Polarization of Electrons in Muon Decay with General Parity-Nonconserving Interactions*

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The polarization of electrons in muon decay is studied in the case of the four-component neutrino theory with general parity-nonconserving interactions. The spectrum for each of the two states of longitudinal polarization is described by a set of three parameters of which only two are independent. By observing the electron polarization in arbitrary directions, one can determine ten parameters of which six are related to the two longitudinal polarizations and the rest to the transverse part. It is seen that the prediction of the two-component neutrino theory about the electron polarization in muon decay is quite specific and can be distinguished from that of the general case, if the latter involves the interactions of the S, T, P types in charge-retention order. However, from the study of the polarization of electrons, it is impossible to distinguish the two-component theory from the four-component theory which involves only the vector-type interactions. It is noticed that the TCP theorem determines a detailed correspondence between the polarization dependence of the spectra for the μ^+ and μ^- decays. This provides us with an opportunity to determine experimentally whether weak interactions actually obey the TCP theorem or not.

1. INTRODUCTION

STRONG interest in the muon decay has been aroused recently by the discovery of parity nonconservation[1,2] in β decay and $\pi-\mu-e$ decay and by the renewed attention to the two-component neutrino theory[3] which gives us very definite and promising predictions on the processes involving neutrinos. Attempts have been made to determine the extent to which the validity of the two-component theory can be tested by the observation of the muon decay spectrum.[4,5] If one neglects the mass of the decay electron in comparison with its momentum, the decay spectrum of the muon with its spin completely polarized can be described by a three-parameter formula of the form

$$dN\propto x^2dxd\Omega[3(1-x)+2\rho(\tfrac{4}{3}x-1) \mp\xi\cos\theta\{1-x+2\delta(\tfrac{4}{3}x-1)\}], \quad (1.1)$$

where the upper and lower signs of the $\cos\theta$ term refer to the μ^- and μ^+ decays, respectively. ρ, δ, and ξ are parameters which depend on the ten coupling constants of the general parity-nonconserving interaction as follows:

$$\rho=\frac{3b+6c}{a+4b+6c},\quad \delta=\frac{3b'-6c'}{-3a'+4b'-14c'},\quad \xi=\frac{3a'-4b'+14c'}{a+4b+6c}, \quad (1.2)$$

where

$$\begin{aligned} a&=|g_S|^2+|g_S'|^2+|g_P|^2+|g_P'|^2,\\ b&=|g_V|^2+|g_V'|^2+|g_A|^2+|g_A'|^2,\\ c&=|g_T|^2+|g_T'|^2,\\ a'&=2\,\mathrm{Re}(g_S^*g_P'+g_P^*g_S'),\\ b'&=2\,\mathrm{Re}(g_V^*g_A'+g_A^*g_V'),\\ c'&=2\,\mathrm{Re}(g_T^*g_T'). \end{aligned} \quad (1.3)$$

The spectrum of the two-component theory is characterized by $\rho=\delta=\frac{3}{4}$ and $0\leqq|\xi|\leqq1$. Study of the formula (1.1) shows us that a spectrum exactly identical with that of the two-component theory can be obtained in infinitely many ways by using the general interaction of the four-component neutrino theory (including the S, T, P interactions), and thus the observation of the decay spectrum alone cannot lead to any definite information about the validity of the two-component theory.

According to the two-component theory, electrons in muon decay will be strongly polarized along the direction of their motion.[6–8] Furthermore, the shape of the

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[1] T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956).

[2] Wu, Ambler, Hayward, Hoppes, and Hudson, Phys. Rev. **105**, 1413 (1957); Garwin, Lederman, and Weinrich, Phys. Rev. **105**, 1415 (1957); J. I. Friedman and V. L. Telegdi, Phys. Rev. **105**, 1681 (1957); L. A. Page and M. Heinberg, Phys. Rev. **106**, 1220 (1957).

[3] T. D. Lee and C. N. Yang, Phys. Rev. **105**, 1671 (1957); A. Salam, Nuovo cimento **5**, 299 (1957); L. Landau, Nuclear Phys. **3**, 127 (1957).

[4] T. Kinoshita and A. Sirlin, Phys. Rev. **107**, 593 (1957). *Note added in proof.*—There is a mistake in the sign of c' of this paper; c' must be replaced by $-c'$ in the formulas (2.8), (2.10), and (2.30).

[5] C. Bouchiat and L. Michel, Phys. Rev. **106**, 170 (1957); T. Kotani (to be published); Larsen, Lubkin, and Tausner, Phys. Rev. **107**, 856 (1957).

[6] T. Kinoshita and A. Sirlin, Phys. Rev. **106**, 1110 (1957).

[7] T. D. Lee and C. N. Yang (private communication). T. D. Lee, lectures given at Brookhaven National Laboratory, January, 1957 (to be published) and lectures given at Harvard University, March, 1957 (unpublished). These lecture notes have been brought to our attention after this and related work (reference 6) were finished.

[8] T. Kotani (to be published); H. Überall, Nuovo cimento **6**, 376 (1957).

spectrum is completely fixed for both right-handed and left-handed electrons. Thus the observation of the electron polarization is expected to give us much more insight into the decay mechanism than the study of the decay spectrum itself. The purpose of this paper is to investigate the polarization of electrons in muon decay in detail under the assumption of the general parity-nonconserving interactions of the four-component neutrino theory. The longitudinal polarization of electrons in muon decay is discussed in Sec. 2. The electron polarization in arbitrary directions is treated in Sec. 3. The relation between the polarization dependence of the spectra of the μ^- and μ^+ decays is studied in Sec. 4. It is pointed out that such a relation is quite general and can be derived from the *TCP* theorem alone.

