Selection Rules Implied by CP Invariance

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Some consequences of the rigorous invariance under the product of charge conjugation and reflection are discussed. The restrictions on the interactions of real boson fields with one spinor field, and of complex boson fields with charge symmetric spinor fields are derived and compared with the restrictions implied by invariance under the separate operations. Two selection rules for transitions among such boson fields are given. Some possible applications to known particles are mentioned.

I. INTRODUCTION

RECENT experiments have shown that neither parity[1] nor charge conjugation[2] are conserved in processes involving the emission of neutrinos. It has been suggested that the product CP of these quantities is conserved in these processes.[3,4] According to the Pauli-Lüders theorem,[5] the conservation of CP is equivalent to time-reversal invariance, for interactions invariant under proper Lorentz transformations, at least within the framwork of the usual local field theory. Experimental tests of the invariance under time reversal of neutrino processes have been suggested,[6] and information regarding this point may be forthcoming soon.

In this paper the consequences of the proposed CP invariance concerning the interactions of particles will be examined. Our major concern will be with the abstract consequences of this invariance and the conditions required for its application, without regard to whether these conditions are satisfied by the known elementary particles.

It is well known that the conservation of P or C places some restrictions on the interaction of fields. For instance, if parity is conserved, than a real or complex spinless boson field, ϕ, may have either of the interactions

$$g\bar{\psi}\psi\phi+\text{H.c.},\quad \text{or}\quad g'\bar{\psi}\gamma_5\psi\phi+\text{H.c.}, \tag{1}$$

with a particular fermion field ψ, but not both simultaneously. Alternatively, these restrictions may be stated in the form of transitions which are forbidden among particles whose transformation properties under reflection are already known from some other interaction. An example of this is the statement that if parity is conserved, a pseudoscalar particle cannot decay into 2 scalar particles.

The restrictions in the case of invariance under charge conjugation have been treated by, among others, Pais and Jost,[7] and Pauli.[5] The results here generally refer only to real boson fields and their interactions with single spinor fields, although under certain circumstances they apply to charged fields as well,[7] as we shall see. An illustration of a mixing of interactions forbidden by charge conjugation invariance is the following[5]: let ϕ be a real spinless boson field and ψ a spinor field. Then ψ and ϕ may interact by

$$g\bar{\psi}\psi\phi+\text{H.c.},\quad \text{or}\quad g'\bar{\psi}\gamma_\mu\psi\partial_\mu\phi+\text{H.c.}, \tag{2}$$

but not both.

If C and P are not conserved, it is to be expected that the restrictions mentioned above will not exist. Nevertheless, it will be shown that the invariance under the product operation CP implies somewhat weaker restrictions on the interactions. In Sec. II these are discussed for real boson fields and in Sec. III for charged boson fields whose interactions satisfy the further condition of charge symmetry. Finally, in Sec. IV some of the implications of the selection rules will be discussed.

II. REAL BOSON FIELDS

A. Conditions on Interactions

We consider a real boson field interacting with a single spinor field ψ through a linear interaction. The significance of the restriction to real boson fields and to a single spinor field is that the operation of C or CP will simply multiply the boson field and the quadratic Dirac covariants by phase factors. The operations on the spinor fields are defined by:

$$C\psi(x)C^{-1}=\mathcal{C}\bar{\psi}^T(x),\quad P\psi(x)P^{-1}=\gamma_4\psi(-x), \tag{3}$$

where $\mathcal{C}\gamma_\mu\mathcal{C}^{-1}=-\gamma_\mu{}^T$.

The question of a phase factor on the right-hand side of Eq. (3) is irrelevant if we consider quadratic covariants formed from ψ. Table I gives the transformation property of these covariants under C, P, and CP.

