Production of Λ^0 Particles in Lead and Carbon by 2.5-Bev Protons*

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In a systematic study of 18 000 pictures taken at the Cosmotron with the Yale diffusion cloud chamber, two Λ^0 decays were found having origin in a lead plate at the upstream edge of the chamber. With corrections for the geometric efficiency of the chamber and for the scanning efficiency, this leads to an upper limit of approximately 8 mb per lead nucleus for the production of Λ^0 particles.

Thirty-seven nuclear interactions, found in the methane gas of the chamber, indicate a cross section of 155 ± 45 mb for star production in carbon. A two-dimensional Monte Carlo calculation of the internal pion-nucleon cascade in carbon was carried out for 45 events. Computed values for the π^+/π^- ratio and for the number of emerging charged pions per event are compared with those for the experimental carbon stars. It is concluded that the Monte Carlo calculation exaggerates the cascade and thus provides an upper limit to the multiplicity of cascade-induced events. A lower limit is provided by considering only the first collision of each Monte Carlo event. A count of the Monte Carlo fast internal pion collisions for which the production of a hyperon is energetically possible leads to an upper limit of 0.7 mb and a lower limit of 0.1 mb per carbon nucleus for Λ^0 production via the intermediate pion mechanism. Correction by Jastrow's lead-to-carbon production ratio yields Monte Carlo cross sections for lead which indicate that this mechanism may dominate over production in direct collisions between the incoming proton and internal nucleons.

INTRODUCTION

EARLIER work with cosmic rays[1] and at the Cosmotron[2] on the production of Λ^0 and θ^0 particles indicated that such production in proton-nucleus collisions might well be considerably less than in pion-nucleus collisions. And indeed only very few examples of hyperon production in single nucleon-nucleon collisions have been observed.[3,4] But even if production in nucleon-nucleon collisions did not occur at all, production in the case of nucleon-nucleus collisions could still proceed by another, indirect process. If the interaction between a high-energy nucleon and a nucleus is assumed to proceed, as discussed by Serber,[5] as a series of individual collisions between elementary particles, then pions produced in nucleon-nucleon collisions within the nucleus may carry sufficient energy to produce hyperons in subsequent internal collisions within the same nucleus.

In this experiment we undertook to measure the cross section for Λ^0 production per lead nucleus for incident protons of about 2.5 Bev and to compare these results with those obtained by the Columbia cloud chamber group for production by 1.9-Bev pions.[6] If one assumes the usual associated-particle modes of production, these energies are about twice the threshold energy in each case.

A Monte Carlo calculation of the internal pion-nucleon cascade in carbon was carried out for 45 events in order to estimate the indirect production cross sections for Λ^0 particles in both carbon and lead.

EXPERIMENTAL PROCEDURE

The experiment was carried out at the Brookhaven Cosmotron with a diffusion cloud chamber[7] of sensitive layer $25\times25\times5$ cm in a magnetic field of 8000 gauss. The external proton beam was obtained by shorting the pole face winding in one quadrant of the Cosmotron, causing the circulating beam to "blow up" after it reached 3 Bev. It was brought out by means of a 3×1 inch channel through 12 feet of Cosmotron shielding, was deflected 3.4 degrees by a steering magnet, and finally entered the cloud chamber ninety feet from the shielding. When operating at about 1/1000 of full machine intensity, a proton beam was available with low background and nearly optimum beam intensity. The horizontal spread of this beam was two or three feet at the cloud chamber. Measurements with a current-carrying wire allowed a determination of the magnet current needed to steer protons of the full energy into the chamber. The small deflection angle allowed entrance to particles over a considerable range of momentum. The current was set so that only those particles having momenta between the full beam momentum of 3.8 Bev/c and a minimum of 3.0 Bev/c could enter the chamber. With this current setting no pions from inside the shielding could reach the chamber without scattering. The small solid angle subtended by

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1 W. D. Walker and N. M. Duller, Phys. Rev. **90**, 320 (1953); Fretter, May, and Nakada, Phys. Rev. **89**, 168 (1953); Bridge, Peyrou, Rossi, and Safford, Phys. Rev. **91**, 362 (1953).

2 Walker, Preston, Fowler, and Kraybill, Phys. Rev. **97**, 1086 (1955). See also Cookson, Bowen, Tagliaferri, Werbrouck, and Moore, Bull. Am. Phys. Soc. Ser. II, **2**, 19 (1957); and Blumenfeld, Boldt, Bridge, Caldwell, Pal, and Leavitt, Bull. Am. Phys. Soc. Ser. II, **2**, 19 (1957).