Recently the longitudinal polarization of electrons from β decay has been measured by using the Mott scattering of the electrostatically deflected β-ray beam,[9] the circular polarization of the bremsstrahlung produced by polarized electrons,[10] and the Møller scattering.[11] The results are in qualitative agreement with the predictions of the two-component theory for pure Gamow-Teller transitions but not always in other cases.[12] Experiments are now in progress at Columbia on the longitudinal polarization of electrons from muon decay using the bremsstrahlung method.[13] Therefore, some consequences of the theoretical considerations may be compared in the near future with the experimental results.

2. LONGITUDINALLY POLARIZED ELECTRONS FROM MUON DECAY

Let us consider the muon decay process

$$\mu^-\rightarrow e^-+\nu_1+\nu_2, \tag{2.1}$$

where no specification is made for the moment about the particle or antiparticle nature of ν_1 and ν_2. This process may be described by the Fermi-type Hamiltonian

$$H=\sum_i(\bar{\psi}_e\Gamma_i\psi_\mu)(\bar{\psi}_{\nu1}\Gamma_i(g_i+g_i'\gamma_5)\psi_{\nu2})+\text{H.c.}, \tag{2.2}$$

where H.c. means the Hermitian conjugate and the summation is taken over the five types of interaction.[14]

In this section we are interested in the probability that an electron is emitted with longitudinal polarization. For this purpose, it is convenient to rewrite (2.2) in the form

$$H=H_++H_-, \tag{2.3}$$

where

$$\begin{aligned}H_\pm=&\left(\bar{\psi}_e\frac{(1\pm\gamma_5)}{2}\Gamma_S\psi_\mu\right)[(g_S\mp g_P')(\bar{\psi}_{\nu1}\Gamma_S\psi_{\nu2})\\&+i(g_S'\mp g_P)(\bar{\psi}_{\nu1}\Gamma_P\psi_{\nu2})]+\left(\bar{\psi}_e\frac{(1\pm\gamma_5)}{2}\Gamma_V\psi_\mu\right)\\&\times[(g_V\pm g_A')(\bar{\psi}_{\nu1}\Gamma_V\psi_{\nu2})+i(g_V'\pm g_A)(\bar{\psi}_{\nu1}\Gamma_A\psi_{\nu2})]\\&+\left(\bar{\psi}_e\frac{(1\pm\gamma_5)}{2}\Gamma_T\psi_\mu\right)(g_T\pm g_T')(\bar{\psi}_{\nu1}\Gamma_T\psi_{\nu2})+\text{H.c.}\end{aligned} \tag{2.4}$$

Now, in the limit where the electron mass m is negligible compared with its energy E, $\bar{\psi}_e(1+\gamma_5)/2$ [or $\bar{\psi}_e(1-\gamma_5)/2$] represents a creation operator of an electron in a state in which its spin is parallel (or antiparallel) to the direction of its motion. H_+ and H_- are therefore parts of the Hamiltonian which create the right-handed and left-handed electrons, respectively. The spectrum for the electron emitted in a definite state of longitudinal polarization when the muon is at rest with its spin completely polarized is given by

$$\begin{aligned}dN_\pm=&\frac{\mu^5x^2dxd\Omega}{3\times2^9\pi^4}\{\tfrac{3}{2}a_\pm(1-x)(1\pm\cos\theta)+b_\pm[3-2x\\&\mp\cos\theta(1-2x)]+c_\pm[3-x\mp\cos\theta(1+x)]\},\end{aligned} \tag{2.5}$$

with

$$\begin{aligned}a_\pm&=|g_S\mp g_P'|^2+|g_S'\mp g_P|^2,\\b_\pm&=|g_V\pm g_A'|^2+|g_V'\pm g_A|^2,\\c_\pm&=|g_T\pm g_T'|^2,\end{aligned} \tag{2.6}$$

when the electron rest mass m is neglected compared with its energy E. Here, μ is the muon mass, x is the electron momentum measured in units of its maximum value $\mu/2$, and θ is the angle between the muon spin and the electron momentum. The upper (lower) sign of (2.5) corresponds to the case where the electron spin is parallel (antiparallel) to its direction of motion. Equation (2.5) does not contain the cross terms of (2.4) since an average is taken over the spin states of neutrinos.

If one observes the electrons irrespective of their polarization, one obtains the spectrum $dN=dN_++dN_-$ which is easily transformed into the three-parameter expression (1.1).[4]

For the detailed consideration of the longitudinal polarization of electrons in muon decay, it is again convenient to rewrite (2.5) in terms of the Michel-like parameters. In fact, one can express (2.5) in exactly

[9] Frauenfelder, Bobone, von Goeler, Levine, Lewis, Peacock, Rossi, and DePasquali, Phys. Rev. **106**, 386 (1957).

[10] Goldhaber, Grodzins, and Sunyar, Phys. Rev. **106**, 826 (1957).

[11] Frauenfelder, Hanson, Levine, Rossi, and DePasquali, Phys. Rev. **107**, 643 (1957).

