$(d\bar{\nu}/dE_n \cong 1/E_0)$ is also reasonably consistent with this value of E_0.

ACKNOWLEDGMENTS

The author is indebted to a number of persons for the privilege of using and quoting their data before publication and wishes to thank Dr. B. C. Diven, Dr. R. B. Leachman, Mr. W. E. Stein, Dr. R. F. Taschek, and Dr. L. Wilets for many valuable discussions, and Mr. Paul E. Harper for many calculations in which an IBM 704 computer was used.

PHYSICAL REVIEW VOLUME 108, NUMBER 3 NOVEMBER 1, 1957

Stripping-Type Nuclear Reactions*†

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(Received June 27, 1957)

An exact formal expression is derived which treats the initial and final states of a nuclear reaction in a symmetric manner. The specific example treated is the (d,n) reaction. The Coulomb interaction is not taken into account. Estimates of some of the terms of the general expression which is obtained provide some indication of the reasons for the similarity of the final results of the various approaches to direct interaction. One of the terms of the development includes effects of compound-nucleus formation. Suggestions are made for a single-particle model of the intermediate state. The process of exchange and heavy-particle stripping is incorporated into the formalism in the Appendix.

I. INTRODUCTION

THE purpose of the present work is to derive an exact, though formal, expression for the cross section of a nuclear reaction of the stripping type. An attempt has been made to strive for conciseness and simplicity in the derivations. Rather than make specific and detailed predictions, the principles behind the argument are stressed and briefly outlined. With this aim in mind, the discussion has been restricted to the (d,n) reaction, though it would apply equally well to the (d,p), (He^3,p), etc., cases. Furthermore, the Coulomb interaction[1] has not been taken into account. Inclusion of this effect does not present any serious difficulties, but the results become somewhat cumbersome to write down. For similar reasons, no detailed treatment of spins or angular momenta has been undertaken either. Appendix A presents a brief discussion of the formal methods by which exchange and heavy-particle stripping[2] can be taken into account.

The point of view in the calculations has been to treat the initial and final states involved in the nuclear reaction on an equal footing. The results obtained in the present paper can be derived by a series of purely formal manipulations from previously known expressions.[3] The present point of view enables one, however, to suggest a reason for the success of the various approximate treatments of deuteron stripping[4] in fitting experimental data. Finally, our results lead naturally to suggestions for ways of including effects of compound-nucleus formation on the cross section.

II. GENERAL EXPRESSION FOR THE (d,n) CROSS SECTION

The notation of the present section follows in its general features that of Lippmann and Schwinger[5] and Chew and Goldberger.[6] We consider only the outgoing-wave solutions $\Psi^{(+)}$, but suppress the superscript $(+)$. We also assume that when the energy E appears in a Green's function, it contains a small positive imaginary part $(E \to E+i\epsilon,\ \epsilon>0)$. Such a choice[5] is equivalent to a boundary condition which selects only outgoing waves at infinity.

From here on, we shall always refer explicitly to the (d,n) reaction. The complete Hamiltonian for this reaction can be written in the form:

$$H=H_0{}^i+H_1{}^i=H_0{}^f+H_1{}^f, \tag{1}$$

where H_0 is the zeroth-order Hamiltonian, and H_1 represents a "perturbation." The superscripts i and f refer to the initial and final state configurations. The operators depend on all the variables of the system, which consists of the target nucleus and the neutron and proton in the deuteron. We have not written down

* Supported in part by the U. S. Air Force, Office of Scientific Research.
† Supported by the U. S. Atomic Energy Commission.
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the explicit dependence of H on these variables, but this dependence is specifically implied.