Let us now consider a real spinless field ϕ. If we do not allow interactions with derivatives of the fermion field, then the interaction must be constructed from the

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[1] Wu, Ambler, Hayward, Hoppes, and Hudson, Phys. Rev. **105**, 1413 (1957).
[2] Garwin, Lederman, and Weinrich, Phys. Rev. **105**, 1415 (1957).
[3] T. D. Lee and C. N. Yang, Phys. Rev. **105**, 1671 (1957).
[4] L. D. Landau, Nuclear Phys. **3**, 127 (1957).
[5] W. Pauli, in *Neils Bohr and the Development of Physics*, edited by W. Pauli (McGraw-Hill Book Company, Inc., New York, 1955), p. 30. Earlier references are given in this paper.
[6] R. B. Curtis and R. R. Lewis, Phys. Rev. **107**, 1381 (1957).
[7] A. Pais and R. Jost, Phys. Rev. **87**, 871 (1952).

first four covariants of Table I, and the quantities ϕ, $\partial_\mu\phi$. The field ϕ will transform under CP by

$$\begin{aligned} CP\phi(x)(CP)^{-1} &= n\phi(-x),\\ CP\partial_i\phi(x)(CP)^{-1} &= -n\partial_i\phi(-x), \quad (i=1,2,3)\\ CP\partial_4\phi(x)(CP)^{-1} &= n\partial_4\phi(-x), \end{aligned} \tag{4}$$

where $|n|^2=1$.

The interaction will then transform by:

$$\begin{aligned} CP\bar\psi\psi\phi(CP)^{-1} &= n\bar\psi\psi\phi,\\ CP\bar\psi\gamma_5\psi\phi(CP)^{-1} &= -n\bar\psi\gamma_5\psi\phi,\\ CP\bar\psi\gamma_\mu\psi\partial_\mu\phi(CP)^{-1} &= -n\bar\psi\gamma_\mu\psi\partial_\mu\phi,\\ CP\bar\psi\gamma_\mu\gamma_5\psi\partial_\mu\phi(CP)^{-1} &= -n\bar\psi\gamma_\mu\gamma_5\psi\partial_\mu\phi. \end{aligned} \tag{5}$$

Conservation of CP means that the Lagrangian can be made invariant under CP by a unique choice of n. Thus if CP is to be conserved, such a field may interact either by

$$\bar\psi\psi\phi,$$

or by any combination of

$$\bar\psi\gamma_5\psi\phi;\quad \bar\psi\gamma_\mu\psi\partial_\mu\phi;\quad \bar\psi\gamma_\mu\gamma_5\psi\partial_\mu\phi,$$

but not both classes.

If the spinor field ψ undergoes only the kind of interaction treated here, the vector coupling $\bar\psi\gamma_\mu\psi\partial_\mu\phi$ can always be transformed away by a unitary transformation,[8] and the restriction on interactions then becomes the same as if P were conserved, that is, scalar interaction does not mix with pseudoscalar or pseudovector.

Consider next a real, spin one, field, ϕ_μ. If we again exclude derivatives of the fermion field, the interaction must be formed from spinor covariants and ϕ_μ; $F_{\mu\nu}=\partial_\mu\phi_\nu-\partial_\nu\phi_\mu$. The latter will transform under CP by

$$\begin{aligned} CP\phi_i(x)(CP)^{-1} &= -n'\phi_i(x) \quad (i=1,2,3),\\ CP\phi_4(x)(CP)^{-1} &= n'\phi_4(-x), \end{aligned} \tag{6}$$

TABLE I. Transformation properties of spinor covariants.

