previously[1,4,5] using classical current distributions for some or all of the pions. The only important difference is in the behavior of the correction as $\epsilon\rightarrow 0$. If the classical distribution of the positive pions is used to obtain an enhancement factor for the negative pion,[4] the correction is proportional to ϵ as $\epsilon\rightarrow 0$. In the present calculation the correction is proportional to a constant in this limit. This difference is readily interpreted as due to the nonlocalization of the wave function of the positive pions. This implies a slower variation with the relative positions of the pions than in the localized classical case.

[3] R. H. Dalitz, Phys. Rev. 94, 1046 (1954).
[4] J. Orear (private communication).
[5] Y. Eisenberg (private communication).

The result of this calculation has been applied to a recent analysis of the spin of the τ meson.[6]

ACKNOWLEDGMENTS

I am indebted to Dr. J. Orear for many helpful discussions. I am grateful to Professor R. H. Dalitz for several clarifications, and for bringing to my attention his derivation of the Coulomb effect[7] which is applicable to deriving the above results.

[6] E. Lomon, *Proceedings of the Seventh Annual Rochester Conference on High-Energy Physics, 1957* (Interscience Publishers, Inc., New York, 1957), Session VIII.
[7] R. H. Dalitz, Proc. Phys. Soc. (London) A69, 527 (1956); see footnote on page 537.

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Relativistic Corrections to the Polarization of Protons at High Energies*

SHOROKU OHNUMA
Sloane Physics Laboratory, Yale University, New Haven, Connecticut
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Relativistic corrections to high-energy p-p polarization data are calculated, with a breakdown of contributions into parts corresponding to the anomalous proton magnetic moment and remaining effects. It is found that as much as 70% of the relativistic correction may be caused by the anomalous part of the proton moment. In view of the presence of wave function distortion effects and the difference in sensitivity of different parts of the correction to the appreciable wave function distortion effects recently pointed out by Breit, it is concluded that the values of the relativistic corrections to polarization are at present quite unreliable. On the other hand, the corrections are smaller than the experimental error in many cases and their unreliability does not appear to affect existing fits to data seriously.

I

THE relativistic corrections to the Coulomb interactions of two protons have been investigated by Garren,[1] Breit,[2] and Ebel and Hull.[3] However, as has recently been pointed out by Breit,[4] the effects of wave function distortion due to specifically nuclear forces may give rise to non-negligible additional corrections in such calculations. In particular, the spin-orbit terms of the Coulomb interaction would be modified rather seriously.

Since the corrections have been applied to the analysis of data in the literature[1,5] it appears desirable to analyze contributions having different physical origins because, as shown by Breit,[4] the effects of wave function distortion may be expected to be represented by different factors for these contributions.

* This research was supported by the Office of Ordnance Research, U. S. Army.
[1] A. Garren, Phys. Rev. 96, 1709 (1954); 101, 419 (1956).
[2] G. Breit, Phys. Rev. 99, 1581 (1955).
[3] M. E. Ebel and M. H. Hull, Jr., Phys. Rev. 99, 1596 (1955).
[4] G. Breit, Phys. Rev. 106, 314 (1957).
[5] Stapp, Ypsilantis, and Metropolis, Phys. Rev. 105, 302 (1957).

II

The relativistic Coulomb amplitude for p-p scattering obtained by Garren,[1] apart from its phase factor, is of the form

$$\mathcal{R}=-(\eta/2k)\mathbf{s}^{-2}\left\{1-i\nu\frac{\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2}{2}\cdot\frac{[\mathbf{p}_i\times\mathbf{p}_f]}{|\mathbf{p}_i\times\mathbf{p}_f|}\sin\theta+O(\theta^2)\right\}, \tag{1}$$

where $\nu=(\epsilon-1)(2\epsilon^2-1)^{-1}[(2\epsilon+1)+2\epsilon(\epsilon+1)\mu_p]$; ϵMc^2 is the energy of each proton in the center-of-mass system; μ_p is the anomalous magnetic moment of proton in Bohr magnetons; $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ are Pauli's spin matrix vectors; $\mathbf{p}_i$ and $\mathbf{p}_f$ are, respectively, initial and final momenta of each proton in the center-of-mass system; the remaining quantities are the same as those given by Breit and Hull.[6]

The relativistic corrections to the polarization of

[6] G. Breit and M. H. Hull, Jr., Phys. Rev. 97, 1047 (1955). Hereafter, unless especially mentioned, the same notation as in this paper will be used.