3 Block, Harth, Fowler, Shutt, Thorndike, and Whittemore, Phys. Rev. **98**, 248 (1955).

4 Wright, Saphir, Powell, Maenchen, and Fowler, Phys. Rev. **100**, 1802 (1955).

5 R. Serber, Phys. Rev. **72**, 1114 (1947).

6 Blumenfeld, Booth, Lederman, and Chinowsky, Phys. Rev. **98**, 1203 (1956); and Bull. Am. Phys. Soc. Ser. II, **1**, 63 (1956).

7 Designed by T. W. Morris, Yale Ph.D. thesis, 1955 (unpublished).

the cloud chamber then precluded any appreciable pion contamination of the beam.

The beam entered the chamber through a re-entrant window designed to allow easy placement of a lead or carbon plate at the edge of the camera's field of view with minimum disruption of the sensitive layer. In order to provide good sensitivity near the re-entrant window it was found necessary to provide additional cooling at the bottom of the window. This succeeded in improving the sensitive layer in this region in spite of a downward convection adjacent to the face of the re-entrant window. This convection was quite localized and did not appreciably affect the stability of the gas elsewhere in the sensitive region.

Photographs of the chamber were made directly and through two mirrors, mounted one on each side of the hole through the magnet yoke. The effect of optical distortions on track curvatures was measured on pictures taken with the cloud chamber replaced by a series of taut nylon filaments. Corrections determined from these measurements were subsequently applied to all momentum measurements. As a check on the beam momentum, curvature measurements were made on 111 beam tracks in the cloud chamber. When corrected for optical distortions, they yield a mean momentum of 3.3 Bev/c or a kinetic energy of 2.5 Bev.

Λ^0 PRODUCTION IN LEAD BY PROTONS

Of all pictures taken, 17 000 were used for a study of neutral V events. Of these, 12 000 were taken with a lead plate in the re-entrant window upstream of the chamber and 5000 with a similar carbon plate. These groups correspond, respectively, to 160 000 and 90 000 beam protons through the plates. All possible unstable particle decays were analyzed according to the procedure of Podolanski and Armenteros[8] to test their consistency with Λ^0 and θ^0 decay dynamics and were also checked for possible consistency with charged K and pion decay modes.

No neutral V events were found in the carbon-plate pictures. In the lead-plate pictures, two events were definitely identified as the decays of Λ^0 particles originating in the lead plate. A third Λ^0 particle seems to originate in the wall above the plate; however, origin in the plate is not excluded. There were in addition one Λ^0 (with θ^0 not excluded) and five other events (two Λ^0, two θ^0 and one Λ^0 or θ^0) with origins in the bottom of the chamber. No cases of associated production or charged V decay were observed.

The over-all efficiency for detecting V decay can be treated as a combination of three separate factors: The first factor is the geometric efficiency of the chamber, which depends on the angular and momentum distributions of the V particles and on the size and shape of the sensitive layer; the second is the loss of efficiency resulting from the gap in the sensitive layer close to the lead plate; and the final factor is the scanning efficiency.

In order to evaluate reliably the first two efficiency factors, a Monte Carlo calculation was made in which 100 Λ^0 particles were assumed to be created at uniformly spaced intervals throughout the lead plate, allowing for the attenuation of the incident proton beam in the plate. The laboratory azimuthal angle and momentum of each Λ^0 were chosen by random number to fall into one of five momentum intervals and one of four angular intervals which were weighted according to the experimental distributions found by Gayther and Butler for Λ^0 particles produced by cosmic ray primaries averaging between 5 and 10 Bev.[9] Only angles less than 90° were used in the calculation and a correction factor was later introduced to allow for backward production in the laboratory system. The results of the Columbia cloud chamber group for the 1.9-Bev π^- beam show only four events produced backward in the laboratory system in a total of 76 decays from the lead plate.[10] With correction for the probability of observation, this amounts to less than 10% of the total number of events or a reduction in geometric efficiency by a factor of only 0.9 due to backward production.[11]

