Maser Noise Considerations

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The contribution of the saturation field to the noise figure of a three-level maser is calculated. It is shown that the effect is very small, under ordinary circumstances.

Certain aspects of spontaneous-emission noise are considered. It is shown that the spontaneous-emission equivalent temperature for a free-electron vacuum-tube amplifier is one-half that of a maser.

The possibility of eliminating spontaneous-emission noise is considered and it is concluded that more general quantum-mechanical amplifiers can in principle be constructed which will not have spontaneous-emission noise.

INTRODUCTION

SEVERAL years ago research on a new method for amplification of microwaves started independently at the University of Maryland,[1] Columbia University,[2] and the Lebedev Institute[3] in Moscow.

Proposals for coherent microwave amplification by the stimulated emission of radiation (maser) were published in 1953.[1] This early paper calculated the intrinsic gain (without feedback) for a maser amplifier employing a gas. The small intrinsic gain of a gas device severely limits the gain band-width product which can be obtained, and it was suggested[1] at that time that solids would have to be used. The experiments of Gordon, Zeiger, and Townes[2] demonstrated, in 1954, that a gas device could be made to operate, and confirmed the extreme limitations on gain band-width product. These limitations enable the ammonia molecular beam maser to be employed as a spectrometer of very high resolution, but prevent its use as a practical amplifier. Bloembergen's suggestion[4] for a solid state maser now offers the possibility of low-noise continuous amplification with useful gain-band-width product. A solid state maser oscillator has been successfully operated by Scovil, Feher, and Seidel.[5]

NOISE IN A THREE-LEVEL MASER

Bloembergen's[4] three-level maser employs a saturation radio-frequency field to achieve nearly equal populations of states 1 and 3 (Fig. 1). Amplification then results between either states 1 and 2 or 3 and 2. One might be inclined to believe that such a device would be noisy because if saturation is obtained, we have essentially an infinite temperature associated with levels 1 and 3. However, at saturation the absorption coefficient is small so the noise emission should also be small. It therefore requires analysis to see how the saturation field will affect the noise figure.

The discussion of Strandberg[6] is convenient at this point. We consider first a two-level negative-temperature maser in the form of a travelling-wave amplifier. Let p_ν be the energy for a given mode. Then we can write for the derivative of power with respect to distance along the transmission system, for a range of frequencies $d\nu$,

$$V_G A R_\nu d\nu dp_\nu/dx = V_G A R_\nu d\nu[Kn_2h\nu(N+1) - Kn_1h\nu N - \alpha_c N h\nu + \alpha_c p_\nu(T_c)]. \quad (1)$$

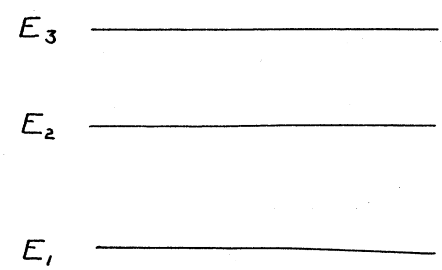

FIG. 1. Energy levels in a three-level maser.

[1] J. Weber, Transactions of the Institute of Radio Engineers Professional Group on Electron Devices, PGED-3 June, 1953. This appears to be the earliest publication in the open literature on masers. The principle of amplification by stimulated emission of radiation was given and it was noted that the radiation would be coherent. The intrinsic gain (without feedback) was calculated for an electric-field-reversal symmetric-top-molecule amplifier utilizing $\Delta m = \pm 1$ transitions. Most of this work was done in March, 1951, and discussed at seminars in the Washington and Princeton areas, and at the 1952 Institute of Radio Engineers Electron Tubes Conference in Ottawa. Publication was delayed until 1953, mainly because the small intrinsic gain of maser gas molecule amplifiers made the use of solids essential for usable gain bandwidth products. No promising method for a solid state maser was available in 1951. [Maser-type amplification was presumably observed by E. M. Purcell and R. V. Pound in 1950 in connection with their nuclear resonance experiments which were discussed in Phys. Rev. **81**, 279 (1951).]

[2] Gordon, Zeiger, and Townes, Phys. Rev. **95**, 282 (1954). Professor Townes has kindly informed us that some informal discussion of certain aspects of molecular beam type masers was given by A. H. Nethercot on behalf of Professor Townes at a meeting at the University of Illinois on submillimeter waves, in May, 1951. The Columbia work was also outlined in unpublished Columbia Radiation Laboratory Progress Reports starting in December, 1951.