[12] Ambler, Hayward, Hoppes, Hudson, and Wu, Phys. Rev. **106**, 1361 (1957); F. Boehm and A. H. Wapstra, Phys. Rev. **106**, 1364 (1957); Frauenfelder, Bobone, von Goeler, Levine, Lewis, Peacock, Rossi, and DePasquali, Phys. Rev. **107**, 909 (1957); Frauenfelder, Hanson, Levine, Rossi, and DePasquali, Phys. Rev. **107**, 910 (1957).

[13] L. M. Lederman, *Proceedings of the Seventh Annual Rochester Conference on High-Energy Nuclear Physics, April, 1957* (Interscience Publishers, Inc., New York, 1957).

[14] We define the Γ_i as follows: $\Gamma_S=1$, $\Gamma_V=\gamma_\mu$, $\Gamma_T=(i/2\sqrt{2})(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)$, $\Gamma_A=-i\gamma_\mu\gamma_5$, $\Gamma_P=-i\gamma_5=\gamma_1\gamma_2\gamma_3\gamma_0$. The γ_μ's are defined by $\gamma_\mu\gamma_\nu+\gamma_\nu\gamma_\mu=2\delta_{\mu\nu}$, where $\delta_{00}=-\delta_{11}=-\delta_{22}=-\delta_{33}=1$, $\delta_{\mu\nu}=0$ for $\mu\neq\nu$.

the same form as (1.1) in the following manner:

$$dN_\pm = A_\pm x^2 dx d\Omega [3(1-x)+2\rho_\pm(\tfrac{4}{3}x-1) - \xi_\pm \cos\theta\{1-x+2\delta_\pm(\tfrac{4}{3}x-1)\}], \quad (2.7)$$

where

$$A_\pm = \frac{\mu^5}{3\times 2^{10}\pi^4}(a_\pm+4b_\pm+6c_\pm),$$
$$\rho_\pm = \frac{3b_\pm+6c_\pm}{a_\pm+4b_\pm+6c_\pm},$$
$$\delta_\pm = \frac{3b_\pm-6c_\pm}{3a_\pm+4b_\pm-14c_\pm}, \quad (2.8)$$
$$\xi_\pm = \pm\left(\frac{-3a_\pm-4b_\pm+14c_\pm}{a_\pm+4b_\pm+6c_\pm}\right).$$

The parameters $\rho_\pm$, $\delta_\pm$, and $\xi_\pm$ are subject to a set of relations which derive from the inequalities $a_\pm$, $b_\pm$, $c_\pm \geqq 0$. Some of the useful relations and their consequences are considered in the following.

(a) When a neutrino and an antineutrino are emitted in (2.1), one obtains the relations

$$0 \leqq \rho_\pm \leqq 1, \quad (2.9)$$

$$|\xi_\pm \delta_\pm| \leqq \rho_\pm, \quad (2.10)$$

$$3(3-4\rho_\pm) \pm \xi_\pm(3-4\delta_\pm) = 0. \quad (2.11)$$

Using (2.11), one can eliminate the parameters $\delta_\pm$ in (2.7). Thus, from the experimental distribution of energy and angle of electrons, one can determine two independent parameters for each of the longitudinal polarizations (say $\rho_\pm$, $\xi_\pm$). The ratio

$$R = A_+/A_- \quad (2.12)$$

of the total probabilities of finding right-handed and left-handed electrons and the lifetime of the muon give additional information. Thus, in principle, one can determine the six constants $a_\pm$, $b_\pm$, and $c_\pm$. This is the most one can expect from the observation of the longitudinal polarization of decay electrons.

From (2.10) and (2.11) one finds

$$-3+(8/3)\rho_\pm \leqq \pm\xi_\pm \leqq -3+(16/3)\rho_\pm, \quad (2.13)$$

for nonidentical neutrinos; this relation may be useful for testing consistency.

(b) If the two neutrinos are identical in the muon decay, one obtains

$$g_V = g_T = g_T' = g_A' = 0. \quad (2.14)$$

Thus the relations (2.9)–(2.11) are replaced by the stronger conditions

$$0 \leqq \rho_\pm \leqq \tfrac{3}{4}, \quad (2.15)$$

$$\xi_\pm \delta_\pm = \mp \rho_\pm, \quad (2.16)$$

$$\mp \xi_\pm = 3-(8/3)\rho_\pm. \quad (2.17)$$

In this case, the spectrum for the right-handed or left-handed electron is described by only one independent parameter (ρ_+ or ρ_-).

(c) If there is no interaction other than the vector type interactions,[15] one obtains a definite prediction

$$\rho_+ = \rho_- = \tfrac{3}{4}, \quad \delta_+ = \delta_- = \tfrac{3}{4}, \quad \xi_+ = -1, \quad \xi_- = 1, \quad (2.18)$$

because $a_\pm = c_\pm = 0$. The electron spectrum is therefore given by

$$dN_\pm = \tfrac{1}{2}A_\pm x^2 dx d\Omega [3-2x \mp \cos\theta(1-2x)], \quad (2.19)$$

whose shape is completely fixed. Only the ratio R of (2.12) between the right and left polarizations remains to be determined.

Conversely, if one assumes that the observed values of the parameters $\rho_\pm$, $\delta_\pm$, and $\xi_\pm$ are given by (2.18), one can see easily that the most general interaction must satisfy the condition

$$a_\pm = c_\pm = 0. \quad (2.20)$$

In other words, if the observed parameters satisfy (2.18), one can say without ambiguity that, in charge retention order, only the vector-type interactions are involved in muon decay. It is, however, impossible to tell whether the two neutrinos are identical or not since Eqs. (2.15)–(2.17) are compatible with (2.18).