In the next step the polar angle of production of each Monte Carlo Λ^0 was chosen at random with uniform weighting for all angles. The point of decay was then fixed by taking a two-digit random number as the value of $e^{-l/\lambda}$ and calculating l from this by use of a table of the exponential function for the particular value of λ associated with the momentum interval chosen for the given event. In this manner, 100 intervals of the decay distance were available. For each such particle, projections were made of the decay point onto both the horizontal and the vertical planes containing the incident beam proton. Since the position of the zero polar angle is completely arbitrary, both projections were plotted as projections onto the vertical plane in order to double the number of events. Figure 1 shows this projection for most of the 200 points. The thin line indicates the probable extent of the sensitive region exclusive of any correction for the gap in the sensitive layer near the lead plate. Only 30% of the 200 points fall within this region. Applying the factor of 0.9 for Λ^0 particles produced at laboratory angles greater than 90°, we obtain a geometric efficiency of 25%.

In order to estimate the loss of efficiency due to the gap in the sensitive layer, all points of decay of the Monte Carlo Λ^0 events were projected on to the line

[8] J. Podolanski and R. Armenteros, Phil. Mag. 45, 13 (1954).

[9] D. B. Gayther and C. C. Butler, Phil. Mag. 46, 467 (1955).

[10] H. Blumenfeld (private communication).

[11] Gayther and Butler (reference 9) find that 41% of their observed decays are produced at laboratory angles greater than 90°. This does not seem applicable in view of the Columbia results quoted above, especially for our case of incident nucleons. When one treats internal collisions singly, backward production requires higher values of Fermi momentum for nucleon-nucleon than for pion-nucleon collisions.

parallel to the beam direction and the density of decays along this line was fitted to an exponential with a decay parameter 3.9 cm. A series of 148 pictures was chosen by a fixed procedure from the whole group of 12 000 lead-plate pictures. An exponential grid, with the 3.9-cm decay parameter, then allowed a quick evaluation of the loss in efficiency due to the gap for each of these pictures. This procedure together with the requirement that an event must be at least 1 cm inside the sensitive region to be identifiable, yielded a 60% residual efficiency.

The scanning efficiency is quite difficult to evaluate because of the paucity of events. We know, however, that the efficiency for detecting multiprong stars approaches 100% and for detecting two-prong stars is approximately 60–80%. On this basis we can conclude that the scanning efficiency is most likely $\geq 50\%$. In this connection there is a bias in the detection of decays which would affect the estimation of over-all efficiency. None of the observed events, including those produced in the bottom, decay in a plane having less than a 55° dihedral angle (Δ) with the plane defined by the vertical direction and the path of the Λ^0 before decay. That is, the observed decay planes tend to be horizontal, indicating a bias against observing events with nearly vertical decay planes. Tabulation of the Monte Carlo Λ^0 decays shows that there is no appreciable chamber bias against observing Λ^0 events whose production planes are less than 45° from the vertical plane. Therefore, this bias, which amounts to as much as a factor of two, must be connected with the scanning efficiency. If this additional correction factor of two is used, then the scanning efficiency (which would then be only for events decaying in planes having $45° < \Delta < 90°$) would be very probably $\geq 90\%$. Therefore, under this assumption the net scanning efficiency becomes $\geq 40\%$.

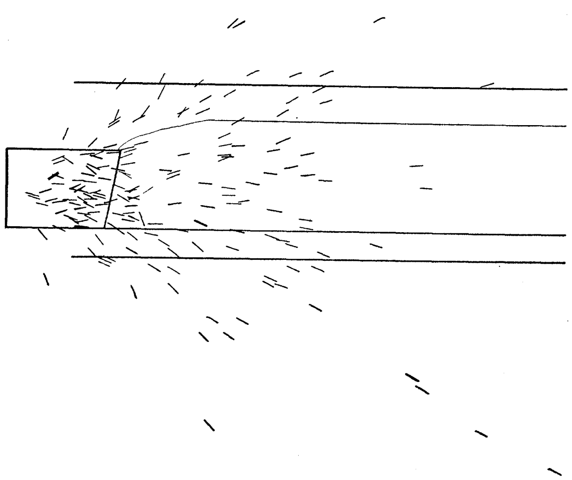

FIG. 1. Distribution of Monte Carlo Λ^0 decays, projected upon a vertical plane parallel to the direction of incident proton beam. Cross section of diffusion chamber and of target are indicated by heavy lines. Outline of sensitive region is depicted by fine lines.

The lower limit of the over-all efficiency for detecting Λ^0 decays then becomes $0.26 \times 0.60 \times 0.40 = 0.06$.