The separation of H into H_0 and H_1 is different for the initial and the final state. From the point of view of the initial state, we have

$$H_0{}^i=H_T+T_N+T_P+V_{NP}, \tag{2a}$$

and

$$H_1{}^i=V_{NT}+V_{PT}. \tag{2b}$$

In the above expressions, H_T represents the Hamiltonian of the target nucleus, T_N and T_P the kinetic-energy operators of the neutron and proton, and V_{NP}, V_{NT}, V_{PT} the neutron-proton, neutron-target, and proton-target interactions respectively.

From the point of view of the final state, H_0 and H_1 can be defined as

$$H_0{}^f=H_T+T_N+T_P+V_{PT}, \tag{3a}$$

and

$$H_1{}^f=V_{NP}+V_{NT}. \tag{3b}$$

We next define the Green's function and wave function corresponding to $H_0{}^i$ by

$$(E-H_0{}^i)G_i(E)=1, \tag{4}$$

and

$$(E-H_0{}^i)\Psi_i=0. \tag{5}$$

Similar equations are used to define G_f and Ψ_f. The functions Ψ_i are a complete set of eigenfunctions (E can be taken to go over all possible eigenvalues) for the problem involving a neutron-proton system and a target nucleus which do not interact with each other. Analogously, Ψ_f describes all possible states of the system involving a target nucleus plus a proton which have no interaction whatever with a free neutron. We can rewrite Eq. (4) in the form

$$(E-H_0{}^f+V_{PT}-V_{NP})G_i=1. \tag{6}$$

Equation (6) leads immediately to the identity,

$$G_i=G_f+G_f(V_{NP}-V_{PT})G_i. \tag{7}$$

The equations satisfied by Ψ, and the corresponding G, both associated with the complete Hamiltonian H, are

$$(E-H)\Psi=0, \quad (E-H)G=1. \tag{8}$$

We consider now a specific value of the energy, and call it $\mathcal{E}$. $\mathcal{E}$ is the sum of the kinetic energy of the incident deuteron, the deuteron binding energy $-\epsilon_D$, and the target energy $-\epsilon_T$.

The solution for Ψ in which the incident deuteron wave is explicitly present can be written in the two forms

$$\Psi=\psi_i+G_iH_1{}^i\Psi, \tag{9a}$$

and

$$\Psi=[1+GH_1{}^i]\psi_i. \tag{9b}$$

The function ψ_i above is a solution Ψ_i of Eq. (5) for the energy $\mathcal{E}$. Use of Eqs. (9a), (7), and (4) gives

$$\Psi=\psi_i+G_f(V_{NT}+V_{PT})\Psi+G_f(V_{NP}-V_{PT})(\Psi-\psi_i);$$

$$\Psi=\psi_i-G_f(\mathcal{E}-H_0{}^f)\psi_i+G_f(V_{NT}+V_{PT}+V_{NP}-V_{PT})\Psi,$$

or

$$\Psi=G_f(V_{NT}+V_{NP})\Psi=G_fH_1{}^f\Psi. \tag{10}$$

Finally, use of Eq. (9b) yields:

$$\Psi=G_fH_1{}^f[1+GH_1{}^i]\psi_i, \tag{11}$$

or

$$\Psi=G_f(V_{NT}+V_{NP})[1+G(V_{NT}+V_{PT})]\psi_i.$$

In order to obtain the scattering amplitude, we consider the final-state asymptotic limit in which there is an outgoing spherical wave of neutrons and the proton is bound to the nucleus with a binding energy $-\epsilon_P$.

G_f can be explicitly written as

$$G_f=\sum_n \frac{\Psi_f{}^n\Psi_f{}^{n\dagger}}{E-E_n}. \tag{12}$$

We write ψ_f in the form

$$\psi_f=(2\pi)^{-\frac{3}{2}}e^{i\mathbf{k}_N\cdot\mathbf{r}_N}\,\Xi(TP). \tag{13}$$