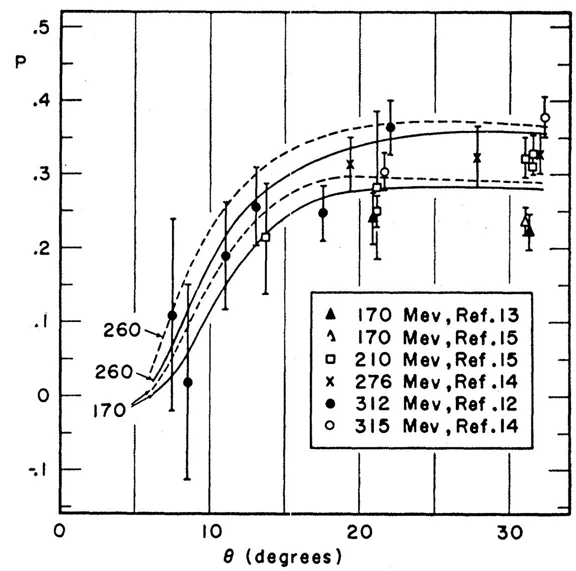

FIG. 1. Polarization for p-p scattering at 170 and 260 Mev calculated from the phase shifts by Saperstein and Durand (reference 10). Curves drawn by means of broken and solid lines represent, respectively, values of the polarization with and without the relativistic corrections [Eq. (2)]. The nonrelativistic values of σ have been used to estimate P from $P\sigma$. The center-of-mass scattering angle θ is in degrees. Designations of experimental points are: ▲ 170 Mev, reference 13; △ 170 Mev, reference 15; □ 210 Mev, reference 15; × 276 Mev, reference 14; ● 312 Mev, reference 12; ○ 315 Mev, reference 14.

protons in double and triple scattering are then given by

$$\Delta(P\sigma)=2\nu k^{-2}\sin\theta\cos\varphi \times\mathrm{Im}[\alpha_c{}^{+}(\alpha_2+\alpha_5-\alpha_3\sin^2\theta+2\alpha_c{}^{-})^*], \quad (2)$$

$$\Delta(D\sigma)=2\nu k^{-2}\sin^2\theta \times\mathrm{Re}[\alpha_c{}^{+}(\alpha_1-\alpha_4)^*]+2\nu^2k^{-2}\sin^2\theta|\alpha_c{}^{+}|^2, \quad (3)$$

$$\Delta(R\sigma)=2\nu k^{-2}\sin\theta\cdot\mathbf{s} \times\mathrm{Re}[\alpha_c{}^{+}(\alpha_0+\alpha_2+\alpha_3\sin^2\theta+\alpha_c{}^{+}+\alpha_c{}^{-})], \quad (4)$$

$$\Delta(A\sigma)=\Delta(R\sigma)\cdot(\mathbf{c}/\mathbf{s}). \quad (5)$$

Here P is polarization and σ is the elastic differential scattering cross section for unpolarized protons. Three triple-scattering parameters $D(\theta)$, $R(\theta)$, and $A(\theta)$ are as defined by Wolfenstein[7]; also

$$\alpha_c{}^{\pm}=-\tfrac{1}{4}\eta[\mathbf{s}^{-2}\exp(-i\eta\ln\mathbf{s}^2)\pm\mathbf{c}^{-2}\exp(-i\eta\ln\mathbf{c}^2)].$$
$$\alpha_0=k(s_{00}-S^c)e^{-i\Phi}.$$

As the phase factor of the Coulomb scattering amplitude (1), $\exp[i(\Phi-\eta\ln\mathbf{s}^2)]$ with the relativistic value of η was used in this calculation. The results of Breit[2] and of Ebel and Hull[3] have indicated that the principal effect of treating the Coulomb interaction relativistically is contained simply in the relativistic value of η for energies under consideration, i.e., $\lesssim 300$ Mev.

One qualitative feature of $\Delta(P\sigma)$ can be seen by taking only terms of Eq. (2) which are linear in η and

[7] L. Wolfenstein, Phys. Rev. **96**, 1654 (1954).

