^2 term takes up part of whatever slack might be left over for L_0^2. A useful first approximation to (20) then is obtained by omitting L_0 entirely:

$$3L_1^2 + 5L_2^2 \approx \alpha + \beta - (15/2)L_f^2. \quad (22)$$

Equations (20) and (21) may be solved for L_0 by successive approximations, using (22) as the first step. It will be seen later that the result of the first step is, on the whole, sufficiently accurate to be taken as final. Several values of the ratio

$$x = L_1/L_2$$

will be considered, the solution being carried through completely for each such value. In terms of the new symbol x, Eq. (22) may be solved for L_2, giving

$$\Lambda^2 = [\alpha+\beta-(15/2)L_f^2]/(5+3x^2), \quad (23)$$

the symbol Λ also being introduced to denote the first approximation to L_2. In (23) the positive square root must be taken, as already was decided in III(a).

Using (23), the solution of (21) is

$$L_0 = \Lambda\cos\delta_{02} + \tfrac{3}{2}L_f\cos\delta_0 - \{\alpha - (9/4)|L_f + x\Lambda e^{i\delta_1} - \Lambda e^{i\delta_2}|^2 - (9/4)L_f^2(1-\cos^2\delta_0) - \Lambda^2(1-\cos^2\delta_{02}) - 3\Lambda L_f(\cos\delta_2 - \cos\delta_0\cos\delta_{02})\}^{\frac{1}{2}}, \quad (24)$$

where $\delta_{02} \equiv \delta_0 - \delta_2$. Here the negative square root is taken. This choice is required for the reason previously mentioned, that in the medium-energy region the total cross section is very little different from the SMG value, so that L_0 several times greater than L_{SMG} cannot be tolerated. (Furthermore the Zachariasen analysis[8] also already suggests that the meson contribution to L_0 is negative, leading $L_0 < \Lambda$ to be an expected result.) That (24) actually will give $L_0 < 0$ over much of the medium energy range is a consequence of the appearance of α in the square root, dominating over the other terms.

Table II shows the solutions of (24) and (25), for the circumstance that all the phase shifts are zero. The most striking facts to be observed in this table, from a physical point of view, are that L_0 changes sign near 40 Mev, becoming strongly negative towards the higher energies, that L_0 does not change rapidly as a function of x, particularly at the higher energies, and that for $x=1$ we do have Λ not much different from L_{SMG}, although consistently larger. The slow variation of L_0 with x must be attributed mostly, we may note again, to the large value of α in (24), with the presence of L_f in (24) also somewhat damping the dependence of L_0 upon x. The values of L_0 and Λ will be discussed at length in the next section.

From a mathematical point of view, Table II is interesting insofar as it gives an *a posteriori* justification for several steps of the calculations. First, we note that choosing the positive sign for the square root in (24) indeed would give a very large positive value for L_0, essentially just $\sqrt{\alpha}$. For this circumstance (20) shows that at the higher energies the $^3P_1$ and $^3P_2$ waves would receive only about $\frac{1}{3}$ of the photodisintegration total cross section, violating the assumption that L_1 and L_2 obey the rigid-nucleon theory. Second, upon inserting the numbers of Table II into (24) it is seen that the dependence of L_0 upon the phases is sufficiently weak so that no set of phase shifts which is at all reasonable could modify the qualitative conclusions about L_0, with the quantitative value of L_0 also varying slowly with the phases. Thus Table II is a good representation of the physical circumstances, even though the finer details remain unknown. The values of Λ are particularly insensitive to the phases, as Λ does not depend much upon L_0. Third, a test of the approximation $L_2 \approx \Lambda$ is indicated in Table II, in the form of the values of Λ', the second approximation to L_2. This Λ' is computed by again solving (20) for L_2, now having substituted for L_0 the approximate value just found. Within the accuracy of interest the values of L_0 certainly would not be changed if Λ' instead of Λ then were used in

TABLE II. Solutions of Eq. (24). At 25 Mev the value of L_0 becomes complex when $x=2$.