The part of G_f we are interested in is its projection to the $\chi(T,P)$ state:

$$\mathfrak{G}_f(TPN;\,T'P'N')$$

$$=\chi(TP)\int\chi^\dagger(T''P'')G_f(T''P''N;\,T'P'N'). \tag{14}$$

$\chi(T,P)$ is the specific bound state solution $\Xi(T,P)$ with eigenvalue $-\epsilon_P$. $\mathfrak{G}_f$ has the explicit form

$$\mathfrak{G}_f(TPN;\,T'P'N')$$

$$=\chi(TP)\chi^\dagger(T'P')\frac{2M}{\hbar^2}(2\pi)^{-3}\int\frac{e^{i\mathbf{k}'\cdot(\mathbf{r}_N-\mathbf{r}_{N'})}}{k_N{}^2-k'^2}d^3k', \tag{15}$$

where M is the nucleon mass and[7]

$$\mathcal{E}=(\hbar^2/2M)k_N{}^2-\epsilon_P-\epsilon_T. \tag{16}$$

In the asymptotic limit, $\mathfrak{G}$ becomes

$$\lim_{r_N\to\infty}\mathfrak{G}_f(TPN;\,T'P'N')$$

$$=\frac{M}{2\pi\hbar^2}\chi(TP)\left\{\lim_{r_N\to\infty}\frac{e^{ik_Nr_N}}{r_N}\right\}\psi_f{}^\dagger(T'P'N'). \tag{17}$$

The scattering amplitude $\mathcal{T}$ is by definition the coefficient of $\chi(TP)(e^{ik_Nr_N}/r_N)$ or

$$\mathcal{T}=(2\pi\hbar^2)^{-1}M(T_O+T'), \tag{18}$$

[7] Corrections due to the recoil of a finite-mass nucleus are neglected in Eqs. (15), (17), and (21). They can be included rather simply, and do not change the basic argument. For example, $\mathbf{k}_N$ and $\mathbf{r}_N$ simply become the wave vector of the relative motion and the relative displacement of the neutron and residual nucleus in the c.m. system.

where

$$T_0=\int \psi_f^\dagger V_{NP}\psi_i dv=\langle f|V_{NP}|i\rangle, \tag{19}$$

and

$$T'=\int \psi_f^\dagger (V_{NT}+H_1{}^f G H_1{}^i)\psi_i dv =\langle f|V_{NT}+H_1{}^f G H_1{}^i|i\rangle. \tag{20}$$

The differential cross section is:

$$\frac{d\sigma}{d\Omega}=\frac{v_f}{v_i}|\mathcal{T}|^2, \tag{21}$$

or

$$\frac{d\sigma}{d\Omega}=\frac{M^2}{(2\pi\hbar)^2}\frac{v_f}{v_i}|\langle f|V_{NP}|i\rangle+\langle f|V_{NT}|i\rangle +\langle f|H_1{}^f G H_1{}^i|i\rangle|^2,$$

where v_f and v_i are the velocities of the neutron and the deuteron respectively.[7]

Equations (18)–(21) are derivable by a few formal manipulative steps using the approach of Gerjuoy or Tobocman.[3] The stress here, however, has been on the point of view of treating the initial and final states symmetrically. Carrying this point of view further, we observe that T_0 can be written in the form:

$$T_0=\langle f|\mathcal{E}-(H_T+T_N+T_P)|i\rangle. \tag{22}$$

If we now assume that a shell-model picture of the residual nucleus is valid, the initial and final states can be written in the c.m. system as

$$\psi_i=(2\pi)^{-3}\phi_D(\mathbf{r}_N-\mathbf{r}_P)\exp[\tfrac{1}{2}i\mathbf{k}_D\cdot(\mathbf{r}_N+\mathbf{r}_P)] \times\exp[i\mathbf{k}_T\cdot\mathbf{r}_T]\zeta_T(\xi), \tag{23}$$