(d) The muon decay interaction in the two-component neutrino theory can be regarded as a special case of the interactions discussed above, where we have an additional restriction

$$g_V' = -g_V, \quad g_A' = -g_A, \quad (2.21)$$

when the two neutrinos emitted are different. Thus, if the condition (2.18) is not satisfied, the two-component theory with nonidentical neutrinos is ruled out. However, when (2.18) is satisfied, the experiments on polarized electrons cannot produce definite information on whether (2.21) is also valid or not.

(e) At the high-energy end ($x=1$), where the two neutrinos are emitted in the same direction, one obtains

$$dN_\pm(x=1) \propto b_\pm(1\pm\cos\theta) + 2c_\pm(1\mp\cos\theta) \quad (2.22)$$

from (2.5). This angular dependence can be understood easily if one remembers that the muon spin has components $\cos(\theta/2)$ and $\sin(\theta/2)e^{i\phi}$ along and opposite to the direction of the electron momentum, respectively. It must also be taken into account that the two neutrinos are emitted with opposite spins in the V- and A-type interactions and parallel spins in the S, P, and T cases.[16] If the two neutrinos are identical [case (b)] or if there are only vector-type interactions [cases (c)

[15] In this paper, we shall use the phrase "vector-type interactions" to represent the four interaction terms with the coupling constants g_V, g_V', g_A, and g_A'. "Scalar type" and "tensor type" are also used in the same manner.

[16] This follows from the following property of the matrix Γ_i: $(1\pm\gamma_5)\Gamma_i(g_i+g_i'\gamma_5)(1\pm\gamma_5)=0$ for $i=V, A$; $(1\pm\gamma_5)\Gamma_i(g_i+g_i'\gamma_5)(1\mp\gamma_5)=0$ for $i=S, T, P$.

and (d)], the electrons emitted with maximum energy ($x=1$) in the direction parallel or antiparallel to the muon spin ($\theta=0$ or $\theta=\pi$) must be fully polarized in the direction of the muon spin according to (2.22). The converse occurs in the tensor case. In this connection, it is interesting to note that ξ_+ must be negative and ξ_- positive for both the two-component theory and the hypothesis of identical neutrinos as is seen from (2.17) and (2.18).

(f) If A_- (or A_+) vanishes in the general case, it is obvious that the decay electron is completely polarized in the direction of its motion (or opposite direction) for all values of energy and angle θ (as long as $E\gg m$). Even if the electron polarization is complete, however, the value of ξ is quite arbitrary ($-3\leqq\xi\leqq 7/3$ in the case $A_-=0$). Conversely, it is impossible to obtain information on the degree of electron polarization from the asymmetry term of the spectrum (1.1). In fact, if the experiments show that $\rho=\delta=\frac{3}{4}$, which would be the case if the two-component theory is correct, and further $\xi=-1$, the ratio R of right-handed and left-handed electrons still depends on a and b as

$$R=b/a, \qquad (2.23)$$

which can take any positive value in the framework of the general four-component theory.

If there are only vector-type interactions, on the other hand, the quantity R is related uniquely to ξ by

$$R=(1-\xi)/(1+\xi), \qquad (2.24)$$

where

$$\xi=\frac{-b'}{b}=\frac{-2\,\mathrm{Re}(g_V{}^*g_A{}'+g_A{}^*g_V{}')}{|g_V|^2+|g_V{}'|^2+|g_A|^2+|g_A{}'|^2}. \qquad (2.25)$$

Thus, it is possible to predict the degree of longitudinal polarization of electrons from knowledge about the value of ξ. This was already discussed in the case of the two-component neutrino theory.[6]

As we have seen in the analysis of this section, the observation of the longitudinal polarization of electrons will not lead to unambiguous proof of any particular type of interaction. Nonetheless, experimental verification of conditions like (2.18) would be a strong argument in favor of the two-component theory, because this seems to be the only theory that can explain in a simple and natural way the absence of the S, T, and P interactions of charge-retention order in the decay of the muon.

3. POLARIZATION OF ELECTRONS IN ARBITRARY DIRECTIONS

We shall now discuss the case where the electron is emitted in some arbitrary state of polarization in the muon decay. Neglecting the electron mass compared with its momentum, the probability of finding an electron whose spin points up along certain direction **s** in the decay of polarized μ^- at rest is calculated from (2.2) as follows[17]:

$$\begin{aligned}dN_s=\frac{\mu^5x^2dxd\Omega}{3\times2^9\pi^4}\{&[\tfrac{3}{2}a_+(1+\cos\theta)(1-x)+b_+(3-2x\\&-\cos\theta(1-2x))+c_+(3-x-\cos\theta(1+x))]\cos^2(\theta_s/2)\\&+[\tfrac{3}{2}a_-(1-\cos\theta)(1-x)+b_-(3-2x+\cos\theta(1-2x))\\&+c_-(3-x+\cos\theta(1+x))\sin^2(\theta_s/2)\\&+[\tfrac{3}{2}\alpha(1-x)+\beta]\sin\theta\sin\theta_s\cos\varphi_s\\&+[\tfrac{3}{2}\alpha'(1-x)+\beta']\sin\theta\sin\theta_s\sin\varphi_s\},\end{aligned} \qquad (3.1)$$