The beam tracks in each of the 12 000 lead plate pictures were counted by the scanners. This count was rechecked by a special scanning of the series of 148 pictures mentioned previously. A correction was applied for the slightly different criteria for beam tracks used in the two cases. An additional correction was needed to allow for those beam tracks passing through the sensitive layer but above the lead plate. The corrected value of the total flux is 127 000 beam protons through the 4.3-cm lead plate. As a check, it was shown that this flux is consistent with the number of negative, high-momentum tracks found emerging from the lead target in the set of 148 pictures. The 40 negative tracks found in this set correspond to 320 tracks for the complete set of lead-plate pictures. If we assume that the π_+/π^- ratio is not a sensitive function of the laboratory angle, then the value of 1.7 from the carbon stars of the present experiment (consistent with the value of 2 obtained by Yuan and Lindenbaum[12] for beryllium) leads to an estimate of approximately 900 for the number of pions of both signs produced in our lead-plate pictures. This number can be shown to be consistent with a total flux of 127 000 beam protons through the lead plate when allowance is made for the plate thickness and for the geometric criteria used in making the count of negative tracks.

Using the two definite Λ^0 decays and the over-all efficiency of 6%, we now obtain a mean interaction length for Λ^0 production in lead of 16 400 cm or a cross section per lead nucleus of approximately 2 mb. Of more significance is the upper limit obtained when we include the third, less probable Λ^0 decay mentioned previously, and when the Poisson distribution statistics are considered.[13] Then we find that for any cross section higher than 8 mb, the probability of obtaining three events or less falls below 5%. This upper limit amounts to about 0.3% of the geometric cross section for lead. The Columbia data indicate a cross section for Λ^0 production by pions of 1.9 Bev amounting to several percent of geometric.[6] Thus the present experiment indicates that the cross section for the production of Λ^0 particles by protons on lead nuclei is appreciably less than that for incident pions, when the incident energies are in each case roughly twice the threshold energy.

STARS PRODUCED IN THE GAS OF THE CHAMBER

A total of 37 nuclear interactions in the methane-gas filling of the cloud chamber were found in 18 000 pictures. The classification of these events by the number of secondaries emitted in each case is given in Table I along with the average number of minimum

[12] L. C. L. Yuan and S. J. Lindenbaum, Phys. Rev. **103**, 404 (1956).
[13] V. H. Regener, Phys, Rev. **84**, 161 (1951).

TABLE I. Size distribution of stars produced in carbon by 2.5-Bev protons.

No. of secondary prongs	<2	2	3	4	5	6	7	8	9	
No. of stars	0	16	5	1	8	3	2	0	2	
No. of minimumly ionizing prongs per star	0	1.25	1.2	2.0	1.25	1.33	2.0	0	3.0	Av. = 1.4

tracks per star for each category. The two stars with nine secondary prongs represent complete disintegration of the struck carbon nucleus with simultaneous production of positive and negative pions. Several criteria were used in separating the hydrogen and carbon events. First of all, any stars with an odd number of prongs were automatically excluded as hydrogen events. Of the even-pronged events, the three with six prongs had four or more dense secondaries while the one four-prong event contained no negative secondary. These are all thus inconsistent with interpretation as hydrogen events. Consequently, all the stars with more than two prongs are events in carbon. The probability is less than 17% that any of the observed stars were produced in oxygen or nitrogen nuclei of the alcohol vapor or residual trace of air in the chamber.

The classification of the sixteen two-prong events is more uncertain. However, two of these events are characterized by dense blobs at the star center and are therefore interpreted as carbon events. The remaining two-prong events can be consistently interpreted as hydrogen events. The final assignment is then 23 carbon and 14 hydrogen events.

Small-angle elastic events, which would have one emerging prong, are not observed at all. As interpreted by the theory of Fernbach, Serber, and Taylor[14] such scattering would correspond to the diffraction scattering of the incoming proton wave which would, for 2.5-Bev incident protons, be essentially confined to within less than 4° of the forward direction in the laboratory system. For such small scattering angles and with nothing else to distinguish the event, it is not surprising that the efficiency of detection is low.

The scanning efficiency was checked by rescanning 50% of the pictures. In these rescanned pictures, all the events with more than two prongs (12 events) were found by both scanners, indicating a high efficiency (close to 100%) while for the eight two-prong events the efficiencies of the scanners were found to be approximately 60% and 80%. Assuming that all two-prong stars are equally easy to detect, this means that a double scanning would be 85–95% efficient.