and

$$\psi_f^\dagger=(2\pi)^{-3}\eta_P^\dagger(\mathbf{r}_P-\mathbf{r}_T)\zeta_T^\dagger(\xi)\exp\left[-i\mathbf{k}_f\cdot\left(\frac{M_T}{M_f}\mathbf{r}_T +\frac{M}{M_f}\mathbf{r}_P\right)\right]\exp[-i\mathbf{k}_N\cdot\mathbf{r}_N], \tag{24}$$

where $\mathbf{k}_T$ and $\mathbf{r}_T$ are the wave vector and coordinate of the center of mass of the target, ξ represents the internal coordinates of the target, and $\mathbf{k}_f$ and M_f represent the c.m. wave vector and the mass of the residual nucleus. Further, in the c.m. system, $\mathbf{k}_T=-\mathbf{k}_D$ and $\mathbf{k}_f=-\mathbf{k}_N$.

T_0 can now be considerably simplified by utilizing the Fourier transforms of ψ_f and ψ_i. In fact, T_0 has the form of a simple overlap integral between the initial and final states, multiplied by an expression which is a known function of momentum:

$$T_0=\left(\mathcal{E}+\epsilon_T-\frac{\hbar^2}{2M}[k_D{}^2-2\mathbf{k}_D\cdot\mathbf{k}_N+2k_N{}^2]\right)\langle f|i\rangle. \tag{25}$$

Equations (19) and (25) are of course identical in content. If one is given the solution to Schrödinger's equation, as one is when ψ_f and ψ_i are explicitly required, the potentials involved can be obtained by simple differentiation. The only claim we make is that Eq. (25) is in somewhat more transparent form than (19), may be evaluated more simply than Eq. (19) in specific cases, and together with Eq. (20) represents the achievement of our aim: the presentation of expressions in the amplitude for the (d,n) reaction in which the initial and final interactions appear symmetrically.

III. DISCUSSION

In order to clarify somewhat the role T' plays in relation to the overlap integral T_0, it is useful to make an arbitrary division of G into two parts. G can be thought of as an expression of the form

$$G=\sum_n \frac{\Psi_n(x)\Psi_n^\dagger(x')}{E-E_n}, \tag{26}$$

where the Ψ_n are solutions of the complete Hamiltonian H. One can classify these solutions into scattering and compound-nucleus (resonant scattering) solutions and write G as a sum of two terms:

$$G=G_S+G_C. \tag{27}$$

Correspondingly, one can split T' into its constituent parts, T_S+T_C:

$$T_S=\langle f|V_{NT}+H_1{}^f G_S H_1{}^i|i\rangle, \tag{20a}$$

and

$$T_C=\langle f|H_1{}^f G_C H_1{}^i|i\rangle. \tag{20b}$$

The term with the function G_S can be conceived of as corresponding to intermediate states of the deuteron-target system for which, in the asymptotic limit, some parts of the system are well separated, and all constituent parts of the system are found near each other only for a rather short time. More specifically, one can think of the intermediate states as appropriately distorted waves representing the scattering of deuterons on the target, neutrons on the target plus proton, etc. The term $\langle f|V_{NT}|i\rangle$ is naturally to be grouped with G_S. Some rather crude and qualitative estimates involving a plane-wave approximation to G_S, indicate that the ratio T_S/T_0 is of the order of 15% or less.

A possible suggestion for a model of compound nuclear states would be to consider the various resonance scattering terms which may arise in the deuteron-target system. As in the analysis of α-particle decay, a potential barrier of some kind is required for such resonant terms. Two ways in which such a barrier can arise in the (d,n) reaction intermediate states are immediately apparent. One type of barrier is the Coulomb barrier presented by the nucleus to the deuteron, leading to resonant terms in deuteron scattering. A somewhat less obvious effective barrier can arise in the following way: Suppose the proton of the

deuteron is captured by the target, and the residual nucleus is left in a state of sufficiently high excitation so that it can bind the neutron. This state would be highly unstable. Decay to the ground state of the residual nucleus can be expected to be accompanied by re-emission of the neutron.