	$x=\frac{1}{2}$			$x=1$			$x=2$		
$\hbar\omega$ (in Mev)	Λ	L_0	Λ'	Λ	L_0	Λ'	Λ	L_0	Λ'
25	0.843	0.381	0.827	0.715	0.172	0.714	0.490	...	...
40	0.515	0.011	0.515	0.437	−0.064	0.437	0.299	−0.034	0.299
65	0.331	−0.145	0.326	0.281	−0.180	0.271	0.193	−0.261	0.182
90	0.261	−0.147	0.253	0.220	−0.174	0.212	0.152	−0.256	0.138

(24). However Λ' also is of some interest in itself, as a very close approximation to the exact solution for L_2, and will be discussed as such in the next section.

Some numbers which test the dependence of L_0 upon the phase shifts may be mentioned at this point, even though the form in which (24) is arranged already clearly exhibits this dependence. The calculations at 90 Mev were repeated using in (24) the Gammel-Thaler phases, $\delta_0=0.09°$, $\delta_1=-19.8°$, $\delta_2=15.0°$. With these phases the values of L_0 corresponding to $x=\frac{1}{2}$, 1, 2 are found to be -0.126, -0.130, -0.084. These results do support the general remarks made earlier about the dependence of L_0 upon the δ_J, inasmuch as the qualitative result that L_0 is negative has remained unchanged. Explicit calculations with the Gammel-Thaler potentials also support the general remarks about the rigid-nucleon predictions for L_1 and L_2.

IV. CONCLUSIONS

The large isotropic term in the medium-energy photodisintegration was seen in Sec. III to be the result of a modification of the $^3P_0$ amplitude, apparently of the sort discussed by Wilson, namely, the virtual production and reabsorption of a pion in a fashion which violates the low-energy Siegert theorem. In magnitude the meson contribution probably is best measured by the difference $(L_0-\Lambda')$, taken for the case $x=1$ (see Table II), and giving at the energies 25, 40, 65, 90 Mev, respectively, the values -0.542, -0.501, -0.451, -0.386. Within the accuracy of the present work these values indicate that $(L_0-\Lambda')$ may be regarded as constant.

Comparison with the Wilson prediction is achieved by considering the curve marked "S" in Fig. 2 of his paper. Of course, his "S" curve is somewhat wrong, for the meson process does not make an additive contribution to the total cross section but rather to the amplitude L_0. This error does not invalidate Wilson's results in the high-energy region, which he principally considered, but does require that his "S" curve be reinterpreted for the medium-energy region. The correct interpretation is

$$\sigma_{``S"}=(4\pi/9)B(k)(L_0-\Lambda')^2.$$

Numerical calculation then gives for the Wilson prediction of $(L_0-\Lambda')$ the values -0.359, -0.377, -0.385, at the energies 25, 65, 90 Mev. The agreement with the phenomenological results is very suggestive of the correctness of the Wilson idea, although further investigation really is required.

The origin of the isotropic term now may be summarized. In the high-energy range, although for energies below where the isobar dominates, the $^3P_0$ amplitude is very much enhanced by a meson-reabsorption process. Its quadratic contribution in the cross section is large and isotropic and is responsible for a major part of the total cross section, as suggested by Wilson. Towards lower energies the quadratic contribution of the $^3P_0$ amplitude becomes much less important, the new process and the rigid-nucleon process being of opposite sign. Nevertheless the isotropic part of the cross section remains large, for the $^3P_0$ amplitude is substantially altered and interferes with the other waves in such a way as to produce an isotropic cross section. This effect persists to extremely low energies.

It would be very interesting to measure the mesic modification of L_0 at much lower energies than those considered here, so as to see just how L_0 goes over to the rigid-nucleon prediction. The ratio (a_e/b) does become smaller at low energy (a_e is the *electric* dipole part of a), but L_1 and L_2 become easier to predict. With very accurate data, a useful analysis may be possible.

A possible alternative explanation of the negative values of L_0 may be mentioned at this point. We note that the D-wave term in (4) is much larger than that in (5) and (6), and is negative. Furthermore W is rather more sharply peaked than is U [see Fig. 4(b)], and rather strange forms occasionally have been considered for v_0. So perhaps the L_0 effect is entirely a non-central force effect, and no new process is required to explain it. Numerical examination shows this explanation to be unlikely,‡ even when the large Feshbach-Lomon values for δ_0 are considered. The departure from rigid-nucleon values simply is too great to be explained only with the aid of the forces. Of course, the negative D-wave contribution in (4) is present, and must *help* in driving L_0 negative. At 65 Mev its magnitude is about 0.06, and just happens to equal the difference between the Wilson value for $(L_0-\Lambda')$ and the phenomenological value.