where x and θ are defined in the last section, θ_s is the polar angle of **s** with respect to the electron momentum **p**, and φ_s is the azimuthal angle of **s** with respect to the plane defined by **p** and the direction of the muon spin. (In defining θ_s and φ_s, **p** is regarded as the polar axis.) The quantities α, β, α', and β' are defined by

$$\begin{aligned}\alpha&=|g_S|^2+|g_S{}'|^2-|g_P|^2-|g_P{}'|^2,\\\beta&=|g_V|^2+|g_V{}'|^2-|g_A|^2-|g_A{}'|^2,\\\alpha'&=2\,\mathrm{Im}(g_S{}^*g'_P-g_P{}^*g_S{}'),\\\beta'&=2\,\mathrm{Im}(g_A{}^*g_V{}'-g_V{}^*g_A{}').\end{aligned} \qquad (3.2)$$

Obviously the best one can expect from the most detailed study of the muon decay spectrum is the determination of ten coefficients of (3.1), insofar as the neutrinos are not observed. This is not enough to determine the ten *complex* coupling constants of the Fermi-type interaction (2.2) from experiments.

In (3.1), the first two terms are proportional to dN_+ and dN_- of (2.5), respectively. Thus these terms do not give any information more than that of the longitudinal polarization. The factors $\cos^2(\theta_s/2)$ and $\sin^2(\theta_s/2)$ can be interpreted in the same manner as in case (e) of Sec. 2.

The last two terms of (3.1) contain the only new information derivable from the arbitrary polarization (the transverse polarization in particular). Experimental detection of the last term of (3.1) is of particular interest since its existence means the violation of the time reversibility of the muon decay process. Unfortunately, the Mott scattering cannot be used to observe the transverse polarization of such high-energy electrons. Development of methods of detecting the transverse polarization of high-energy particles would be desirable.†

[17] The vector **s** gives the direction of the electron spin in its rest system.

† *Note added in proof.*—There is a slight possibility that the Møller scattering can be used to detect the transverse polarization of high-energy electrons. In the relativistic limit, the scattering cross section of an incident electron polarized in the direction θ, φ (in its rest system) by an electron at rest (polarized in the direction θ_0, φ_0) is proportional to $1-7/9\cos\theta_0\cos\theta-1/9\sin\theta_0\sin\theta\cos(\varphi+\varphi_0)$, where θ and θ_0 are the polar angles of spins with respect to the momentum of the incident electron and φ and φ_0 are the azimuthal angles with respect to the scattering

The electron polarization in the direction **s**,

$$P=\frac{dN(\theta_s,\varphi_s)-dN(\pi-\theta_s,\pi+\varphi_s)}{dN(\theta_s,\varphi_s)+dN(\pi-\theta_s,\pi+\varphi_s)},\tag{3.3}$$

has a maximum for

$$\tan\varphi_s=D/C,\quad \tan\theta_s=\pm\frac{\sin\theta(C^2+D^2)^{\frac{1}{2}}}{A+B\cos\theta},\tag{3.4}$$

which determines the "direction" of the electron spin. The corresponding maximum value is given by

$$|P_{\max}|=\frac{[(A+B\cos\theta)^2+(C^2+D^2)\sin^2\theta]^{\frac{1}{2}}}{E+F\cos\theta},\tag{3.5}$$

where

$$\begin{aligned}
A&=-3a'(1-x)+2b'(3-2x)+2c'(3-x),\\
B&=3a(1-x)-2b(1-2x)-2c(1+x),\\
C&=3\alpha(1-x)+2\beta,\\
D&=3\alpha'(1-x)+2\beta',\\
E&=3a(1-x)+2b(3-2x)+2c(3-x),\\
F&=-3a'(1-x)-2b'(1-2x)-2c'(1+x).
\end{aligned}\tag{3.6}$$

The maximum polarization of the electron reaches 100% under various circumstances. For instance, $|P_{\max}|$ will be 1 for $x=1$ and any θ if (i) either $b_+=c_+=\beta=\beta'=0$ or $b_-=c_-=\beta=\beta'=0$, or (ii) $c=0$ and $g_Vg_A=g_V'g_A'$. In both cases, a_+ and a_- can be chosen arbitrarily. As is seen immediately from (ii), $P_{\max}$ can be 100% for $x=1$ and any θ in the two-component theory in which $g_V=-g_V'$, $g_A=-g_A'$ and $a=c=0$ hold. This has been pointed out by Überall in the particular case of real coupling constants.[8] It must be stressed however that $|P_{\max}|=1$ can also occur in other cases and thus is not useful to distinguish the two-component theory.

It is even possible to obtain $|P_{\max}|=1$ for any x and θ. This occurs, for instance, when (iii) $b=c=0$ and $g_Sg_P=g_S'g_P'$, or (iv) $a_+=b_+=c_+=0$ or $a_-=b_-=c_-=0$. It may be noted that case (iv) is essentially the same as that discussed under Sec. 2 (f).