The net result of both of these processes is resonant scattering: the probability amplitudes have large values for small distances between constituents of the scattering system, in the language of stationary states. Alternatively, one may say that in a deuteron-target resonant scattering state, for example, if the incoming deuterons have a kinetic energy near a certain value, they will spend a rather long time in the vicinity of the target. As is well known from the α-particle problem, for energies far from the resonant energy, the state essentially becomes an ordinary scattering state.

If the resonances are well separated, and a resonance occurs at E_0, with an associated half-width Γ, G_C may be approximated by

$$G_C \cong \frac{\psi_0(x)\psi_0^\dagger(x')}{\mathcal{E}-E_0+\frac{1}{2}i\Gamma}, \tag{28}$$

where ψ_0 is the bound state eigenfunction near the energy E_0, when the barrier is made infinitely high. The way in which this approximation arises in a simple specific case is illustrated in Appendix B. The term T_C is expected to make a large contribution to the (d,n) amplitude relative to T_O in the vicinity of a resonance. There are two reasons for this: the energy denominator in G_C becomes quite small, and the overlap of the intermediate state wave functions with the potentials and the initial and final state wave functions is large.

The above considerations may provide some explanation for the agreement between experiment and the various approximate calculations.[3,4] In one way or another these approximations all consider only the overlap integral T_O. The simple overlap integral of T_O exhibits the behavior of the angular distributions. In order to fit the experimental distributions in detail it is customary to use a cutoff procedure which limits the integration to the regions exterior to the nucleus.

The qualitative discussion above indicates that far from resonances T_O would be expected to be the dominant term. Further, the presence of the Coulomb repulsion added to the fact that the other processes constitute an absorption from the incident beam at points inside the nuclear radius would lead one to expect that the contribution of the core region to T_O may be small, at least for low energies.

In conclusion, we would like to emphasize that the material in the present section is necessarily qualitative and discursive. The developments of Sec. II are of course independent of the present considerations, and may perhaps find a calculational use quite aside from the illustrations which we have used.

ACKNOWLEDGMENTS

We would like to thank Dr. Carl Levinson, Dr. L. Madansky, and Dr. L. Rodberg for many helpful discussions.

APPENDIX A

The processes by which a neutron other than the neutron from the deuteron constitutes the outgoing particle have been called "heavy-particle stripping."[2] This mode of reaction can be included formally by incorporating the exchange wave functions for several separate processes.

We illustrate the method by considering a (d,n) reaction for the case of a target nucleus which is described by a neutron and a core. For the sake of simplicity we neglect all exchange processes which take place in the core. These processes can be considered, but the expressions become somewhat cumbersome.

The initial state in this formulation is described in somewhat more detail than in Sec. II. It consists of a deuteron with a neutron labeled 1 plus the associated proton; a core, and a neutron, labeled 2, which is bound to the core.

The complete Hamiltonian for the process can be written in the form

$$H=H_C+T_{N(1)}+T_{N(2)}+T_P+V_{N(1)P}+V_{N(2)P}+V_{N(1)C}+V_{N(2)C}+V_{N(1)N(2)}+V_{PC}. \tag{A1}$$

As expected, (A1) is symmetric in neutrons 1 and 2.

The deuteron stripping process described in Sec. II will be indicated in the Hamiltonian by the superscripts 1 and 2 for the initial and final states respectively (in place of the superscripts i and f), with

$$\begin{aligned} H_T&=H_C+T_{N(2)}+V_{N(2)C}, \\ V_{N(1)T}&=V_{N(1)C}+V_{N(1)N(2)}, \\ V_{PT}&=V_{N(2)P}+V_{PC}. \end{aligned} \tag{A2}$$

