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Inelastic Scattering of 4-Mev Protons and Deuterons from Separated Isotopes of Dy^{161}, Dy^{163}, Yb^{171}, and Yb^{173}

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The inelastic scattering of protons and deuterons from the odd isotopes of Dy and Yb has been measured at energies around 4 Mev in a heavy-particle spectrometer at a scattering angle of 145°. Isotopically pure targets were prepared in an electromagnetic isotope separator. Inelastic groups corresponding to the two lowest rotational states were observed in Dy^{161} at 44 kev and 102 kev, in Dy^{163} at 74 kev and 170 kev, in Yb^{171} at 67 kev and 76 kev, and in Yb^{173} at 78 kev and 181 kev. The reduced electric quadrupole transition probabilities for these levels were determined.

INTRODUCTION

WHEN a heavy element is bombarded with low-energy particles, the only inelastic events correspond to Coulomb excitation of low-lying nuclear levels.[1] In spite of the preponderance of elastic scattering it is possible to observe the groups of inelastically scattered particles in magnetic spectrometers of high resolution.[2]

This method of detection has several advantages as compared with other techniques used in Coulomb excitation experiments: The level order is directly established; the inelastic cross sections are easily obtained by comparison with the Rutherford scattering, and it is possible to use minute quantities of the target material. In the experiments reported here, only about 15 μg of each isotope were used for the target.

Until recently the Coulomb excitation of the odd isotopes of dysprosium and ytterbium has been studied only by means of natural elements.[1] Separated targets of these isotopes have been produced in the isotope separator at this Institute and their low-lying levels have been studied by means of the inelastic scattering technique.

APPARATUS

Protons and deuterons were accelerated in a 4-Mev electrostatic generator. Energy analysis was performed in a 90° deflecting magnet of 50-cm radius. The settings of the entrance slit (0.8 mm) and the exit slit (0.3 mm) were such that the energy spread in the analyzed beam at 4 Mev was less than 6 kev. The analyzed beam current reaching the target was around 0.1 microampere.

After energy analysis the beam was made to strike the target in the heavy-particle spectrometer used for the detection of the scattered particles. This spectrometer is of the uniform-field type described by Hafner, Donoghue, and Snyder,[3] and others.[4] It has a maximum radius of 36 cm and a minimum radius of 20 cm. A range of energies covering more than a factor of 3 is thus simultaneously recorded on photographic plates extending 40 cm along the focal line. The photographic plates used are Ilford C2 with an emulsion thickness of 25 or 50 μ. By means of a screw, the plateholder can be shifted laterally in the spectrograph, so that three different exposures can be made on the same set of plates without breaking the vacuum. The number of tracks in the plates is counted in a microscope as a function of the distance along the plates.

The spectrograph field is measured by means of a nuclear-induction magnetometer. Calibration of the spectrograph was carried out in a way similar to that described by Browne and Buechner,[5] using the known momenta of Bi^{212} and Po^{210} alpha particles.

A more complete description of the present installation will be given elsewhere.

[1] See Alder, Bohr, Huus, Mottelson, and Winther, Revs. Modern Phys. **28**, 432 (1956).
[2] B. Elbek and C. K. Bockelman, Phys. Rev. **105**, 657 (1957).
[3] Hafner, Donoghue, and Snyder, Brookhaven National Laboratory (privately circulated report).
[4] D. J. Prowse and W. M. Gibson, J. Sci. Instr. **33**, 129 (1956).
[5] C. P. Browne and W. W. Buechner, Rev. Sci. Instr. **27**, 899 (1955).

TARGETS

All targets for the investigation were prepared in the electromagnetic isotope separator.[6] For the separation of the rare earth elements the furnace of the ion source[7] was charged with the water-free chloride of the element.

The chlorides were prepared by dissolving the rare earth oxide in hydrochloric acid, evaporating to dryness in an excess of ammonium chloride, and fuming at 600°.[8]

One charging of the ion source required about 100 mg of the chloride. For operation of the ion source the furnace temperature was approximately 800°C. At this temperature the pressure in the discharge chamber is estimated to be 0.01 mm Hg. The separated-beam current was of the order of 30 microamperes, and the total efficiency in the separation was about 1%. A higher yield could be obtained, but only at the expense of resolution. Two different types of targets were employed. In some cases the separated beams were allowed to impinge directly onto a 0.2-mm aluminum foil. The atoms are then deposited in a thin surface layer of the aluminum. This layer is sufficiently thin to 4-Mev protons to make an excellent target in the heavy-particle spectrometer. The layer thickness is about 20 μg Al/cm^2 as determined from the line profile of elastically scattered protons. This corresponds to an energy loss of 3 kev for protons scattered in the backward direction.