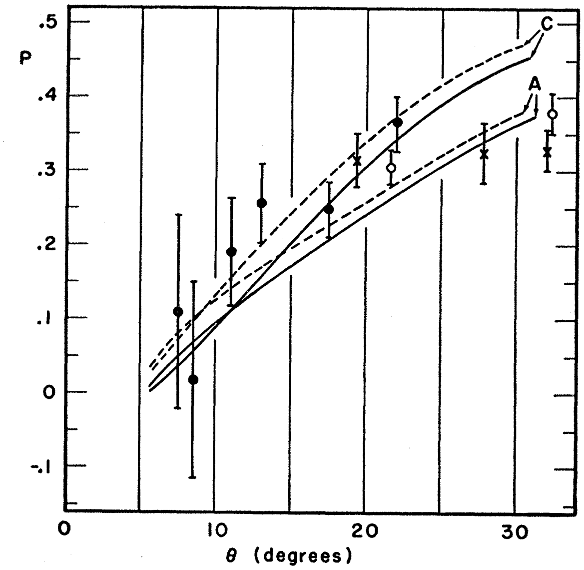

FIG. 2. Polarization for p-p scattering at 310 Mev calculated from the phase shifts of sets A and C by Hull *et al.* (reference 9, Table I). Designations of experimental points are the same as in Fig. 1.

by retaining those nuclear phase shifts with $L\leq 3$. Since $\eta\leq 0.017$ for $E\leq 100$ Mev, the linear approximation in η is justified. A brief calculation shows that

$$\Delta(P\sigma)=k^{-2}\eta\nu\cot\theta[a_0+a_2\cos^2\theta], \quad (6)$$

with

$$a_0=7(\cos^2\epsilon\sin^2\delta_2{}^P+\sin^2\epsilon\sin^2\delta_2{}^F)+3\sin^2\delta_1{}^P +2\sin^2\delta_0{}^P+\sqrt{6}\sin 2\epsilon(\sin^2\delta_2{}^P-\sin^2\delta_2{}^F) -17(\sin^2\epsilon\sin^2\delta_2{}^P+\cos^2\epsilon\sin^2\delta_2{}^F) -(7/4)\sin^2\delta_3{}^F-(93/4)\sin^2\delta_4{}^F, \quad (7)$$

and

$$a_2=25(\sin^2\epsilon\sin^2\delta_2{}^P+\cos^2\epsilon\sin^2\delta_2{}^F) +(35/4)\sin^2\delta_3{}^F+(145/4)\sin^2\delta_4{}^F, \quad (8)$$

where ϵ is the coupling parameter of two states, $^3P_2$ and $^3F_2$, defined by Blatt and Biedenharn.[8] Equations (6), (7), and (8) show that $\Delta(P\sigma)$ is always positive for small scattering angles θ, where this correction is expected to be important, regardless of the values of nuclear phase shifts.

III

The relativistic corrections to P based on some of the available phase-shift data[9–11] have been calculated and are presented below in Figs. 1–3. The experimental

[8] J. M. Blatt and L. C. Biedenharn, Phys. Rev. **86**, 399 (1952) and Revs. Modern Phys. **24**, 258 (1952).

[9] Hull, Ehrman, Hatcher, and Durand, Phys. Rev. **103**, 1047 (1956).

[10] A. M. Saperstein and L. Durand, III, Phys. Rev. **104**, 1102 (1956).

[11] H. P. Stapp, University of California Radiation Laboratory Report UCRL-3098 (unpublished). Phase shifts used here are of what he calls his best fit.