O	COC^{-1}	POP^{-1}	$(CP)O(CP)^{-1}$	
$\bar\psi\psi$	$\bar\psi\psi$	$\bar\psi\psi$	$\bar\psi\psi$	
$\bar\psi\gamma_5\psi$	$\bar\psi\gamma_5\psi$	$-\bar\psi\gamma_5\psi$	$-\bar\psi\gamma_5\psi$	
$\bar\psi\gamma_\mu\psi$	$-\bar\psi\gamma_\mu\psi$	$-\bar\psi\gamma_i\psi$	$\bar\psi\gamma_i\psi$	$(i=1,2,3)$
		$\bar\psi\gamma_4\psi$	$-\bar\psi\gamma_4\psi$	
$\bar\psi\gamma_\mu\gamma_5\psi$	$\bar\psi\gamma_\mu\gamma_5\psi$	$\bar\psi\gamma_i\gamma_5\psi$	$\bar\psi\gamma_i\gamma_5\psi$	$(i=1,2,3)$
		$-\bar\psi\gamma_4\gamma_5\psi$	$-\bar\psi\gamma_4\gamma_5\psi$	
$\bar\psi\sigma_{\mu\nu}\psi$	$-\bar\psi\sigma_{\mu\nu}\psi$	$\bar\psi\sigma_{ij}\psi$	$-\bar\psi\sigma_{ij}\psi$	$(i,j=1,2,3)$
		$-\bar\psi\sigma_{i4}\psi$	$\bar\psi\sigma_{i4}\psi$	$(i=1,2,3)$
$\bar\psi\gamma_5\sigma_{\mu\nu}\psi$	$-\bar\psi\gamma_5\sigma_{\mu\nu}\psi$	$-\bar\psi\gamma_5\sigma_{ij}\psi$	$\bar\psi\gamma_5\sigma_{ij}\psi$	$(i,j=1,2,3)$
		$\bar\psi\gamma_5\sigma_{i4}\psi$	$-\bar\psi\gamma_5\sigma_{i4}\psi$	$(i=1,2,3)$

$\sigma_{\mu\nu}=[\gamma_\mu,\gamma_\nu]$

[8] F. J. Dyson, Phys. Rev. **73**, 929 (1948).

and

$$\begin{aligned} CPF_{ij}(x)(CP)^{-1} &= n'F_{ij}(-x) \quad (i,j=1,2,3),\\ CPF_{i4}(x)(CP)^{-1} &= -n'F_{i4}(-x) \quad (i=1,2,3). \end{aligned}$$

The interactions will then transform by

$$\begin{aligned} CP\bar\psi\gamma_\mu\psi\phi_\mu(CP)^{-1} &= -n'\bar\psi\gamma_\mu\psi\phi_\mu,\\ CP\bar\psi\gamma_\mu\gamma_5\psi\phi_\mu(CP)^{-1} &= -n'\bar\psi\gamma_\mu\gamma_5\psi\phi_\mu,\\ CP\bar\psi\sigma_{\mu\nu}\psi F_{\mu\nu}(CP)^{-1} &= -n'\bar\psi\sigma_{\mu\nu}\psi F_{\mu\nu},\\ CP\bar\psi\sigma_{\mu\nu}\gamma_5\psi F_{\mu\nu}(CP)^{-1} &= n'\bar\psi\sigma_{\mu\nu}\gamma_5\psi F_{\mu\nu}. \end{aligned} \tag{7}$$

The interaction can therefore be either a mixture of $\bar\psi\gamma_\mu\psi\phi_\mu$, $\bar\psi\sigma_{\mu\nu}\psi F_{\mu\nu}$, and $\bar\psi\gamma_\mu\gamma_5\psi\phi_\mu$, or it can be

$$\bar\psi\sigma_{\mu\nu}\gamma_5\psi F_{\mu\nu}.$$

This differs from the case when both C and P are conserved in that the pseudovector coupling $\bar\psi\gamma_\mu\gamma_5\psi\phi_\mu$ could not mix with the vector and tensor coupling when both are conserved. For the electromagnetic field A_μ, the additional requirement of gauge invariance eliminates the possibility of a pseudovector coupling and therefore CP invariance leads to the same restriction as C and P invariance. The experimental indication is that both C and P are indeed conserved.[3]

B. Selection Rules

For the purpose of deriving selection rules about transitions, we consider a set of real boson fields, whose transformation under CP is specified, as for example if they have interactions, of the type described in A, whose form is known. That is, we assume that the quantities n, n' of Eqs. (4) and (6) are prescribed for all of the fields. We further assume that CP commutes with the total Hamiltonian for all the bosons, and all particles with which they interact.

Let N be the number of particles occurring in a particular transition, for which n or n' is negative.