[3] N. G. Basov and A. M. Prokhorov, J. Exptl. Theoret. Phys. (U.S.S.R.) **27**, 431 (1954), and Proc. Acad. Sci. (U.S.S.R.) **101**, 47 (1955). This work was in progress in 1952 and was somewhat similar to the Columbia research.

[4] N. Bloembergen, Phys. Rev. **104**, 324 (1956). A different approach was used by Combrisson, Honig, and Townes, Compt. rend. **242**, 2451 (1956).

[5] Scovil, Feher, and Seidel, Phys. Rev. **105**, 762 (1957).

[6] M. W. P. Strandberg, Phys. Rev. **106**, 617 (1957).

In (1), $R_\nu d\nu$ is the number of modes per unit volume in a range $d\nu$ which can propagate, A is the cross sectional area, V_G is the power propagation velocity, K is a quantity involving squared matrix elements, which can be calculated from quantum mechanics, n_2 is the number of particles per unit volume in the upper state, n_1 is the number of particles per unit volume in the lower state, and N is the number of radio-frequency quanta in each mode. In the first term of (1) the factor N is for stimulated emission while the 1 which follows it is for spontaneous emission.* The second term on the right side of (1) gives the effect of the power absorbed by the particles, which are employed in amplification.

The third term of the right side of (1) gives the power absorbed by the walls, and the last term is due to the power emitted by the walls if the wall temperature is T_c. The quantity $p_\nu(T_c)$ is the average energy per mode at temperature T_c. The last term can be deduced by considering a section of transmission system in equilibrium and balancing the absorbed power against the emitted power from the walls. All terms in (1) are power per unit length. We make use of Strandberg's β which is defined by

$$\beta = K(n_2 - n_1) \tag{2}$$

and is called the quantum mechanical power gain. Making use of (2) and dividing through by $AV_G R_\nu d\nu$ gives us

$$\frac{dp_\nu}{dx} = p_\nu\beta + \frac{h\nu n_2\beta}{n_2 - n_1} - \alpha_c p_\nu + \alpha_c p_\nu(T_c). \tag{3}$$

Expression (3) was given by Strandberg and is correct for a two-level maser. We now proceed to modify (3) so that the noise contributed by the saturation field can be taken into account.

Consider then a three-level maser. We assume that in the presence of the saturation field a steady state is reached. We also assume that a temperature T_{13} can be assigned to the system of particles associated with levels 1 and 3 in the steady state. If the steady-state quantum states are maintained, no energy will be exchanged with an environment at temperature T_{13}. Let the power-absorption coefficient associated with the 1,3 system at frequency ν be $\alpha_{13}(\nu)$. This frequency ν is different from the resonant frequency $(E_3 - E_1)/h = \nu_{31}$ and is in fact the frequency which it is desired to amplify. The noise power emitted per unit length by the 1,3 system can be taken into account by addition of a term $\alpha_{13}p_\nu(T_{13})$ to expression (3). Also a term $\alpha_{13}p_\nu$ has to be subtracted for the power absorbed from transitions to state 3. The result is then

$$\frac{dp_\nu}{dx} = p_\nu\beta + \frac{h\nu n_2\beta}{n_2 - n_1} - \alpha_c p_\nu - \alpha_{13}p_\nu + \alpha_c p_\nu(T_c) + \alpha_{13}p_\nu(T_{13}). \tag{4}$$

Levels 1 and 2 are assumed to be used for amplification. The temperature T_x is defined by setting

$$n_1/n_2 = e^{h\nu/kT_x}. \tag{5}$$

Employing (5) and integrating (4) along the amplifier gives

$$(p_\nu)_{\text{out}} = (p_\nu)_{\text{in}} g^2 + \left[\frac{\beta p_\nu(T_x) - \alpha_c p_\nu(T_c) - \alpha_{13}p_\nu(T_{13})}{\beta - \alpha_c - \alpha_{13}}\right](1 - g^2). \tag{6}$$

In (6), g^2 is the power gain of the amplifier. Let the source temperature be T_s, let the load temperature be T_0, and let the transmission line have a power loss factor t and a temperature T_t. We can then write the noise figure in the forms

$$\text{Noise figure} = \frac{(p_\nu)_{\text{out}} + p_\nu(T_0)}{tg^2 p_\nu(T_s)},$$

$$\text{Noise figure} = 1 + \frac{1}{tp_\nu(T_s)}\Bigg\{(1-t)p_\nu(T_t) + (1 - g^{-2}) \times \left[\frac{\alpha_c p_\nu(T_c) + \alpha_{13}p_\nu(T_{13}) - \beta p_\nu(T_x)}{\beta - \alpha_c - \alpha_{13}}\right] + g^{-2}p_\nu(T_0)\Bigg\}. \tag{7}$$

In (7) we must insert for the average energy per mode the quantity $p_\nu(T) = h\nu/(e^{h\nu/kT} - 1)$.