Equations (2) and (3) become

$$\begin{aligned} H_0{}^1&=H_T+T_{N(1)}+T_P+V_{N(1)P}, \\ H_1{}^1&=V_{N(1)T}+V_{PT}, \end{aligned} \tag{A3}$$

and

$$\begin{aligned} H_0{}^2&=H_T+T_{N(1)}+T_P+V_{PT}, \\ H_1{}^2&=V_{N(1)P}+V_{N(1)T}. \end{aligned} \tag{A4}$$

Deuteron stripping is the process in which the outgoing stripped neutron is the neutron initially in the deuteron. Neutron 2 remains bound. The complete solution $\Psi(1,2)$ of the Hamiltonian H, written in a form which makes the presence of the deuteron stripping apparent, is

$$\begin{aligned} &\Psi(1,2)=G_2(1,[2];1',[2']) \\ &\quad\times(H_1{}^2[1+GH_1{}^1])(1',2';1'',2'')\psi_i(1'',[2'']), \end{aligned} \tag{A5}$$

or

$$\Psi(1,2)=G_2(1,[2];1',[2'])H_1{}^2(1',2';1'',2'')\Psi(1'',2'').$$

The indices of neutrons 1 and 2 are explicitly given in Eq. (A5). The bracketed numerals indicate the neutron bound to the core. G_2 is the Green's function corresponding to H_0^2.

An exchange process is also possible in the present case. Both the neutron and the proton in the deuteron are captured in this process and neutron 2 goes out. It should be noted that this exchange component vanishes when one computes

$$\lim_{r_{N(1)}\to\infty}\Psi(1,2).$$

One can calculate the wave function in a form which exhibits the exchange most clearly by formal manipulations analogous to those involved in Eqs. (7)–(10). The results, however, can be written down immediately if one recognizes this process as the so-called "heavy-particle stripping." The deuteron captures the core C; the neutron 2 is stripped off from the core and is left free. In analogy with (A2), we require

$$\begin{aligned} H_D&=T_{N(1)}+T_P+V_{N(1)P},\\ V_{N(2)D}&=V_{N(1)N(2)}+V_{N(2)P},\\ V_{CD}&=V_{N(1)C}+V_{PC}. \end{aligned} \tag{A6}$$

The analogs of Eqs. (2) and (3) are now

$$\begin{aligned} H_0^3&=H_D+H_C+T_{N(2)}+V_{N(2)C}=H_0^1,\\ H_1^3&=V_{CD}+V_{N(2)D}=H_1^1; \end{aligned} \tag{A7}$$

and

$$\begin{aligned} H_0^4&=H_D+H_C+T_{N(2)}+V_{CD},\\ H_1^4&=V_{N(2)C}+V_{N(2)D}. \end{aligned} \tag{A8}$$

It follows immediately that the wave function can be written in the form

$$\begin{aligned} \Psi(1,2)=&G_4(2,[1];2',[1'])\\ &\times(H_1^4[1+GH_1^3])(1',2';1'',2'')\psi_i(1'',[2'']), \end{aligned} \tag{A9}$$

or

$$\Psi(1,2)=G_4(2,1;2',1')H_1^4(1',2';1'',2'')\Psi(1'',2'').$$

G_4 is the Green's function corresponding to H_0^4. Incidentally we note that H_0^2 goes to H_0^4 under the exchange of 1 and 2. It should be emphasized that Eq. (A5), since it represents an exact solution to the total Hamiltonian, contains the exchange solution just obtained, but in a somewhat obscure form, as well as other terms which are not of present concern.

The symmetry of the total Hamiltonian with respect to the exchange of 1 and 2 must now be taken into account. The final form of the solution is

$$\Upsilon(1,2)=\Psi(1,2)-\Psi(2,1). \tag{A10}$$

We have assumed that the functions Ψ contain the appropriate spin functions for 1 and 2. Thus Υ is totally antisymmetric for the exchange of 1 and 2.

For reasons which will be apparent shortly, (A5) is the most convenient form to use for $\Psi(1,2)$ and (A6) is the best choice for $\Psi(2,1)$. If the Ψ's are normalized to unity, the normalization of Υ is chosen to express the fact that two physical processes can now take place: deuteron stripping and heavy-particle stripping.