These targets are easily prepared, but there are some disadvantages inherent in the use of a thick backing. To obtain a low background it is necessary to use highly pure aluminum, since common impurities such as iron, copper, and zinc will give rise to elastic scattering from the backing with a recoil energy loss that brings the particles into a region on the photographic plate where many of the inelastic groups are expected. When a thick backing is employed there will be an elastic scattering edge, and the region from this point toward decreasing particle energy is more or less obscured. Even with the very purest aluminum available to us (99.99%) this effect was troublesome when weak groups were to be observed.

We also had difficulties with the deterioration of the targets. As the proton beam is completely stopped in the backing, up to 1 watt is dissipated in the very small target spot. This is sufficient to melt the aluminum, and thereby badly damage the target.

Both these difficulties can be avoided by using very thin backings. To this purpose Formvar is in many respects an ideal material in heavy-particle spectrometry, but it is not very resistant to the beam, unless covered by a proper metal layer. A metal-covered Formvar foil can stand the proton beam well,

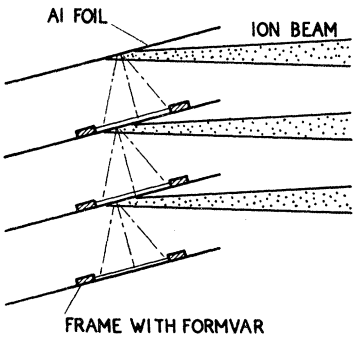

FIG. 1. Arrangement for collecting the separated isotopes sputtered from an aluminum foil.

but not the beam in the isotope separator, which is completely stopped even in a very thin foil.

To obtain a deposit of the separated isotope on a Formvar backing, the following technique was employed. The foil was first strengthened by vacuum evaporation of a thin aluminum layer; then, the sputtering technique illustrated in Fig. 1 was used. The separated beams impinge under an oblique angle on pieces of the above-mentioned pure aluminum foil. The separated material will sputter from the bombarded surface during the bombardment and finally be caught on the aluminized Formvar target which faces the beam spot, but is protected from the direct beam. Some aluminum will also sputter along with the separated material. Preliminary measurements indicate that about 5 aluminum atoms are deposited for each atom of the rare earth material. This, of course, gives an unwanted contribution to the target thickness. A detailed investigation of this method of target preparation is now under way, using heavy-particle spectrometry as an analyzing tool.

The amount of each isotope collected was of the order of 30 μg. At least 50% was deposited on the Formvar target within an area of 5×5 mm. As the dimensions of the beam spot in the heavy-particle spectrometer are only $\frac{1}{4}$×3 mm, a slight nonuniformity of the scattered material is unimportant.

The purity of the separated isotopes was not directly determined. Judging from earlier experience with the isotope separator, the abundant isotopes should contain less than 1% of the neighboring isotopes.

EXPERIMENTAL PROCEDURE

Each target was given an exposure of about 2000 microcoulombs in the heavy particle spectrometer. Since the elastic peak from such an exposure is much too intense to be counted, a short exposure of about 20 microcoulombs was made before and after each long run. These exposures were taken on the two adjacent zones on the same set of plates under exactly the same conditions as the long exposure.

Normalization was accomplished by means of the beam integrator. If significant changes in the beam conditions or the target had occurred during exposure,

[6] J. Koch, National Bureau of Standards Circular No. 522, Washington, 1956, p. 165.
[7] O. Almén and K. O. Nielsen (to be published); K. O. Nielsen (to be published).
[8] We are indebted to B. Skytte Jensen for making these preparations.

this would be shown in the intensities of the two normalization peaks.