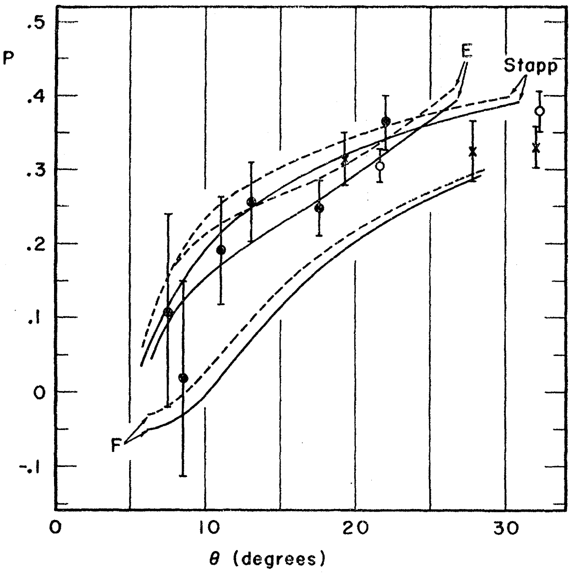

FIG. 3. Polarization for p-p scattering at 310 Mev calculated from the phase shifts of sets E and F by Hull *et al.* (reference 9, Table I), and from the phase shifts by Stapp (reference 11). Designations of experimental points are the same as in Fig. 1.

values[12–15] are designated by circles, triangles, crosses, etc., as described more fully in the figure captions. The statistical errors are indicated by vertical lines. Curves drawn by means of broken and solid lines show, respectively, values of the polarization with and without the relativistic corrections. From these figures, it may be concluded that, in view of the experimental errors of the data, the relativistic corrections to the polarizations will not be of great importance in determining phase shifts before more accurate data are available. It is also worth mentioning that the terms in Eq. (2) arising from the anomalous part of the proton magnetic moment contribute about 70% of the total corrections. Since

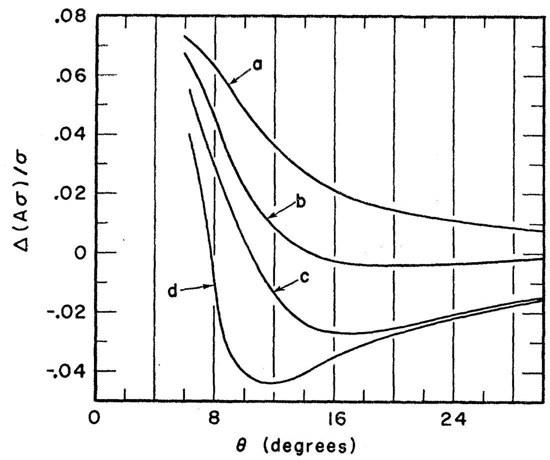

FIG. 4. Relativistic corrections for triple-scattering parameter A calculated from Eq. (5), using the phase shifts by: (a) Hull *et al.* (reference 9), set A, 310 Mev; (b) Stapp (reference 11), 310 Mev; (c) and (d) Saperstein and Durand (reference 10), 260, and 310 Mev, respectively. Nonrelativistic values of σ have been used. The center-of-mass angle θ is in degrees.

these terms have been found to be especially affected by the wave function distortion mentioned above,[4] more detailed consideration than Garren's of the effects of the anomalous moment seems necessary to derive quantitative conclusions.

Of the relativistic corrections for three triple-scattering parameters, Eqs. (3), (4), and (5), $\Delta(A\sigma)$ is the only one to be considered, the other two being entirely negligible. Also the relativistic corrections for the unpolarized cross section σ calculated from the amplitude (1) is equal to $\Delta(D\sigma)$, Eq. (3). Therefore, the nonrelativistic values of σ have been used to estimate values of $\Delta(P\sigma)/\sigma$ and $\Delta(A\sigma)/\sigma$ from $\Delta(P\sigma)$ and $\Delta(A\sigma)$. Figure 4 shows $\Delta(A\sigma)/\sigma$ for sets of phase shifts obtained by Hull *et al.*,[9] Saperstein and Durand,[10] and Stapp.[11] Except at very small angles, i.e., $\theta \lesssim 10°$, this quantity does not contribute significantly in estimates of corrections to the nonrelativistic values of A and may not be needed in a phase-shift analysis.

The author wishes to thank Professor G. Breit for suggesting this calculation and for his continued advice during its course.

[12] Chamberlain, Pettengill, Segrè, and Wiegand, Phys. Rev. **95**, 1348 (1954).
[13] D. Fischer and J. Baldwin, Phys. Rev. **100**, 1445 (1955).
[14] Chamberlain, Segrè, Tripp, Wiegand, and Ypsilantis, Phys. Rev. **105**, 288 (1957).
[15] Baskir, Hafner, Roberts, and Tinlot, Phys. Rev. **106**, 564 (1957).