4. *TCP* THEOREM

We have so far restricted ourselves to the case of the μ^--e^- decay. In order to describe the μ^+-e^+ decay, it is convenient to rewrite the Hamiltonian (2.2) in terms of the charge conjugate field operators

$$\psi_\mu{}^C=C\bar{\psi}_\mu{}^T,\quad \psi_e{}^C=C\bar{\psi}_e{}^T,\tag{4.1}$$

where the superscript T means transposition and the unitary matrix C has the property

$$C^{-1}\gamma_\mu C=-\gamma_\mu{}^T,\quad C^T=-C,\quad C^{-1}=C^\dagger=-C^*.\tag{4.2}$$

As is seen easily, this procedure is equivalent to the simultaneous substitution

$$\begin{aligned}
&\psi_{\mu(e)}{}^C\to\psi_{\mu(e)},\\
&g_S{}^*\to g_S,\quad -g_S{}'^*\to g_S',\\
&g_P{}^*\to g_P,\quad -g_P{}'^*\to g_P',\\
&-g_V{}^*\to g_V,\quad -g_V{}'^*\to g_V',\\
&g_A{}^*\to g_A,\quad g_A{}'^*\to g_A',\\
&-g_T{}^*\to g_T,\quad g_T{}'^*\to g_T',
\end{aligned}\tag{4.3}$$

applied to the Hamiltonian (2.2). From this, it is easy to find the relation between the spectrum $d\bar{N}$ for the μ^+-e^+ decay and dN of (2.5) or (3.1) for the μ^--e^- decay. One thus obtains the equations

$$dN_\pm(\theta)=d\bar{N}_\pm(\pi-\theta),\tag{4.4}$$

for the longitudinal polarization and

$$dN_s(\theta,\theta_s,\varphi_s)=d\bar{N}_s(\pi-\theta,\pi-\theta_s,\varphi_s),\tag{4.5}$$

for the polarization in the general direction **s**, respectively.

It is interesting to notice that the relation (4.4) or (4.5) between the spectra of the μ^- and μ^+ decays is a quite general one and holds only if the interaction Hamiltonian is invariant with respect to the product of space reflection (P), time reversal (T), and particle-antiparticle conjugation (C). Namely, it is just one of the consequences of the TCP theorem.[18,19] In order to see this clearly, it is convenient to write the general μ^--decay spectrum in the following form:

$$\begin{aligned}dN\sim C_0&+C_1(\mathbf{p}_e,\boldsymbol{\sigma}_e)(\mathbf{p}_e,\boldsymbol{\sigma}_\mu)+C_2(\mathbf{p}_e\times\boldsymbol{\sigma}_e,\,\mathbf{p}_e\times\boldsymbol{\sigma}_\mu)\\&+C_3(\mathbf{p}_e,\,\boldsymbol{\sigma}_e\times\boldsymbol{\sigma}_\mu)+C_4(\mathbf{p}_e,\boldsymbol{\sigma}_e)+C_5(\mathbf{p}_e,\boldsymbol{\sigma}_\mu),\end{aligned}\tag{4.6}$$

which is the most general form insofar as the neutrinos are not observed.[20] The coefficients C_i are real functions of $(\mathbf{p}_e,\mathbf{p}_e)$ and the coupling constants. In the particular

plane. The scattering of the longitudinally polarized electron is described by the first two terms which agrees with the recent calculation of A. M. Bincer (to be published). The detection of transverse polarization by this method is not easy because of the small coefficient of the last term. For lower energies, however, this term is not small and may be used as an alternative to the Mott scattering.

[18] J. Schwinger, Phys. Rev. **82**, 914 (1951); **91**, 713 (1953); G. Lüders, Kgl. Danske Viedenskab. Selskab, Mat.-fys. Medd. **28**, No. 5 (1954); W. Pauli, in *Niels Bohr and the Development of Physics*, edited by W. Pauli (Pergamon Press, London, 1955), p. 30.

[19] The mass and lifetime equality of particle and antiparticle follows from the TCP theorem [see Lee, Oehme, and Yang, Phys. Rev. **106**, 340 (1957); G. Lüders and B. Zumino, Phys. Rev. **106**, 385 (1957)]. The angular distribution of e^+ in $\pi^+-\mu^+-e^+$ decay is exactly the same as that of e^- in $\pi^--\mu^--e^-$ decay if the decay interaction is TCP-invariant. (See Lee and Yang, reference 3). Formulas (4.4) and (4.5) are simple generalizations of this result to the electron polarization in muon decay.

[20] Formulas (4.6) and (4.8) are given as the density matrices for spin states of muon and electron. The polarization spectrum in some particular directions can be obtained by taking the expectation value with respect to the corresponding state vector.

case of the Hamiltonian (2.2), the C_i are given by

$$\begin{aligned} C_0&=\tfrac{3}{2}a(1-x)+b(3-2x)+c(3-x),\\ C_1&=p_e^{-2}[\tfrac{3}{2}a(1-x)-b(1-2x)-c(1+x)],\\ C_2&=p_e^{-2}[\tfrac{3}{2}\alpha(1-x)+\beta],\\ C_3&=-p_e^{-1}[\tfrac{3}{2}\alpha'(1-x)+\beta'],\\ C_4&=p_e^{-1}[-\tfrac{3}{2}a'(1-x)+b'(3-2x)+c'(3-x)],\\ C_5&=p_e^{-1}[-\tfrac{3}{2}a'(1-x)-b'(1-2x)-c'(1+x)]. \end{aligned} \tag{4.7}$$