With such a value for the scanning efficiency, the corrected ratio of events in carbon to events in hydrogen is essentially the same as the ratios of the geometric cross sections.

In an analysis of the star prongs, eight negative and nine positive pions were identified by momentum *versus* ionization. Using a rough extrapolation of the pion momentum spectrum found for 2.3-Bev protons on beryllium[12] and assuming π^+ of greater than 700 Mev/c to be indistinguishable from protons, we would expect that $\frac{1}{5}$ to $\frac{1}{7}$ of the π^+ actually produced have not been identified as such. Correction for this leaves us with 8 π^- and 11 π^+. Another method of making this correction (assuming π^+ and π^- momentum spectra to be identical) is to count the fraction of the negative pions whose identification could not be accomplished on the basis of momentum and ionization alone but required recognition of the negative charge. Making a similar fractional correction to the number of positive pions leaves us with 8 π^- and 14 π^+. These numbers of pions are based on 20 carbon stars and thus lead to values of π^+/π^- from 1.4 to 1.8 and pion multiplicities of 0.95 to 1.1 charged pions per star. The statistical error in the ratio is ± 0.45 and in the multiplicity, ± 0.2.

The following procedure was used to estimate the total cross section for star production in carbon. The total length of beam tracks was measured in 182 pictures chosen from the total set at equal intervals. By comparing with the number of tracks counted by the scanner in the same 182 pictures and with the total track count for all 18 000 pictures, the total track length for all pictures was calculated. When repeated, using the track count of another scanner, this procedure yielded the same result within 3%. In order to be identifiable as a proton-induced star, a segment of beam track two centimeters long must be visible upstream of the interaction. With allowance for this criterion, the total track length becomes 3.98×10^6 cm. The average pressure of the methane was 5.0 psig and the mean temperature throughout the sensitive region was estimated as -40°C. Table II shows the corresponding cross sections, subject to various assumptions about the scanning efficiencies and classification of

TABLE II. Comparison of proton-induced carbon and hydrogen stars in methane.

	Raw data	Corrected for scanning efficiency: Eff. (2 prongs) = 70%, Eff. (>2 prongs) = 100%	Corrected for scanning efficiency: Eff. (2 prongs) = 60%, Eff. (>2 prongs) = 90%
No. of hydrogen events	14	20.0	21.7
No. of 2-prong carbon events	2	2.8	5.0
No. of carbon events of >2 prongs	21	21.0	23.3
σ (star production in carbon) mb	136 ± 28	141 ± 29	167 ± 31
Ratio of carbon to hydrogen events[a]	1.6	1.19	1.30

[a] The ratio of carbon to hydrogen events in methane, if one uses geometric cross sections, would be 1.36.

[14] Fernbach, Serber, and Taylor, Phys. Rev. **75**, 1352 (1949).

two-prong events. The cross section for star production in carbon at 2.5 Bev is then 155±45 mb when both statistical and scanning efficiency variations are included.

MONTE CARLO CALCULATION OF THE PION-NUCLEON CASCADE IN CARBON

By making a Monte Carlo calculation to follow the cascade produced by a 2.5-Bev proton in carbon, we are able, first of all, to compare some of the features of the resulting stars (such as π^+/π^--ratio and charged-pion multiplicity) with those of the experimental stars, thus roughly checking the validity of the cascade model at these high energies. Moreover, we obtain information on the number of fast internal pion collisions in which hyperon production is energetically possible. This will then allow an estimate of the cross section for production of hyperons in proton-nucleus collisions via intermediate pions.

According to the cascade model of Serber[5] the process of star formation takes place in two stages, the first (duration $\sim 10^{-23}$ sec) consisting of a series of single nucleon-nucleon and pion-nucleon collisions, in which some particles leave the nucleus and the second stage, lasting several orders of magnitude longer, during which the excitation energy is distributed among the nucleons of the nucleus resulting in a rise in the nuclear temperature and subsequent boil-off of additional particles. The Monte Carlo calculations of Goldberger[15] for 86-Mev neutrons and Bernardini, Booth, and Lindenbaum[16] for 400-Mev protons, show that in this energy region the cascade calculation is consistent with experimental results. A general procedure somewhat similar to that of Bernardini was used for this present, higher energy calculation. For such a calculation to be feasible, the following assumptions must be used: (1) The nucleus consists of a gas of noninteracting nucleons. (2) The de Broglie wavelengths of the particles involved in the cascade are less than the mean internucleon distances, so that a series of individual nucleon-nucleon and pion-nucleon collisions can be considered. (3) The density of nucleons is uniform within the nuclear radius and zero outside. (4) The isotopic spin formalism and charge symmetry are correct in their application to all interactions.