The S matrix for the transition is constructed from P brackets, derivatives, boson field operators which annihilate the initial state and create the final state, and the invariant tensors $\delta_{\mu\nu}$, $\epsilon_{\mu\nu\sigma\tau}$. If all interactions are invariant under CP, then so is the S matrix. Upon explicit transformation of S by CP we obtain a factor of -1 for each field appearing in the initial or final state which is among the N, and a factor of -1 for each $\epsilon_{\mu\nu\sigma\tau}$ which appears in the S matrix. Therefore

$$S=CPS(CP)^{-1}=(-1)^{N+n(\epsilon)}S,$$

where $n(\epsilon)$ is the number of times $\epsilon_{\mu\nu\sigma\tau}$ appears in the S matrix. Thus, for neutral bosons, a transition with

$$N+n(\epsilon)\ \text{odd is forbidden.} \tag{I}$$

If n and n' are prescribed for the fields by interactions of the kind discussed, we have

$$N=n_{PS}+n_{PV}+n_V+n_T,$$

where n_{PS}, n_{PV}, n_V, n_T are the number of fields concerned in the transitions which undergo pseudoscalar, pseudovector, vector, or tensor interactions respectively.

The selection rule may then be written

$$n_{PS}+n_{PV}+n_V+n_T+n(\epsilon) \quad \text{odd is forbidden. (Ia)}$$

This may be compared with the selection rules following from separate P invariance:

$$n_{PS}+n_{PV}+n_{PT}+n(\epsilon) \quad \text{odd is forbidden,}$$

and from separate C invariance:

$$n_V+n_T+n_{PT} \quad \text{odd is forbidden.}$$

The appearance of the quantity $n(\epsilon)$ implies that one cannot in general determine whether a process is forbidden by CP invariance just by looking at the intrinsic quantum numbers of the particles involved, but rather one must examine the space states involved as well, a characteristic which is shared by P invariance.

There are two simple cases when one can draw conclusions about the forbiddenness of transitions from I, because no $\epsilon_{\mu\nu\sigma\tau}$ will appear in S.

(a) A transition involving 3 real fields, two of which are spinless, is forbidden if

$$n_{PS}+n_{PV}+n_V+n_T \quad \text{is odd.} \tag{8}$$

Thus a PS-field cannot decay into two PS-fields.

(b) A transition involving 4 real fields, three of which are identical spinless fields, is forbidden if (8) holds.

From this it follows that a real $S(S)$ particle cannot decay into $3\pi^0$.

In some cases, the selection rule I can be used to determine the states involved in a reaction. Thus in the decay of a π^0 into 2 γ rays, the selection rule implies that if CP is conserved, and the π^0 has a well-defined phase n under CP, the reaction proceeds as if P were conserved, in particular, right- and left-handed polarized photons must occur equally in the decay.

III. CHARGED FIELDS

In this section we examine the linear interactions of a complex boson field ϕ with two spinor fields ψ_1, ψ_2. The operation of C or CP will now not relate the spinor covariants or the boson fields to themselves, but rather to their Hermitian conjugates, and will relate a process to another process in which the charges are changed. Because of this, invariance under C or CP will not in general lead to restrictions of the kind above, but rather to relations among the phases of the coupling constants for the several types of interactions.[9] Nevertheless, Pais and Jost have pointed out that the additional requirement of charge symmetry is sufficient to lead to restrictions of the type derived for neutral fields.

To see this we write the interaction as

$$\mathcal{L}_{\text{int}}=g\bar{\psi}_1O\psi_2\phi+g^*\bar{\psi}_2\bar{O}\psi_1\phi^*, \tag{9}$$

where $\bar{O}=\beta O^+\beta$. This interaction is said to be charge-symmetric if the transformation

$$\psi_1\rightarrow\psi_2, \quad \psi_2\rightarrow\psi_1, \quad \phi\rightarrow\epsilon\phi^*, \tag{10}$$

Leaves $\mathcal{L}$ invariant.[10] We can take $\epsilon=1$ without loss of generality. Under this transformation,

$$\mathcal{L}_{\text{int}}\rightarrow g\bar{\psi}_2O\psi_1\phi^*+g^*\bar{\psi}_1\bar{O}\psi_2\phi. \tag{11}$$