All exposures for the present experiment were taken at 145° with respect to the beam, as this maximum obtainable angle gives the best ratio between the inelastic groups and the background of accidentally scattered particles. Apart from the already mentioned scattering from impurities in the target, this background originates mostly in scattering from slit edges in the analyzer and the spectrograph. The total number of tracks in the background amounts to less than 1% of the number of tracks in the elastic peak. Nevertheless, this background is the most serious limitation to the sensitivity of the method. For the deuteron exposures the absolute background seems systematically to be somewhat lower than for the proton exposures.

The bombarding energy chosen varied between 3.5 and 4.3 Mev, being accurately determined for each exposure from the position of several elastic peaks on the plate.

In exposures with deuterons the targets on thick backing could not be used because of a heavy background from the Al(d,p) reaction. In the case of thin backings, the protons occur in sharp groups, which are not so troublesome. It was possible to count the elastically scattered deuterons in the presence of a rather heavy proton group at the same momentum, as the deuteron tracks are then shorter and denser than the tracks of protons.

Normally the plates were only surveyed down to the aluminum elastic peak. This allows inelastic groups with an energy up to 400 kev to be observed in the proton exposures of rare earths, and up to twice that value in the exposures with deuterons. The elastic groups from contaminants such as Fe, Cu, Zn, Cl, and S found in this region could be identified as being elastic or shifted away from an inconvenient position by energy or angle shifts, but the most powerful method of identification proved to be the successive bombardment by both protons and deuterons. All energy losses due to recoil effects then change by a factor of two, whereas all inelastic losses remain unaltered.

After a preliminary scanning of the plates, the regions around the inelastic groups were carefully recounted. The intensity of the groups was then determined by integrating the number of tracks counted in 0.14-mm wide strips across the peak. In the same way the background was determined from a number of strips to either side of the group in question. The inelastic groups each contained between 100 and 1000 tracks, whereas the background to be subtracted varied between 20 and 200 tracks.

RESULTS

Assuming the validity of the Rutherford scattering law for the elastic scattering, the cross section for inelastic scattering was determined from the ratio between the inelastic and elastic groups. From the inelastic cross section the value of the reduced transition probability for $E2$ excitation[1] $B(E2)$ was readily obtained from the expression for the differential cross section for Coulomb excitation:

$$\frac{d\sigma_{E2}}{d\omega}=\frac{m^2v_f^2}{Z_2^2e^2\hbar^2}B(E2)f_{E2}(\theta,\xi),$$

$$\xi=\frac{Z_1Z_2e^2}{\hbar}\left(\frac{1}{v_f}-\frac{1}{v_i}\right),$$

where v_i and v_f are the initial and final relative velocities of the projectile, respectively, m is its reduced mass, Z_1 and Z_2 are the atomic numbers of the projectile and nucleus, respectively, and θ is the scattering angle. The function $f(\theta,\xi)$ has been calculated by Alder and Winther in the classical approximation.[1] The quantum-mechanical function f for the total cross section depends on the additional parameter $\eta=Z_1Z_2/\hbar v_i$. The difference between the classical and quantum-mechanical total function is a few percent for the bombarding conditions in the present experiment ($\eta=5$–8). One could not know *a priori* whether the differential cross section would show a larger deviation from the classical value, although measurements did not indicate any large difference for $\eta=5$.[2] For the reliability of the transition probabilities derived from the differential cross sections, it is therefore important that the quantum-mechanical angular distribution in the limit of zero energy loss ($\xi=0$) has now been evaluated for different values of η between 0 and 8.[9] The calculations show that the difference between the classical and the quantum-mechanical distribution is less than 2% for $\eta=4$, in good agreement with the difference between the corresponding total cross-section functions. In the evaluation of the transition probabilities, we have therefore felt it justified to use the classical values $f(\theta,\xi)$ and afterwards to apply a correction as calculated for the integrated function.[1]

The level schemes for the nuclei investigated are shown in Fig. 2. The spin sequences are based on the rotational description. The values of the transition probabilities and the energies are collected in Table I. The individual cases are discussed below.

Dysprosium-161

The spectrum of deuterons scattered from Dy^{161} is shown in Fig. 3. The two groups labeled (1) and (2)

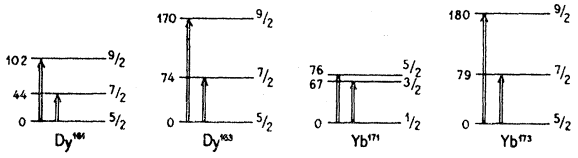

FIG. 2. Levels in the odd isotopes of Dy and Yb as observed by inelastic scattering.