In order to simplify the calculation of those collisions which result in many secondary particles, the collisions were all considered in the laboratory system and the Fermi momentum of the target nucleon was neglected. If the incident momentum is higher, by a factor of five or more, than the Fermi momentum (which is up to about 170 Mev/c for carbon[17]) we can consider the effect of the Fermi momentum on nucleon-nucleon and pion-nucleon interactions to be a smearing-out of the angular and momentum distributions of the secondary particles. Such an approximation will hold roughly throughout the energy regions where inelastic processes are appreciable. Considering the approximations used here, these regions were taken to be greater than 350 Mev for incident nucleons and greater than 400 Mev for pions. Following collisions (particularly those of pions) to much lower energies than these, without including the Fermi momentum in the dynamics of the collisions, is quite crude and may be the weakest approximation involved in the calculation. However, any collision leaving a nucleon in a final state with a momentum less than the Fermi momentum was not used in the Monte Carlo calculation.

A two-dimensional geometry was used in which the secondaries from each collision were immediately rotated, with the incident direction as an axis, into the common plane. The validity of this approximation was tested for a nucleon striking the target nucleus tangentially. The computation was made on the residual path of the scattered nucleon, averaged over all angles, under the assumption of an isotropic distribution in the center-of-mass system. The resulting mean residual path lengths through the nucleus (allowing no further interactions) were, in fractions of the nuclear diameter (a) 0.21 for the three-dimensional case, (b) 0.25 for vertical projection onto the reference plane, and (c) 0.33 for rotation onto the plane as described above. Both methods of mapping the three-dimensional event into a two-dimensional representation overestimate the path length for secondary collisions and will therefore also overestimate the cascade. Although rotation into the reference plane is worse in this respect, it was used because of the simplicity in applying the experimental distributions. The tangential collision is the worst case since as the initial collision is allowed to approach the center, the residual path lengths approach the nuclear radius in all three cases.

The radius of the carbon nucleus used in the Monte Carlo calculation was determined by fitting a classical gray sphere to an absorption cross section of 200 mb and to a value of 39 mb taken as an average of the p-p and p-n interaction cross sections at 2.5 Bev. The calculated value of the radius is not very sensitive to the precise value of this latter parameter and varies as the square root of the absorption cross section, the over-all percentage variation in the calculated radius being less than that in the value used for the absorption cross section. The radius obtained by this method was 2.8×10^{-13} cm for carbon, corresponding to $r_0=1.23\times10^{-13}$ cm.

The calculation was carried out manually with a table of random numbers and with tables giving the appropriate weighting factors for each decision. After each collision, the paths of the particles were reconstructed on paper at the proper angles so that the end result was a picture of each cascade process. All particles were followed (including neutral pions) until

[15] M. L. Goldberger, Phys. Rev. 74, 1269 (1948).
[16] Bernardini, Booth, and Lindenbaum, Phys. Rev. 88, 1017 (1952).
[17] Cladis, Hess, and Moyer, Phys. Rev. 87, 425 (1952).

they escaped from the nucleus, were captured, or in the case of nucleons, until the energy fell below the inelastic threshold. Both nucleons and pions were assumed to be captured if their energy fell below 25 Mev. Forty-five such cascades were calculated.

The impact parameter of the initial proton was chosen by a random number weighted to allow for the variation in total cross-sectional area seen by the incident particle at different impact parameters.

The momentum of a particle initiating a given collision was considered to fall into one of a number of momentum intervals, there being seventeen such intervals for nucleons and ten for pions. The information needed to carry out the calculation was, for each momentum interval, (1) a value of the total interaction cross section so that the mean interaction distance could be determined; (2) a percentage breakdown into elastic scattering and single, double and triple meson production; (3) for each of these categories an estimate of the relative probabilities of different charge states; and finally, (4) the angular distribution in the case of elastic scattering, and the angular and momentum distributions of both nucleons and pions for the inelastic case.