Invariance under charge symmetry then requires

$$g^*\bar{O}=gO. \tag{12}$$

The operators O and $\bar{O}$ will just be related by a factor ±1 where the $+$ sign applies for the scalar and pseudotensor covariants and the $-$ sign for the pseudoscalar, vector, pseudovector, and tensor. If we write $\bar{O}_i=n_iO_i$, where $n_i=\pm1$, the condition for charge symmetry of interaction through a particular invariant O_i is

$$g_i^*n_i=g_i. \tag{13}$$

This condition can be satisfied for any combination of interactions by suitable choice of g_i. In particular, the following interactions in any combination are charge symmetric:

$$\begin{aligned}
&\text{A. } g_S(\bar{\psi}_1\psi_2\phi+\bar{\psi}_2\psi_1\phi^*),\\
&\text{B. } ig_{PS}(\bar{\psi}_1\gamma_5\psi_2\phi+\bar{\psi}_2\gamma_5\psi_1\phi^*),\\
&\text{C. } ig_V(\bar{\psi}_1\gamma_\mu\psi_2\partial_\mu\phi+\bar{\psi}_2\gamma_\mu\psi_1\partial_\mu\phi^*),\\
&\text{D. } ig_{PV}(\bar{\psi}_1\gamma_\mu\gamma_5\psi_2\partial_\mu\phi+\bar{\psi}_2\gamma_\mu\gamma_5\psi_1\partial_\mu\phi^*),\\
&\text{E. } ig_V'(\bar{\psi}_1\gamma_\mu\psi_2\phi_\mu+\bar{\psi}_2\gamma_\mu\psi_1\phi_\mu^*),\\
&\text{F. } ig_{PV}'(\bar{\psi}_1\gamma_\mu\gamma_5\psi_2\phi_\mu+\bar{\psi}_2\gamma_\mu\gamma_5\psi_1\phi_\mu^*),\\
&\text{G. } ig_T'(\bar{\psi}_1\sigma_{\mu\nu}\psi_2F_{\mu\nu}+\bar{\psi}_2\sigma_{\mu\nu}\psi_1F_{\mu\nu}^*),\\
&\text{H. } g_{PT}'(\bar{\psi}_1\gamma_5\sigma_{\mu\nu}\psi_2F_{\mu\nu}+\bar{\psi}_2\gamma_5\sigma_{\mu\nu}\psi_1F_{\mu\nu}^*),
\end{aligned} \tag{14}$$

where all the g's are real.

Thus charge symmetry already leads to a condition on the phases of the coupling constants which as we shall see will combine with the condition coming from CP invariance to restrict the allowed combination of interactions.

We now apply the operation of CP to the interactions A–H, where CP acts as before on the spinor fields, and by

$$\begin{aligned}
&CP\phi(x)(CP)^{-1}=n\phi^*(-x),\\
&CP\phi_i(x)(CP)^{-1}=-n'\phi_i^*(-x), \quad (i=1,2,3)\\
&CP\phi_4(x)(CP)^{-1}=n'\phi_4^*(-x),
\end{aligned} \tag{15}$$

on the boson fields.

This transformation has the effect of multiplying interaction A by $+n$, interactions B, C, D by $-n$, interactions E, F, G by $-n'$ and interaction H by $+n'$.

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[9] For the case of C invariance, see reference 5; for the case of CP invariance see reference 3.

[10] It is also necessary that the free-field Lagrangian and the commutation relation be invariant under the transformation (10), in order that the transformation be useful.

It may be seen from this that invariance under CP leads to the same restrictions for charge-symmetric interactions as it does for the interactions described in Sec. II. That is, the requirement of a unique value for n or n' implies that the scalar interaction does not mix with the vector, pseudoscalar, or pseudovector interactions for a spinless field, and the pseudotensor interaction does not mix with the vector, pseudovector, or tensor interactions for a spin-one field.

By applying the operations CP and charge symmetry to the S matrix, we can derive a selection rule similar to I for transitions involving real boson fields and complex boson fields with charge-symmetric interactions. The selection rule is:

$$n_{PS}+n_{PV}+n_V+n_T+n(\epsilon)+n(\tau_3) \quad \text{odd is forbidden,} \qquad \text{(II)}$$

where n_{PS}, n_{PV}, n_V, n_T, $n(\epsilon)$ are defined as before and refer to real or complex boson fields.