For a well-designed amplifier $t \to 1$, $g \gg 1$, $\beta \gg \alpha_c$, and $\beta \gg \alpha_{13}$. In this case, (7) approaches the value

$$\text{Noise figure} = 1 + \frac{|p_\nu(T_x)|}{p_\nu(T_s)}\left[1 + \frac{\alpha_{13}p_\nu(T_{13})}{\beta|p_\nu(T_x)|}\right]. \tag{8}$$

* *Note added in proof.*—The discussion in this paper and the earlier treatments of noise in a maser assume that incoherent spontaneous emission takes place, corresponding to states which can be described by a temperature. Pound (*Advances in Electronics*, to be published) has pointed out that the coherent effects discussed by Dicke [Phys. Rev. **93**, 99 (1954)] might occur in a maser.

In the three level maser coherent effects due to excitation of "super radiant" states could, at least in principle, be made small by controlling the relaxation times so that a minimum of saturation power is needed. This type of operation is essential anyway if low temperatures are to be maintained. Coherent effects resulting from emission would appear to be unimportant as long as the appropriate relaxation time is substantially shorter than the lifetime for incoherent spontaneous emission. Calculations are in progress to explore these issues further.

If a maser operates in a manner such that coherence effects occur to a significant degree, its noise performance will be very different from that predicted here. We invite attention to the fact that for one set of conditions coherent radiation will produce more noise than the equilibrium value, while for other conditions less noise will be produced. Development of techniques for producing coherent spontaneous emission may, in fact, result in more quiet amplifiers.

We shall now estimate the value of the quantity $\alpha_{13}p_\nu(T_{13})/\beta|p_\nu(T_x)|$, and show that it is very small under ordinary circumstances. $p_\nu(T_{13})$ can be set equal to kT_{13}. The quantity α_{13} can then be obtained from an appropriate line-shape factor. For example, if we have a Lorentz-shaped line we can write (even under conditions of saturation)

$$\alpha_{13}(\nu)=\frac{\chi n_1|\mu_{13}|^2}{kT_{13}}\left[\frac{1/\tau_{13}}{4\pi^2(\nu-\nu_{13})^2+(1/\tau_{13})^2}\right]. \quad (9)$$

In (9), n_1 is the number of particles per unit volume in state 1, τ_{13} is an appropriate relaxation time, μ is a dipole moment matrix element, and χ is a numerical factor which will cancel out. The quantum-mechanical gain β is given by

$$\beta=\chi n_2|\mu_{12}|^2\tau_{12}/|p_\nu(T_x)|. \quad (10)$$

Employing (9) and (10) enables us to write

$$\frac{\alpha_{13}p_\nu(T_{13})}{|\beta p_\nu(T_x)|}=\frac{n_1|\mu_{13}|^2}{n_2|\mu_{12}|^2}\left[\frac{1/(\tau_{13}\tau_{12})}{4\pi_2(\nu-\nu_{13})^2+(1/\tau_{13})^2}\right]. \quad (11)$$

For the types of amplifier now under development $\tau_{12}\approx 10^{-8}$ second, $\tau_{13}\approx\tau_{12}$, $\nu-\nu_{13}\approx 10^9$. Expression (11)† is then of the order 10^{-3}. We see that the effect of saturation calculated here makes an unimportant contribution to the noise figure of these amplifiers. It is clear that this effect will always be small unless $\nu-\nu_{13}\approx 1/(2\pi\tau)$.