The total cross section tends to be *reduced* by the L_0 effect, in the medium-energy region. Nevertheless,

‡ *Note added in proof.*—A recent investigation by Signell, Marshak, and Bilhorn suggests that this explanation of the photoeffect data nevertheless may be the correct one. Preliminary results of a calculation based upon the potential with which they fitted the high-energy nucleon-nucleon scattering [P. S. Signell and R. E. Marshak, Phys. Rev. **106**, 832 (1957)], and using the "rigid-nucleon" theory, show excellent agreement with the 75-Mev photoeffect data. At this energy the combination of circumstances $x=2.1$, $\delta_0=11°$, $\delta_1=-19°$, $\delta_2=11°$ gives for L_0 the value -0.01, essentially zero. The Signell-Marshak work not only produces the numbers just cited for x and the δ_J, but, with 8% deuteron D state, also gives both the low value required for L_0, and the large value required for L_1 in order to fit the total cross section. It should be noted that these results are achieved by means of a very large D state contribution,[20] and, even then, only with extreme values of all the other adjustable parameters we have been considering. That the Signell-Marshak approach leads very naturally to what seems such an unlikely set of circumstances should be interpreted as evidence in favor of their theory.

Actually, it is the large percentage D state which leads to the principal differences between the Signell-Marshak calculation and the one mentioned in the last paragraph of Section III, based upon the Gammel-Thaler potential. It may be that the exchange current implications of the $(\mathbf{L}\cdot\mathbf{S})$ nuclear interaction will make it possible to reconcile 8% D state with the known facts about the deuteron magnetic moment.[20]

I am grateful to R. E. Marshak for many conversations concerning his work.

experiment shows that the total cross section at, say 100 Mev, is slightly more than twice the SMG value. Part of the difference is made up by the F-wave transition, previously neglected. At 100 Mev the F-wave contribution in the total cross section is about 7.5 microbarns. Most of the difference, however, must be found in the numerical values of L_1 and L_2.

Our most reasonable phenomenological determination of L_1 and L_2 is the quantity Λ' of Table II, for the case $x=1$. In Table III this Λ' is compared with L_H, the value of L_{SMG} when a Hulthén form is used for the ground-state S function, and also with L_{RP}, the value of L_{SMG} when a 0.6×10^{-13} cm hard core is used in the ground state.[17] It is seen that Λ' is consistently larger that L_H, with the discrepancy being reduced but not eliminated by the use of the hard core. These observations may be taken as some rather weak evidence to support the belief that there is a repulsive core in the ground-state interaction. The remaining difference between Λ' and L_{RP} either may be regarded as not significant, or as suggesting a larger core radius,[17] or may be removed by suitable adjustment of the phase shifts. A large value for $|\delta_0|$, in particular tends to increase $|L_0|$ and to reduce Λ'. Questions of this kind are best deferred until definite experimental values of the ${}^3P_J$ phase shifts have been obtained.

Actually, of course, even the phenomenological separation of L_0 from the other amplitudes has not been accomplished in this paper in any final sort of way. Not only are the experiments no more accurate than about ten percent but they provide fewer equations than we have unknowns. Unfortunately a polarization measurement would be required in order to get any further data which bear on the L_J. Such a polarization experiment would be difficult to perform, being influenced by interferences with a number of other processes, such as $M1$ disintegration, which do not disturb the angular distribution. It is possible that the experiment can be arranged to distinguish among the various processes. An analysis of this experiment is being conducted by Sawicki and Czyz.[23]

It is interesting that internal consistency in this analysis could not be achieved if the ${}^3F_2$ amplitude were omitted. Without this amplitude, Λ' would be found larger yet and L_0 still would be found negative at the low energy of 25 Mev.

Table III. Comparison of Λ' with rigid-nucleon theory. L_H is the value of L_{SMG} when a Hulthén form is used for the ground-state wave function; L_{RP} is the value of L_{SMG} when a 0.6×10^{-13} cm hard core is used in the ground state.

$\hbar\omega$ (in Mev)	Λ'	L_H	L_{RP}
25	0.714	0.652	0.680
40	0.437	0.396	0.428
65	0.271	0.221	0.247
90	0.212	0.143	0.161

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[23] W. Czyz and J. Sawicki, Nuovo cimento **5**, 45 (1957); also private communication.