As in Sec. II, we are once again interested in the projection to the $\chi(TP)$ state and want to consider the coefficient of $\chi(TP)r_{N(1)}^{-1}\exp[ik_{N(1)}r_{N(1)}]$ as $r_{N(1)}$ is made large.

The scattering amplitude $\mathcal{T}$ thus becomes

$$\mathcal{T}=\int\psi_f^\dagger(1,[2])H_1^2\Psi(1,2)-\int\psi_f^\dagger(2,[1])H_1^4\Psi(1,2), \tag{A11}$$

$$\begin{aligned} \mathcal{T}=&\langle f_{1,[2]}|H_1^2(1+GH_1^1)|i_{1,[2]}\rangle\\ &-\langle f_{2,[1]}|H_1^4(1+GH_1^3)|i_{1,[2]}\rangle. \end{aligned}$$

Finally we note that the overlap amplitude, taking into account the symmetry relation connecting H_1^2 and H_1^4, becomes:

$$\begin{aligned} T_O=&[\mathcal{E}+\epsilon_T-(\hbar^2/2M)(k_D^2-2\mathbf{k}_D\cdot\mathbf{k}_N+2k_N^2)]\\ &\times\int\psi_f^\dagger(1,[2])[\psi_i(1,[2])-\psi_i(2,[1])]. \end{aligned} \tag{A12}$$

The first terms of (A11) can also be written by keeping the antisymmetry in the final state function

$$\begin{aligned} T_O=&[\mathcal{E}+\epsilon_T-(\hbar^2/2M)(k_D^2-2\mathbf{k}_D\cdot\mathbf{k}_N+2k_N^2)]\\ &\times\langle f_{1,[2]}|i_{1,[2]}\rangle-[\mathcal{E}+\epsilon_D-(\hbar^2/2M)\\ &\times(k_D^2+2\mathbf{k}_D\cdot\mathbf{k}_N+2k_N^2)]\langle f_{[2],1}|i_{1,[2]}\rangle. \end{aligned}$$

Since the spins are still contained in the wave functions, the differential cross section for a specific problem is obtained by averaging over the initial spin states and summing over the final spin states.

APPENDIX B

To illustrate the behavior of a Green's function in the vicinity of a scattering resonance, we consider a simple example. We restrict ourselves to the S state only. The Green's function equation is:

$$\{d^2/dr^2+k^2-U\}G(r,r';k^2)=\delta(r-r'). \tag{1B}$$

There is a corresponding equation for the wave function.

The boundary conditions on the Green's function are

$$\lim_{r<\to 0}G(r,r')=0,\quad \lim_{r>\to\infty}\exp(-ikr_>)G(r,r')=f(r_<), \tag{2B}$$

where $r_<$ is the smaller value, and $r_>$ the larger value, of r, r'. The second of Eqs. (2B) corresponds to the outgoing-wave condition at infinity.

We define the function $f(k,r)$ to be the solution of the Schrödinger equation corresponding to (1B) with the properties[8]:

$$f(r,k)\underset{r\to\infty}{\sim}e^{-ikr}, \tag{3B}$$

[8] R. Jost and W. Kohn, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **27**, No. 9 (1953).

and

$$f(r,k)=f^*(r,-k),$$

and define the function $f(k)$ by

$$f(k)\equiv f(0,k). \tag{4B}$$

A function ϕ, which vanishes at the origin and the derivative of which is unity there, can be written in terms of f in the form

$$\phi(r,k^2)=(1/2ik)[f(k)f(r,-k)-f(-k)f(r,k)]. \tag{5B}$$

The Green's function in terms of ϕ and the f's is

$$G(r,r';k^2)=-[f(-k)]^{-1}\phi(r_<,k^2)f(r_>,-k). \tag{6B}$$

As a specific illustration of the explicit form of G for a given example, consider the square barrier:

$$U(r)=\begin{cases}0, & 0\leqslant r\leqslant a\\ U_0, & a\leqslant r\leqslant a+b\\ 0, & a+b\leqslant r.\end{cases} \tag{7B}$$