This same kind of analysis can be carried out for a resonant cavity maser, with the following result [after correction of a minor typographical error in Strandberg's[6] expression (9)]:

$$\text{Noise figure}=\frac{1}{tp_\nu(T_s)}\Bigg\{\frac{(g+1)^2}{g^2}(1-t)p_\nu(T_t)+\frac{(g+1)^2}{g^2}$$
$$\times\Bigg[tp_\nu(T_s)+\frac{Q_e}{Q_0}\{p_\nu(T_c)-[p_\nu(T_x)+\gamma p_\nu(T_{13})]/(1+\gamma)\}$$
$$-\frac{(g-1)}{(g+1)}\left(\frac{p_\nu(T_x)+\gamma p_\nu(T_{13})}{1+\gamma}\right)\Bigg]+g^{-2}p_\nu(T_0)\Bigg\}. \quad (12)$$

Again, in (12), t is the transmission-line power-loss factor, $p_\nu(T)=h\nu/(e^{h\nu/kT}-1)$, T_t is the temperature of the transmission line, g^2 is the power gain, T_s is the source temperature, T_{13} is the temperature associated with the 1,3 system, and T_x is defined by $n_1/n_2=e^{h\nu/kT_x}$. Q_e is the external Q, Q_0 is the unloaded Q, Q_x is the Q associated with[7] the energy levels 1,2 and Q_{13} is the Q associated with the energy levels 1,3. The quantity γ is defined by

$$\gamma=Q_x/Q_{13}, \quad (13)$$

and T_c is the cavity wall temperature.

We can write (13) in the form

$$\gamma=Q_x/Q_{13}=-\alpha_{13}(\nu,T_{13})/\beta. \quad (14)$$

In (14), $\alpha_{13}(\nu)$ and β are again the absorption coefficient and gain for the 1,3 system and the 1,2 system, respectively, at frequency ν. For a well-designed amplifier, γ will be small and the relative importance of the saturation field, according to (12), is given by the factor

$$\frac{\gamma p_\nu(T_{13})}{|p_\nu(T_x)|}=\frac{\alpha_{13}(\nu)p_\nu(T_{13})}{\beta|p_\nu(T_x)|}, \quad (15)$$

which is again the same as (11) which has been shown to be 10^{-3}, in a practical situation.

Another way in which the saturation field can conceivably contribute to the noise is the following one. In the first approximation the spontaneous emission from state 2 down to state 1 is independent of the saturation field of frequency ν_{31}. However, for high values of the saturation field two-quantum transitions may occur in which a particle goes from state 2 to state 1 as an intermediate state, ending up in state 3. This effect will contribute to the quantum-mechanical gain β. An order of magnitude calculation shows that in the high-field limit the value of β is increased a small amount of the order of a few percent. A study of expression (7) shows that the noise figure is almost independent of β if β is large. It follows therefore that with a quiet amplifier (large β) the effect of higher order transitions will not be large.

It is of course conceivable that the saturation radio-frequency field will contribute to the noise in other ways‡ than considered here.

Some Aspects of Spontaneous-Emission Noise

It is now believed that maser amplifier noise performance will ultimately be limited by spontaneous emission. We now consider certain features of this type of noise.

By use of the fluctuation dissipation theorem[8–10] the

† *Note added in proof.*—The evaluation of the last factor of Eq. (8) requires a knowledge of the absorption coefficient of the 1, 3 system at frequency ν in the presence of the saturation field of frequency ν_{13}. In the absence of experimental data, we assume a Lorentz line shape. Employment of the incorrect line shape will have a substantial effect on the magnitude of expression (11), but will not alter the general conclusion that the saturation field has a small effect as long as $|\nu-\nu_{13}|>(\tau)^{-1}$.

[7] Q_{13} and Q_x are defined by

$$Q_{13}=\frac{2\pi\nu\ \text{(energy in the principal cavity mode)}}{\text{net power absorbed by the 1,3 system}},$$

$$Q_x=-\frac{2\pi\nu\ \text{(energy in the principal cavity mode)}}{\text{net power emitted by the 1,2 system}}.$$

‡ *Note added in proof.*—Professor Strandberg has informed us that direct conversion of noise from the saturation oscillator into the amplification band might be an additional source of noise. We thank him for a profitable correspondence and for pointing out an error in an earlier version of Eq. (12).

[8] H. Nyquist, Phys. Rev. **32**, 110 (1928).

[9] H. B. Callen and T. A. Welton, Phys. Rev. **83**, 34 (1951).

[10] J. Weber, Phys. Rev. **101**, 6, 1620 (1956).

well-known expressions for the transition probability for spontaneous emission can be written in the forms

$$P_E=\frac{8\pi^2|\mu_E|^2\nu\mathcal{R}_E}{3h} \tag{16}$$

for electric dipole transitions, and

$$P_M=\frac{2|\mu_M|^2\mathcal{R}_M c^2}{3h\nu} \tag{17}$$

for magnetic dipole transitions.