The first three terms of (4.6) are invariant under both P and T, the term $(\mathbf{p}_e, \boldsymbol{\sigma}_e\times\boldsymbol{\sigma}_\mu)$ changes sign under either P or T, whereas the last two terms change sign under P but not T. Since the operation C simply interchanges particles of the process considered with corresponding antiparticles, the spectrum for the μ^+ decay must be of the form

$$d\bar{N}\sim C_0+C_1(\mathbf{p}_e,\boldsymbol{\sigma}_e)(\mathbf{p}_e,\boldsymbol{\sigma}_\mu)+C_2(\mathbf{p}_e\times\boldsymbol{\sigma}_e,\ \mathbf{p}_e\times\boldsymbol{\sigma}_\mu) +C_3(\mathbf{p}_e,\ \boldsymbol{\sigma}_e\times\boldsymbol{\sigma}_\mu)-C_4(\mathbf{p}_e,\boldsymbol{\sigma}_e)-C_5(\mathbf{p}_e,\boldsymbol{\sigma}_\mu), \tag{4.8}$$

if the interaction Hamiltonian is invariant under TCP. Obviously (4.8) can be obtained from (4.6) by the substitution $-\boldsymbol{\sigma}_e\rightarrow\boldsymbol{\sigma}_e$ and $-\boldsymbol{\sigma}_\mu\rightarrow\boldsymbol{\sigma}_\mu$ which is just what the relations (4.4) and (4.5) imply. It is not difficult to prove these relations rigorously on the basis of transformation properties of field operators under T, C, and P.

No assumption is made in deriving (4.8) from (4.6) except for the invariance of the interaction Hamiltonian under TCP. Thus Eqs. (4.4) and (4.5) may be checked experimentally to see whether weak interactions actually satisfy the TCP theorem or not. In the most general situation, (4.5) means that (a) the polarization of e^- of momentum $\mathbf{p}$ emitted by μ^- polarized in the $+z$ direction must be equal and opposite to that of e^+ of the same momentum $\mathbf{p}$ emitted by μ^+ polarized in the $-z$ direction.

Taking account of the fact that the polarization of μ^- in the π^- decay must be equal and opposite to that of μ^+ in the π^+ decay,[21] one may restate (a) as (b) if the production and subsequent depolarization occur in the same manner for both μ^- and μ^+, e^- and e^+ emitted in the same solid angle relative to the *muon momentum* must be polarized in opposite directions.

Of particular interest experimentally will be the case in which the muons are completely depolarized before they decay. (c) In this case, formulas (4.6) and (4.8) reduce to $C_0+C_4(\mathbf{p}_e,\boldsymbol{\sigma}_e)$ and $C_0-C_4(\mathbf{p}_e,\boldsymbol{\sigma}_e)$, respectively. Thus, insofar as parity is not conserved, e^- and e^+ must be oppositely polarized even if the muons are not polarized.

It is emphasized that the experimental verification of these statements will not itself help to establish the validity of any particular theory. On the other hand, if the experiments disagree with any of them, that would definitely mean that the decay interaction in question cannot satisfy some of the assumptions on which the TCP theorem is established.[22]

5. DISCUSSION

One of the aims of our present investigation of the muon decay is to examine the significance of the two-component neutrino theory in the more general scheme of the four-component theory with parity-nonconserving interactions. In this way, we have seen to what extent the predictions of the two-component theory can be distinguished from others.

It is easy to see that arguments of this paper are valid even if the muon decay proceeds by the emission of both $\nu+\bar{\nu}$ and $\nu+\nu$.[23] (In this case the parameters $a_\pm$, α, etc., in formulas (2.5) and (3.1) must of course be replaced by the sums of parameters describing both channels of the decay.) Thus, if experiments could not be fitted by the formula (2.5) in the case of the longitudinal polarization and the formula (3.1) in the general case, the Fermi-type local Hamiltonian without derivative couplings would have to be ruled out in the muon decay.

It is interesting that the TCP theorem predicts such detailed correspondence between decays of particles and antiparticles as was discussed in Sec. 4. In fact, it is so detailed that it provides us with an excellent opportunity to test the validity of the TCP theorem.[24] It will of course depend on the particular interaction how the momentum and spin of μ^- should be correlated in the π^- decay and whether the electron spectrum should be peaked backwards or not with respect to the muon momentum in the decay of the polarized muon. In the framework of the general four-component neutrino theory, however, it is impossible to explain why some particular situation is preferred to others.

An interesting feature of the two-component theory is that it enables one to predict unambiguously the correlation of spin and momentum of the muon if one assumes in addition lepton conservation in the $\pi-\mu-e$ and β decays.[7] According to this scheme, the spin of μ^+, an antiparticle, emitted in the $\pi^+-\mu^+$ decay must be polarized in the direction of its momentum while

[21] This follows easily from the TCP theorem. See Lee and Yang, reference 3.

[22] Most assumptions of the TCP theorem are very fundamental and hard to dispute. But one assumption, the anticommutativity of kinematically independent fermion fields, may not be essential (see reference 18). Insofar as the interactions of elementary particles discovered to date are concerned, this assumption of anticommutativity is not necessary for the validity of the TCP theorem since it can be replaced by a more general assumption about the consistency of the Lagrangian formalism of relativistic field theories [see T. Kinoshita, Phys. Rev. **96**, 199 (1954)]. On the other hand, it is easy to construct a model of the interaction that violates the TCP invariance if one assumes the consistency of the Lagrangian formalism instead of the anticommutativity of different fermion fields. Thus it is not known yet what are the necessary and sufficient conditions for the TCP theorem to hold.