The total cross sections were estimated from the work of Shapiro, Leavitt, and Chen[18] on nucleons and Lindenbaum and Yuan, and Cool and Piccioni for pions.[19] The total interaction cross section of neutral pions can be shown by the isotopic-spin formalism to be the average of the π^+ and π^- interaction cross sections. However, the relative contributions of direct and charge exchange scattering cannot be uniquely determined by such an argument due to the unknown phase difference between the scattering through the isotopic spin $\frac{3}{2}$ and $\frac{1}{2}$ states.

The relative cross sections for the different types of interactions (elastic scattering, and single, double, and triple meson production) were estimated directly from the available data.[20–22] Information on the relative probabilities of the different charge states resulting form these interactions were taken from the same sources, where available. Although the data are meager, the experimentally observed charge state ratios resulting from π^--nucleon collisions of the same type (e.g., all interactions resulting in single-pion production) are at present consistent with the use of the Fermi statistical weights.[21] Assuming then that all types of pion-nucleon collisions are thus consistent, the Fermi weights as tabulated by Milburn[23] were used for the charged pion interactions.

For the π^0 interactions, we calculated the Fermi weights only in the elastic and single production cases. Charge-state weights for double and triple production interactions were chosen in such a manner that the mean charged meson yield and the ratio π^+/π^- were close to the average of those for the corresponding π^+ and π^- interactions. Only two charge states were used for the triple production since it does not contribute much to the total inelastic pion-nucleon cross section.

These charge-state weights, together with assumptions already made on the relative cross sections for elastic and single production, predict that approximately 23% of π^--p interactions will go to completely neutral states. This is in agreement with the results of Shapiro.[24]

In treating nucleon-nucleon collisions, experimental data were used where available. Although exceptions to Fermi statistical weights are known,[20, 22] they were used where experimental data were absent or inconclusive.

In order to determine the angles and momenta of the secondary particles in an elastic event, the angle of one of the emerging particles was picked by a random number with weighting according to the experimental angular distribution. The dynamics of the individual elastic collision is then, of course, completely and easily determined. Rotation of the event into a reference plane does not disturb the balance of energy and momentum.

For the particles emerging from inelastic events, the angle *vs* momentum plots used by the Brookhaven group were divided into intervals of 200 Mev/c each and 10° each, each of these areas weighted according to the number of secondary particles falling into it. The laboratory distributions of pions from all different charge states and multiplicities were sufficiently similar that they were combined to give one inelastic distribution for each energy. The same was true for emerging nucleons. The angle and momentum of a nucleon or pion emerging from an inelastic event were thus chosen simultaneously by one random number. Such a procedure will obviously construct Monte Carlo events whose secondary particles, although having correct statistical distribution in angle and momentum and in the correlation between angle and momentum, nevertheless do not add up to conserve momentum or energy in a single event.

A method of restoring the balance of energy and momentum by scaling the chosen momenta and adjusting one or more of the angles was used throughout the

[18] Shapiro, Leavitt, and Chen, Phys. Rev. **103**, 211 (1956).

[19] S. J. Lindenbaum and L. C. L. Yuan, Phys. Rev. **100**, 306 (1955); R. Cool and O. Piccioni, Bull. Am. Phys. Soc. Ser. II, **1**, 173 (1956).

[20] Block, Harth, Cocconi, Hart, Fowler, Shutt, Thorndike, and Whittemore, Phys. Rev. **103**, 1484 (1956); Fowler, Shutt, Thorndike, and Whittemore, Phys. Rev. **103**, 1479 (1956); Smith, McReynolds, and Snow, Phys. Rev. **97**, 1186 (1955); **96**, 1167 (1955); A. H. Rosenfeld, Phys. Rev. **96**, 130 (1954); Fields, Reiter, and Sutton, Bull. Am. Phys. Soc. Ser. II, **1**, 71 (1956); Stallwood, Fields, Fox, and Kane, Bull. Am. Phys. Soc. Ser. II, **1**, 71 (1956); Fowler, Shutt, Thorndike, and Whittemore, Phys. Rev. **95**, 1026 (1954); W. A. Wallenmeyer, Phys. Rev. **100**, 1255(A) (1955).

[21] V. P. Kenney, Phys. Rev. **104**, 784 (1956); Eisberg, Fowler, Lea, Shephard, Shutt, Thorndike, and Whittemore, Phys. Rev. **97**, 797 (1955).

[22] Morris, Fowler, and Garrison, Phys. Rev. **103**, 1472 (1956). See also Batson, Culwick, and Riddiford, *Proceedings of the Seventh Rochester Conference on High-Energy Physics.*

[23] R. H. Milburn, Revs. Modern Phys. **27**, 1 (1955).