In these expressions μ_E is the electric dipole matrix element for the appropriate pair of states, c is the speed of light, $\mathcal{R}_E$ is a radiation resistance per unit length squared which we calculate classically according to the following prescription. Place a short dipole antenna of length l carrying uniform current at the position of the particle, in the cavity or wave guide, oriented parallel to the electric field. Calculate its radiation resistance and divide by l^2. The result is $\mathcal{R}_E$. Such calculations for dipoles in wave guides under different conditions have been carried out by Slater.[11] The quantity μ_M is the magnetic dipole moment matrix element. $\mathcal{R}_M$ is calculated in the following way. Place a current loop of area A at the position of the particle in the cavity or wave guide, with the plane of the loop normal to the magnetic field. The radiation resistance of the loop per unit area squared is $\mathcal{R}_M$.

Expressions (16) and (17) can be employed to calculate the total noise, and gain of a maser amplifier. These expressions illustrate the fact that (in the dipole approximation) the spontaneous emission can be made as large or small as we like, but if $\mathcal{R}_E$ or $\mathcal{R}_M$ become zero no power can be coupled out.

Spontaneous emission also contributes noise in conventional vacuum-tube amplifiers. This has been discussed.[12–14] It is of interest to state the effect for free-electron amplifiers in the same terms as for a maser. The maser has a spontaneous-emission-noise[15] equivalent temperature T_{eq} given[16] by

$$T_{eq}=h\nu/k. \tag{18}$$

If a low-density slow electron beam interacts with an electric circuit in equilibrium with a heat bath the electrons will undergo transitions to different states of translational energy. If the transit time is small compared with the circuit period, the changes in electron velocity due to the interaction will correspond to a mean squared circuit electromotive force given[12] by

$$\langle V^2\rangle=\frac{1}{C}\left[\frac{h\nu}{2}+\frac{h\nu}{e^{h\nu/kT}-1}\right]. \tag{19}$$

In (19), C is the capacity.

The first term of (19) gives the effect of spontaneous emission by the electrons. At low temperatures it would be found that certain electrons in the beam had lost a quantum to the circuit. To obtain the equivalent temperature, we set

$$\tfrac{1}{2}CV^2=\tfrac{1}{2}kT_{eq}. \tag{20}$$

Employing the first term of (19) in (20) gives

$$T_{eq}=h\nu/2k. \tag{21}$$

For a free-electron beam, the effect of spontaneous emission is equivalent to a temperature half that in a maser. The factor $\frac{1}{2}$ is a consequence[17] of the fact that the equivalent classical oscillator induces both upward and downward transitions on the electron beam, and both contribute to the electron beam noise. For a maser, only the downward transitions induced by the equivalent oscillator contribute to the noise. Research continues on methods of cooling down an electron beam. Equation (21) indicates that in principle the limiting noise in a vacuum-tube amplifier due to spontaneous emission is even less than that in a maser.

Since spontaneous-emission noise occurs in all amplifiers thus far proposed, it is of interest to inquire whether this type of noise is so fundamental in character that it can never be eliminated. Both vacuum-tube and maser amplifiers can be operated as voltage amplifiers, that is, they need not absorb radio-frequency power in order to operate. A maser such as the Columbia device[2] which has essentially all the interacting molecules in the upper state is an example. The incident radiation merely stimulates excited molecules to radiate without itself being absorbed. In order to avoid spontaneous-emission noise we must start with particles in the lower state, and then arrange to have those which have absorbed incident quanta operate either a detector or an amplifier. Such an amplifier would be a power amplifier. Detectors which operate in such a fashion are devices such as nuclear counters. It is well known that

[11] J. C. Slater, *Microwave Transmission* (McGraw-Hill Book Company, Inc., 1942), Chap. VII.

[12] J. Weber, Phys. Rev. **90**, 977 (1953).

[13] J. Weber, Phys. Rev. **94**, 215 (1954).

[14] I. R. Senitzky, Phys. Rev. **104**, 1486 (1956).

[15] It was first shown by R. H. Dicke that the limiting sensitivity of a maser amplifier is determined by spontaneous emission, corresponding to an equivalent temperature $h\nu/k$ degrees Kelvin. His paper at the Symposium on Amplification by Atomic and Molecular Resonances (Berkeley-Carteret Hotel, Asbury Park, New Jersey, February 29–March 1, 1956) also gave the theory of the noise figure of maser amplifiers.

