If the term in α^2 (or $1/T^2$) is considered, the necessary condition that it shall not vanish is that a triangular relation exist between ν, L and L' where L and L' are tensor indices appearing in (22). Thus $|L-L'|\leqslant\nu\leqslant L+L'$. If we consider $L=L'=1$ (dipole coupling), then the triangular condition is fulfilled for $\nu=2$ but not for $\nu>2$. The cross terms $L=1$, $L'=2$ and $L=2$, $L'=1$ (dipole-quadrupole cross terms) permit $\nu\leqslant 3$ and in alignment (or in α or γ emission) contribute only to the $\nu=2$ term in the angular distribution. The pure quadrupole term ($L=L'=2$) permits $\nu\leqslant 4$ and so may contribute to all terms of practical interest.[12]

ACKNOWLEDGMENTS

It is a pleasure to acknowledge rewarding conversations with Dr. L. D. Roberts, Dr. J. W. T. Dabbs, and Dr. T. A. Green.

[12] In terms of specific calculations of G_1 and G_2 most of these results were already familiar (see reference 5, for example). However, the general principles which are operative in producing these results had not been explicitly stated.

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Polarization Phenomena in the One-Quantum Annihilation of Positrons and the Photoelectric Effect*

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Cross sections are derived for the one-quantum annihilation of longitudinally-polarized positrons and for the photoelectric effect with longitudinally-polarized photons. Simple expressions, quantitatively reliable for light elements, are obtained by considering only K-shell electrons and describing the outgoing electron (incoming positron) by plane waves. In both cases, the incoming and outgoing particles have predominantly the *same* helicity if the free Dirac particle is relativistic.

In a note added in proof, these calculations are extended to the case of elliptically polarized radiation. For linearly polarized photons, the results are compared with those obtained by including lowest-order Coulomb corrections to the continuum wave function of the electron, as given by Sauter in 1931. The difference in the angular distribution is very marked, and indicates a sensitive dependence on the degree of screening of the Coulomb field.

INTRODUCTION

POSITRONS passing through matter emit radiation in flight through the processes of bremsstrahlung, two-quantum annihilation, and one-quantum annihilation with tightly-bound electrons. If the positrons are longitudinally polarized, the emitted photons will be as well, and in all three cases the higher-energy photon (if there is a choice) has predominantly the same helicity as the incoming positron; the degree of circular polarization of the radiation approaches 100% rapidly as the positron becomes relativistic.

The polarization of bremsstrahlung and two-quantum annihilation-radiation has been discussed previously[1,2]; we wish to present a simplified discussion of one-quantum annihilation and the related process, the photoelectric effect. In order to avoid such complications as those introduced by Coulomb wave functions, we shall base the derivation on the following simplifying assumptions:

(1) We assume that the outgoing electron (photoeffect) and incoming positron (annihilation) can be described with sufficient accuracy by plane waves.

(2) Since the cross sections are by far the largest for the most tightly bound electrons, we shall calculate them only for K-shell electrons. Screening is neglected, but otherwise we employ the correct relativistic wave functions for the bound electrons, in order to treat the spin effects properly.

In other words, we shall calculate the cross sections only to lowest order; this is valid for high-energy particles striking low-Z atoms, and will be at least qualitatively correct for heavier elements. ("High energy" merely means large compared to the K-shell binding energy.) We consider first the more straightforward photoelectric effect.

I. Photoelectric Effect

For the (free) outgoing electron, we define the spinors which describe states of complete longitudinal polarization by ($\hbar=c=1$)

$$(\boldsymbol{\alpha}\cdot\mathbf{p}+\beta m)u=Eu,$$
$$(\boldsymbol{\sigma}\cdot\mathbf{p}/p)u_R=+u_R,\ (\boldsymbol{\sigma}\cdot\mathbf{p}/p)u_L=-u_L. \quad (1)$$

We call the electron described by u_R a "right-electron," since its spin and momentum define a right-

* Work performed under the auspices of the U. S. Atomic Energy Commission.
[1] L. A. Page, Phys. Rev. **106**, 394 (1957).
[2] K. W. McVoy, Phys. Rev. **106**, 828 (1957). Note that reference 2 should read "Heitler, second edition" rather than "Heitler, third edition."

hand screw. For consistency, we shall use the *same* convention for longitudinally (circularly) polarized photons: we call a forward-spin photon a right-photon. Although this is a left-circularly polarized photon according to the optical convention, this should cause no confusion, for the present discussion is restricted to photon energies larger than (roughly) the electron rest-mass, where the polarization effects become significant.

If the usual representation of the Dirac matrices is used, with β (and σ_z) diagonal, a convenient choice of phases gives the spinors as

$$u_R(\mathbf{p})=[4pE(p+p_z)(E+m)]^{-\frac{1}{2}}\begin{bmatrix}(E+m)(p+p_z)\\(E+m)p_+\\p(p+p_z)\\pp_+\end{bmatrix},$$

$$u_L(\mathbf{p})=[4pE(p+p_z)(E+m)]^{-\frac{1}{2}}\begin{bmatrix}-(E+m)p_-\\(E+m)(p+p_z)\\pp_-\\-p(p+p_z)\end{bmatrix}. \quad (2)$$

The two bound-state wave functions, corresponding to $j_z=\pm\frac{1}{2}$, are

$$\varphi_1(\mathbf{r})=\begin{bmatrix}-iU\\0\\V\cos\theta\\V\sin\theta e^{i\phi}\end{bmatrix},\quad \varphi_2(\mathbf{r})=\begin{bmatrix}0\\iU\\-V\sin\theta e^{-i\phi}\\V\cos\theta\end{bmatrix},\quad (3)$$

where

$$U(r)=N(1+a)^{\frac{1}{2}}r^{a-1}e^{-\alpha mr},$$
$$V(r)=N(1-a)^{\frac{1}{2}}r^{a-1}e^{-\alpha mr},$$
$$a=(1-\alpha^2)^{\frac{1}{2}},\quad \alpha=Z/137,$$
$$N=(2\alpha m)^{a+\frac{1}{2}}[8\pi\Gamma(1+2a)]^{-\frac{1}{2}}$$

and a nonessential normalization factor has again been neglected.