[24] A. Shapiro (private communication).

first 17 events (Method A). If, however, the events are calculated without worrying about the balance of energy and momentum in a single event, then the average of many events will yield particles distributed in angle and momentum in the same manner as those of the experimental distribution, whereas by Method A these distributions were somewhat distorted. This method (Method B) was used throughout the remainder of the 45 Monte Carlo cascade events.

Table III compares some of the results of the Monte Carlo calculation with those values obtained experimentally in the 37 carbon stars and with those expected from a single nucleon-nucleon collision consisting of 54% p-p and 46% p-n in accordance with the input data used in the Monte Carlo calculation. This corresponds to the limiting case where only the first collision occurs and all secondary particles emerge without further interactions, thus forbidding the development of a cascade. It will be noted that the effect of the internal cascade is to increase the number of pions per event and to decrease the π^+/π^- ratio. For all parameters where a comparison is possible, the experimental values then indicate that the complexity of the cascade has been exaggerated by the Monte Carlo calculation.

Several factors were present which would tend to bring this about. For example, no corrections were made for changes in nuclear density occurring during the cascade process as a result of previous collisions. Therefore, events in which more then seven nucleons emerge are not excluded. Also, as we saw above, the method of reduction to a two-dimensional geometry overestimates the path in nuclear matter that must be traversed by secondary particles. The complexity of the cascade is directly related to the ratio of the nuclear radius to the mean interaction distance in nuclear matter and this ratio is a sensitive inverse function of the nuclear radius used. Thus a somewhat larger value of the radius would have given a reduced cascade effect.

The Monte Carlo events were searched for internal pion-nucleon collisions in which the production of a hyperon by the reaction $\pi+n\sim Y+K$ was energetically possible. Eleven such collisions were found. In each case, the production of a hyperon at zero degrees to the pion direction was assumed and its path length before emerging from the nucleus was measured. After an absorption cross section of 20 mb per nucleon was applied, 6.7 of the 11 hyperons emerged from the nucleus. If one assumes a cross section for Λ^0 production in π^--p collisions of 1 mb[25, 26] out of a total interaction cross section of 30 mb, and assumes that half of the hyperons were either Λ^0 or Σ^0, subsequently decaying to Λ^0, then the cross section for Λ^0 production per carbon nucleus is 0.63 mb with no reabsorption included (see Table IV). Because of the overestimation of the cascade process in the calculation, this must be considered an upper limit.

TABLE III. Results of Monte Carlo calculation of pions produced by 2.5-Bev protons in carbon.

Number of emerging particles per event	Monte Carlo cascade	Single collision	Experiment
π^+	0.95±0.15	0.48	0.63±0.17
π^0	0.91±0.14	0.47	···
π^-	0.62±0.12	0.20	0.40±0.14
Minimally ionizing secondaries	1.58±0.19	···	1.4 ±0.19
ratio π^+/π^-	1.53	2.3	1.6 ±0.45

TABLE IV. Monte Carlo cross sections (in mb) for Λ^0 production via intermediate pions.

Reabsorption cross section	per carbon nucleus: Complete cascade	Pion from initial collisions	per lead nucleus Scaled from carbon values[a]	
0	0.63±0.19	0.23	12.6	4.6
20 mb	0.38±0.12	0.13	4.2	1.4

[a] See reference 27.

By taking only the hyperons produced by collisions of those fast pions which were generated in the initial collisions of the cascade, we obtain a lower limit to the number of hyperons indirectly produced. Only four of the original eleven hyperons satisfy this condition. Now, using a 20-mb reabsorption cross section we obtain a cross section of 0.13 mb per carbon nucleus for the production of Λ^0 particles. This will be a minimum value unless we allow reabsorption cross sections higher than 20 mb.

In order to obtain from this an approximation of the cross section per lead nucleus we made use of a calculation by Jastrow[27] of the ratio of production in lead to production in carbon, computed both with 20 mb reabsorption and with no reabsorption. The resulting lead cross sections (included in Table IV) range from a minimum of 1 mb to a maximum of about 13 mb. On the other hand, the experimental results discussed above assign an upper limit of approximately 8 mb and a most probable value of 2 mb to the Λ^0 production per lead nucleus. We conclude that the intermediate pion mechanism for Λ^0 production in Pb by 2.5-Bev protons may dominate over that by direct collisions between these protons and internal nucleons.

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