[23] Such a mixture has recently been suggested in the case of the two-component neutrino theory by M. H. Friedman, Phys. Rev. **106**, 387 (1957).

[24] It is easy to find relations analogous to (4.5) for other processes like $K_{\mu2}^\pm$, $K_{\mu3}^\pm$, and $K_{e3}^\pm$ decays.

the spin of μ^-, a particle, must point in the direction opposite to its momentum. This seems to be consistent with the present observation. It must be noticed, however, that the $\mu-e$ decay itself is insensitive to which of μ^+ and μ^- is the particle or antiparticle.

Another advantage of the two-component theory is that it enables us to predict the degree of longitudinal polarization of decay electrons if only the $\cos\theta$ dependence of the spectrum (2.19) is known. This was mentioned in reference 6 and also in the paragraph (f) of Sec. 2. It is thus inferred from experimental data that, averaged over the angle θ, at least 88% of the electrons (or positrons) from the muon decay are polarized parallel (antiparallel) to their motion if the two-component theory is valid. The polarization of the muon spin at the moment of decay can also be measured by looking at the electron polarization.[6]

In our discussion, we have neglected completely the effect of the electron mass on the decay spectrum. Comparison of (2.5) with the exact formula[25] shows that the formula (2.5) is a quite good approximation except at the low-energy end of the spectrum. The most important modification to our discussion of the muon decay comes from the effect of the radiative correction. Some parts of this effect have been studied in previous works.[4] But the radiative correction to the electron polarization in muon decay has not yet been explored.

[25] T. Kinoshita and A. Sirlin (unpublished).

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π^--Proton Interactions at 5 Bev*†

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The interaction of 5-Bev negative pions with protons has been studied by exposing a 36-atmosphere hydrogen-filled diffusion cloud chamber to π^- beams from the Berkeley Bevatron. One hundred and thirty-seven interactions producing charged outgoing particles were observed. Of these, 27 were elastic scattering events, 64 were inelastic collisions having two charged outgoing prongs, 39 had four prongs, 3 had six prongs, and 4 involved the production and visible decay of heavy unstable particles. The total cross section is estimated to be 22.5 ± 2.4 mb. The elastic scattering cross section is 4.7 ± 1.0 mb. The angular distribution of the elastic events is consistent with that expected for diffraction scattering from a sphere with radius $(0.9\pm0.15)\times10^{-13}$ cm and opacity 0.6. Analysis of the inelastic events shows that multiple, rather than single, pion production is the predominant process occurring at this energy. An average of 2.3 secondary pions were produced in the inelastic events. This average multiplicity can be fitted by the Fermi statistical theory only by increasing the interaction radius occurring in the theory by 20%. The statistical theory, however, fails to account for the rather marked asymmetry found in the c.m. angular distributions of some of the particles emitted in inelastic events. A combination of the four observed strange-particle production events with 11 similar events obtained in exposures to high-energy neutron and proton beams shows that pion emission accompanies strange-particle production in at least 60% of elementary-particle collisions at Bevatron energies.

I. INTRODUCTION

THIS paper reports a study of π^--p interactions at 5 Bev. Some preliminary reports on this experiment[1] and on the related cloud chamber experiments on n-p and p-p collisions[2,3] at Bevatron energies have already been published.

Investigations of pion-nucleon interactions have tended to fall into several categories according to the range of pion energy. The many experiments at energies up to about 250 Mev have yielded detailed information on elastic scattering and could be analyzed in terms of phase shifts. At higher energies inelastic processes become important and the number of possible angular-momentum states increases. Consequently a phase-shift analysis is no longer feasible. The energy dependence of the total cross section has been measured,[4] and the elastic and inelastic processes in the 1- to 1.5-Bev range have been studied by use of nuclear emulsions[5] and hydrogen-filled diffusion cloud chambers.[6]

Upon successful operation of the Berkeley Bevatron,

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[1] Maenchen, Powell, Saphir, and Wright, Phys. Rev. **99**, 1619 (1955); Maenchen, Fowler, Powell, Saphir, and Wright, Phys. Rev. **100**, 1802 (1955); Fowler, Maenchen, Powell, Saphir, and Wright, Phys. Rev. **103**, 208 (1956); Maenchen, Fowler, Powell, and Wright, Bull. Am. Phys. Soc. Ser. II, **1**, 386 (1956).

[2] Fowler, Maenchen. Powell Saphir, and Wright, Phys. Rev. **101**, 911 (1956); Holmquist, Fowler, and Powell, Bull. Am. Phys. Soc. Ser. II, **1**, 392 (1956).

[3] Wright, Saphir, Powell, Maenchen, and Fowler, Phys. Rev. **100**, 1802 (1955); Wright, Powell, Maenchen, and Fowler, Bull. Am. Phys. Soc. Ser. II, **1**, 386 (1956).

[4] Cool, Piccioni, and Clark, Phys. Rev. **103**, 1082 (1956).

[5] W. D. Walker and J. Crussard, Phys. Rev. **98**, 1416 (1955); Walker, Hushfar, and Shephard, Phys. Rev. **104**, 526 (1956).

[6] Eisberg, Fowler, Lea, Shephard, Shutt, Thorndike, and Whittemore, Phys. Rev. **97**, 797 (1955).