If we define

$$\kappa=(k^2-U_0)^{\frac{1}{2}},\quad A(k)=\{\cos\kappa b-i\kappa^{-1}k\sin\kappa b\}, \tag{8B}$$

and

$$B(k)=i\{\cos\kappa b-i\kappa k^{-1}\sin\kappa b\},$$

the solution $f(r,-k)$ is of the form

$$f(r,-k)=\begin{cases}e^{ik(a+b)}[A(k)\cos k(r-a)\\ \quad +B(k)\sin k(r-a)], & 0\leqslant r\leqslant a\\ e^{ik(a+b)}[\cos\kappa(r-a-b)\\ \quad +i\kappa^{-1}k\sin\kappa(r-a-b)], & a\leqslant r\leqslant a+b\\ e^{ikr}, & r\geqslant a+b.\end{cases} \tag{9B}$$

Consider now a resonant scattering state in the vicinity of $E_0=k_0^2$. The conditions satisfied by a low-lying resonant state are

$$0<k^2\ll U_0;\quad k_0a\cong n\pi,$$

and

$$\kappa=i|\kappa|,\quad |\kappa|=(U_0-k^2)^{\frac{1}{2}}\cong U_0^{\frac{1}{2}}. \tag{10B}$$

More precisely, we can take the condition for the resonance energy E_0 to be

$$\sin k_0a+|\kappa_0|^{-1}k_0\cos k_0a\coth|\kappa_0|b=0. \tag{11B}$$

We limit outselves to the vicinity of the resonance, i.e., to energies for which

$$|E-E_0|\ll\left(\frac{dE}{dk}\right)\frac{4k_0}{U_0^{\frac{1}{2}}a}\exp(-2U_0^{\frac{1}{2}}b), \tag{12B}$$

and define $\frac{1}{2}\Gamma$ to be

$$\tfrac{1}{2}\Gamma=\frac{4k_0^2}{U_0a}\left(\frac{dE}{dk}\right)_0\exp(-2U_0^{\frac{1}{2}}b). \tag{13B}$$

Under these conditions, $f(-k)$ can be approximated by

$$f(-k)\simeq\{U_0^{\frac{1}{2}}a(2k_0)^{-2}\exp(U_0^{\frac{1}{2}}+ik_0)b\}\times\{E-E_0+\tfrac{1}{2}i\Gamma\}, \tag{14B}$$

and the functions $\phi(r,k^2)$ and $f(-k,r)$ become

$$\phi(r,k_0^2)\simeq\begin{cases}k_0^{-1}\sin k_0r, & 0\leqslant r\leqslant a\\ 0, & a\leqslant r,\end{cases} \tag{15B}$$

$$f(r,-k_0)\simeq\begin{cases}-\frac{1}{2}U_0^{\frac{1}{2}}\exp[(U_0^{\frac{1}{2}}+ik_0)b]\phi(r,k_0^2), & 0\leqslant r\leqslant a\\ 0, & a\leqslant r.\end{cases}$$

If we define $\psi(r,k_0^2)$ to be the normalized bound state wave function in the limit as the barrier height goes to infinity, then

$$\psi(r,k_0^2)=\begin{cases}(2/a)^{\frac{1}{2}}\sin k_0r, & 0\leqslant r\leqslant a\\ 0, & a\leqslant r.\end{cases} \tag{16B}$$

The Green's function can be approximately written in the form

$$G(r,r':k^2)\simeq\frac{\psi(r,k_0^2)\psi^*(r',k_0^2)}{E-E_0+\frac{1}{2}i\Gamma}. \tag{17B}$$