$n(\tau_3)$ is the number of real fields whose interaction is of the form

$$\bar{\psi}_1 O\psi_1\phi-\bar{\psi}_2 O\psi_2\phi. \qquad (16)$$

Such fields must transform by

$$\phi\rightarrow-\phi$$

under the charge-symmetry operation if this interaction is to be invariant.

IV. DISCUSSION

It should be emphasized that if the Lagrangian is invariant under the combination of transformations [(3)+(4)+(6)] or under the combination [(3)+(10)+(15)], the selection rules I and II will apply whenever the phases n, n' can be determined by any considerations. It is not necessary that all of the interactions involved be of the type that have been discussed in our consideration of unallowed mixtures. However, the same transformation must be applied to all of the interactions. For example, the covariants which can appear in a hypothetical interaction between Λ^0, Σ^0, and π^0 of the form

$$\mathcal{L}_{\text{int}}=g\bar{\psi}_\Lambda O\psi_{\Sigma^0}\phi_\pi+g^*\bar{\psi}_{\Sigma^0}\bar{O}\psi_\Lambda\phi_\pi, \quad (i=1,2,3) \qquad (17)$$

are not limited by CP invariance, instead there is a condition on the relative phases of the coupling constants. However, if the phase factor in the transformation of the π^0 is determined by the π-nucleon interaction and the interaction (17) is made invariant with this choice of phase, the selection rule (Ia) will still hold. It is clear that this is indeed a property desired of a conservation law, i.e., its application should depend only on the properties of the initial and final states and not on the intermediate details, provided that all of these intermediate states do conserve the quantity in question.

There is evidence that the π-nucleon interaction as well as other strong interactions involving strange particles are invariant under P separately.[3] However, we shall use some of these interactions to illustrate some of the consequences of our considerations regarding CP invariance, even though stronger restrictions may be derived.

The interaction between π mesons and nucleons is charge-symmetric in the sense used here. Our considerations show that the assumption of CP invariance is sufficient to exclude mixing of scalar with pseudoscalar or pseudovector coupling. The vector coupling may be transformed away to first order in its coupling constant. It is perhaps tempting to argue that CP invariance is the general rule for interactions, and that in special cases, such as the π-nucleon interaction or the electromagnetic interaction, there are extra symmetries. charge independence and gauge invariance respectively, which combine with this to give the effect of separate charge conjugation and parity conservation. It does not appear that this argument could be extended to the strong interactions of the strange particles. The reason for this is that charge symmetry in the sense used here is not implied by invariance under rotations in isotopic space. To see this, we consider the hypothetical $KN\Lambda$ interaction, which could be written as

$$g\bar{\psi}_\Lambda\psi_N{}^{(\alpha)}\phi_K{}^{(\alpha)}+g^*\bar{\psi}_N{}^{(\alpha)}\psi_\Lambda\phi_K{}^{(\alpha)}, \qquad (18)$$

where α denotes a spinor index in isotopic spin space which is summed over 1 and 2. This interaction is a scalar in isotopic spin space, and this satisfies the condition usually known as charge independence, whatever the reality property of g. However, it is not charge-symmetric under exchange of Λ and N, and neither is the free-field Lagrangian. Thus there is no restriction on the allowed couplings for this interaction and this interaction cannot be used to define a phase for the K-meson field under C or CP. Therefore CP invariance and charge independence do not imply P invariance for this interaction, which invariance must therefore be assumed separately to account for the experiments cited by Lee and Yang. Similar considerations hold for the other strong K interactions, since the conservation of strangeness forbids the strong interaction of a K meson with two fields of the same strangeness. This does not imply that the transformation properties of the K mesons under CP cannot be determined by other conditions, and indeed the work of Gell-Mann and Pais[11] shows that this may be done for superpositions of the neutral K mesons. In such a case the selection rule (I) will still be applicable to transitions involving real neutral mesons.

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[11] M. Gell-Mann and A. Pais, Phys. Rev. **97**, 1387 (1955).