[9] J. Bang (private communication). We are indebted to J. Bang for the permission to use his results prior to publication.

were identified as inelastic by comparison with the spectrum of scattered protons and also from exposures at other energies. The values for the energies and transition probabilities given in Table I are the averages of 7 exposures, including both protons and deuterons. There was no systematic difference between the values obtained with the two kinds of particles. The fluctuations in energy were ± 2 kev, and the fluctuations in $B(E2)\sim 10\%$ for the first group and $\sim 20\%$ for the second group, corresponding to the statistical fluctuations. Similar remarks apply to the other isotopes investigated.

The ratio between the energies of the two states is 2.28, in agreement with the expectation 2.29 for the two lowest members of a rotational band based on a $\frac{5}{2}$ state. Also the intrinsic quadrupole moment determined from the values of $B(E2)$ is in approximate agreement with the values for nuclei in this region. The ratio $B_2/B_1=0.27$ is a little too low compared with the rotational prediction 0.35, but not definitely outside the limits of error. It is therefore concluded that the two states are the normal rotational states observed in odd nuclei in this region.[10] The energies in the band are surprisingly low as compared with neighboring nuclei, and imply a moment of inertia for Dy^{161} about 1.6 times larger than that of a corresponding even-even nucleus. A significant increase of the moment of inertia as one goes from an even-even to an odd-A nucleus seems a systematic occurrence, but usually the effect is considerably smaller, being on the average only some 20%.[1] The additional moment of inertia of the odd-A nuclei has been ascribed to Coriolis effects in the motion of the last odd particle.[1] It is interesting to note that the effect in Dy^{161} is similar to that of the Np^{237} ground state,[11] in which the odd proton appears to occupy the same binding state as the last neutron in Dy^{161}. The especially large moment of inertia for these nuclei has been attributed to the high angular momentum components in the wave function[12] of the last particle.[11,13]

TABLE I. Coulomb excitation results for the odd isotopes of Dy and Yb obtained by detection of inelastically scattered particles. The energies E_1 and E_2 of the first and the second excited state, respectively, are given in kev. The corresponding values of the reduced electric-quadrupole-transition probability $B(E2)$ are given in units of $e^2\times 10^{-48}$ cm^4. Q_0 denotes the intrinsic quadrupole moment derived from the transition probability B_1, the unit being 10^{-24} cm^2.

Nucleus	I_0	E_1	E_2	E_2/E_1	B_1	B_2	B_2/B_1	Q_0
Dy^{161}	5/2	44	102	2.28	2.54	0.69	0.27	7.3
Dy^{163}	5/2	74	170	2.28	2.52	0.67	0.27	7.3
Yb^{171}	1/2	67	76	···	2.48	3.70	1.49	8.0
Yb^{173}	5/2	78	181	2.32	2.86	0.89	0.31	7.8

[10] These levels have also been found by N. P. Heydenburg and G. F. Pieper, using γ-ray detection, Phys. Rev. **107**, 1297 (1957).
[11] Hollander, Smith, and Rasmussen, Phys. Rev. **102**, 1372 (1956).
[12] S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **29**, No. 16 (1956).
[13] D. Bés and B. Mottelson (private communication).

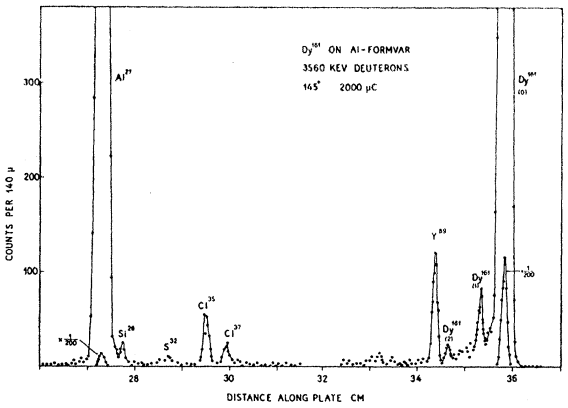

FIG. 3. Groups of deuterons scattered from a target containing Dy^{161}. The groups labeled (1) and (2) correspond to the 44- and 102-kev levels in Dy^{161}. The region around 32 cm on the plates is obscured because of a joint at this point.