Subsequent analysis by Muller [Phys. Rev. **106**, 1, 8 (1957)]; Pound, *Advances in Electronics*, (to be published); Strandberg [Phys. Rev. **106**, 4, 617 (1957)], and Shimoda, Takahasi, and Townes [J. Phys. Soc. Japan, **12**, 6, 686 (1957)] have confirmed Dicke's work. Noise in Molecular Amplifiers was also discussed in the review article by J. P. Wittke [Proc. I.R.E. Vol. **45**, 3, 291, March, 1957)].

[16] This well-known result can be obtained quickly in the following way. The transition probability for downward transitions of a particle in an excited state always has a factor $N+1$, where N is the number of quanta in a given mode of the electromagnetic field. As was remarked earlier, the 1 which follows the N is for the effect of spontaneous emission. In equilibrium we have $N=1/(e^{h\nu/kT}-1)$. To obtain the equivalent temperature for spontaneous emission, set N above equal to 1. Then if $h\nu/kT\ll 1$, we get $T_{eq}=h\nu/k$.

[17] The equivalent classical oscillator is twice as effective in inducing noise on a free-electron beam; therefore it has to be only half as hot to yield the same amount of noise.

such detectors do not have spontaneous-emission noise, and can be made to have no output unless energy is incident on them. A molecular beam can in principle be made to operate in such a way that only molecules which were originally in the lowest state, then absorbed electromagnetic field quanta, will be detected. Future development of molecular beams might make feasible the detection of single radio-frequency quanta with no spontaneous-emission noise.

A power amplifier can in principle be constructed which operates only when ground-state particles are excited. Consider a beam of molecules with total spin of, say, n. In a magnetic field we can have $2n+1$ equally spaced states. First we arrange a molecular beam so that only particles in the lowest state (with $m=-n$) enter an interaction region. Those which absorb microwave quanta will now have $m=-n+1$. All molecules are now removed from the beam except those with $m=-n+1$. These remaining molecules now enter a region in which the magnetic field is slowly dropped to zero, then reversed to its earlier value. The molecules now have $m=n-1$. If these are allowed to enter a second cavity, each molecule can then lose $2n-1$ quanta of the same frequency as the exciting radiation, giving in principle a power gain of $2n-1$. Such an amplifier would not have spontaneous emission noise. However, it would not be a maser. The more general term "quantum mechanical amplifier" should be used to describe it.

CONCLUSION

The saturation field has only a very small effect on the noise of a three-level maser, for the mechanisms considered here. The value $h\nu/k$ which has been given as the limiting equivalent temperature does not appear to be fundamental to all amplifiers, although it does apply to existing maser devices.

It appears that quantum theory does not set a lower limit to the noise temperatures theoretically attainable with microwave detectors and amplifiers, at low temperatures.

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Brownian Movement

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A general analysis of the Brownian movement is given which is not limited to systems having a linear relaxation mechanism. Detailed results are obtained for the case of modest nonlinearity, to which presumably all problems of physical interest are limited.

1. INTRODUCTION

It may perhaps seem presumptuous today to suggest that it is still worthwhile to give an analysis of Brownian movement. However, we wish here also to attempt an analysis of the Brownian motion of a system having a nonlinear relaxation mechanism, and we hope in so doing to give a consistent analysis of Brownian movement which might perhaps be freer of difficulties and possible obscurities than is sometimes the case.

2. ANALYSIS

To be specific, let us consider an elementary electrical circuit with capacity C and resistance R as shown in Fig. 1; the resistance R is assumed to be placed in a thermal bath at temperature T. The condenser is idealized so that its behavior is completely characterized by the charge, q, on it at any instance. We now assume that the resistance R is the seat of random thermal fluctuations of electrical charge, and consequently that the charge q on the condenser will also fluctuate. It follows immediately that the element R must be able to dissipate power at an appropriate rate if a statistical equilibrium is to be maintained; the condenser will then have a mean energy $\bar{E}=\langle q^2\rangle/2C$. The condenser itself being idealized as a purely electromechanical element, this energy is free energy (i.e., available in principle for doing work). It follows then that, no matter what resistance is connected across the condenser, $\bar{E}$ must have the same value, because otherwise we might in principle establish in this way, using a condenser as intermediary, a net flow of power from one resistance to another (both being at the same temperature), which is contrary to the second law of

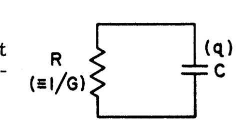

FIG. 1. Simple electrical circuit for discussion of Brownian movement.