The peak in the spectrum labeled Y^{89} is the elastic group due to an yttrium contamination in the target. This contamination originates in a small Y content in the charging material used in the isotope separator. Because of formation of YCl_2^+ ions in the ion source the mass numbers 159, 161, and 163 will be contaminated with Y.

Dysprosium-163

In proton exposures, the above-mentioned Y contamination would partly obscure the first group at 74 kev for all reasonable choices of angle and energy. This group has therefore only been studied in bombardments with deuterons. The numbers given in the table are the averages of three exposures. Although the ground-state spin of Dy^{163} is also $\frac{5}{2}$, the excitation energies are very different from those in Dy^{161}, because of the different intrinsic structure of the states. These states in Dy^{163} have also been seen by Heydenburg and Pieper at 74 and 166 kev.[10]

Ytterbium-171

This nucleus is known to possess spin $\frac{1}{2}$, and one therefore expects an anomalous rotational spectrum[1]

$$E=\frac{\hbar^2}{2\mathcal{I}}[I(I+1)-K(K+1)]+a\frac{\hbar^2}{2\mathcal{I}}[1+(-1)^{I+\frac{1}{2}}(I+\tfrac{1}{2})].$$

Here, I is the total angular momentum, the quantum number K equals $\frac{1}{2}$, while $\mathcal{I}$ denotes the moment of inertia and a the decoupling parameter. In agreement with this expectation, we found two low-lying inelastic groups corresponding to excitation energies of 67 and 76 kev. As the groups were not completely resolved (Fig. 4), the intensities of the two groups were determined by fitting the shape of the doublet with a sum of two groups, using the line profile of the elastic group. The intensity ratio determined was 1.49 as a mean of

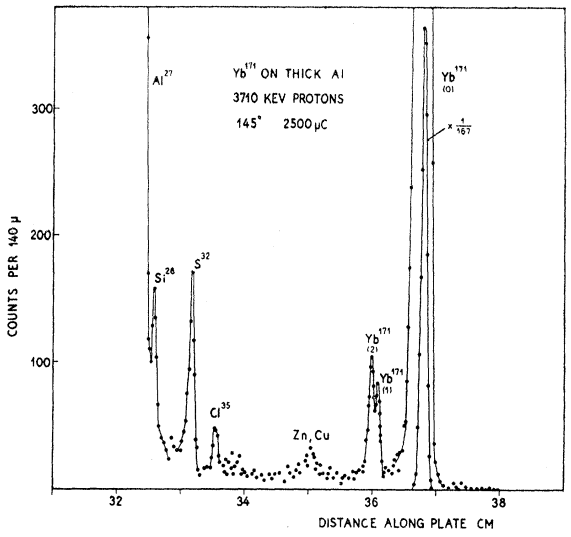

FIG. 4. Groups of protons scattered from a target containing Yb^{171}. The doublet labeled (1) and (2) corresponds to the 67- and 76-kev levels in Yb^{171}.

5 exposures using protons only. This result is in good agreement with the predicted value of 1.50. In contrast to the case for nuclei with spin $\neq\frac{1}{2}$, the level with the highest spin is here most strongly populated in Coulomb excitation. This is the basis for the spin assignments of Fig. 2.

From the measured energies one calculates the decoupling parameter $a=0.86$ and the value of $6\hbar^2/2\vartheta = 72$ kev. The experimental value of a can be compared with the value $a=0.91$ obtained from the wave functions of reference 12.

It is interesting to compare the spectrum of Yb^{171} with that of Tm^{169} which also has spin $\frac{1}{2}$.[14] The value of $6\hbar^2/2\vartheta$ is about the same for both nuclei, but the different sign of the decoupling parameter makes the spectra appear very different.

Ytterbium-173

Two inelastic groups corresponding to levels at 78 and 181 kev were observed. A level at 180 kev had earlier been reported by Temmer and Heydenburg, using natural Yb.[15] The ratio between the energies and transition probabilities corresponds to those predicted for a spin-$\frac{5}{2}$ rotational band within the stated errors.

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[14] B. R. Mottelson and S. G. Nilsson, Z. Physik **141**, 217 (1955).
[15] G. M. Temmer and N. P. Heydenburg, Phys. Rev. **100**, 150 (1956).