In terms of these wave functions, we can take the matrix element for the process to be

$$M=\int d^3re^{i(\mathbf{q}\cdot\mathbf{r})}[\phi^*(\mathbf{r})(\boldsymbol{\alpha}\cdot\mathbf{e}^*)u(\mathbf{p})], \quad (4)$$

with $\mathbf{q}=\mathbf{p}-\mathbf{k}$.[3] We shall choose $\mathbf{k}$ in the Z direction throughout, and take the polarization vector as, e.g., $\mathbf{e}=(\mathbf{e}_x+i\mathbf{e}_y)/\sqrt{2}$ for right-photons as defined above.

The exact expressions for the matrix elements are given in the appendix. However, since our approach has already neglected α^2 corrections, we shall retain only terms of lowest (first) order in α; where the expression $(\alpha^2m^2+q^2)$ occurs, though, we shall not set $\alpha^2m^2=0$, since small momentum transfers can be significant, especially in the forward direction. The matrix elements for states of complete polarization are then:

$$M_{RR}^{(1)}=M_{LL}^{(2)*}=\frac{4\pi\alpha iN}{[pE(E+m)]^{\frac{1}{2}}}\frac{(E+m)(p+p_z)p_-}{(\alpha^2m^2+q^2)^2(p+p_z)^{\frac{1}{2}}},$$
$$M_{LR}^{(1)}=M_{RL}^{(2)*}=\frac{4\pi\alpha iN}{[pE(E+m)]^{\frac{1}{2}}}\frac{[2mp+(E+m)(p_z-k_z)]p_+}{(\alpha^2m^2+q^2)^2(p+p_z)^{\frac{1}{2}}}, \quad (5)$$

$$M_{LL}^{(1)}=-M_{RR}^{(2)*}=-\frac{4\pi\alpha iN}{[pE(E+m)]^{\frac{1}{2}}}\frac{[2mp-(E+m)(p_z-k_z)](p+p_z)}{(\alpha^2m^2+q^2)^2(p+p_z)^{\frac{1}{2}}},$$
$$M_{RL}^{(1)}=-M_{LR}^{(2)*}=-\frac{4\pi\alpha iN}{[pE(E+m)]^{\frac{1}{2}}}\frac{(E+m)(p_-)^2}{(\alpha^2m^2+q^2)^2(p+p_z)^{\frac{1}{2}}}, \quad (6)$$

where the first subscript refers to photon and the second to outgoing electron, and the superscript specifies the initial electron state as φ_1 or φ_2. (In our coordinate system, $k_z=k$, but using the component notation is necessary for the transformation to the annihilation matrix element.) The simple phase relations between matrix elements with opposite spins is a special case of Lenard's theorem.[4]

Remembering the random relative phase between φ_1 and φ_2, which eliminates 1–2 cross terms in the cross section, we may define

$$|M_{AB}|^2\equiv|M_{AB}^{(1)}|^2+|M_{AB}^{(2)}|^2,$$

and we note from (5) and (6) that

$$|M_{RR}|^2=|M_{LL}|^2,\quad |M_{RL}|^2=|M_{LR}|^2, \quad (7)$$

i.e., that there are only two rather than four distinct cross sections, one in which photon and electron spins are "like" and one in which they are "unlike." Consequently we shall restrict further discussion to RR ($=LL$) and RL ($=LR$) cross sections.

The differential cross sections, including a factor 2

[3] To conform with the conventions of references 1 and 2, we are employing the notation of the second edition of *The Quantum Theory of Radiation* by W. Heitler. That is, the initial state is written on the left, and $\boldsymbol{\alpha}\cdot\mathbf{A}^*$ is the *absorption* operator for photons. Heitler's third edition agrees with other modern field-theory texts in reversing these conventions.

[4] Andrew Lenard, Phys. Rev. **107**, 1712 (1957).

for the two K electrons, are

$$\frac{d\sigma_{RR}}{d\Omega}=\frac{C}{(1-b\cos\theta)^4}\times[B_+ +(B_+ - D_+)\cos\theta - D_+\cos^2\theta],$$
$$\frac{d\sigma_{RL}}{d\Omega}=\frac{C}{(1-b\cos\theta)^4}\times[B_- -(B_- - D_-)\cos\theta - D_-\cos^2\theta], \quad (8)$$

where θ is the angle between $\mathbf{k}$ and $\mathbf{p}$, and the coefficients are given by

$$C=\frac{4Z^5 r_0^2}{(137)^4}\frac{m^5}{k(E+m)(p^2+k^2+\alpha^2m^2)^4},$$
$$b=2kp/(p^2+k^2+\alpha^2m^2)\approx\beta,$$
$$A_\pm=2mp\pm k(E+m),\qquad B_\pm=p(A_\pm)^2+p^3(E+m)^2,$$
$$D_\pm=2p^2(E+m)A_\pm,\quad r_0=e^2/m. \quad (9)$$

We note that the cross section for unlike spins is zero for electrons emitted directly forward, and that for like spins is zero for backward electrons. Since almost all electrons are emitted forward if the photon energy is larger than the electron's rest-mass, the electron in this case will have predominantly the *same* helicity as the photon.

The total cross sections are

$$\sigma_{RR}=\frac{2C}{3(1-b^2)^3}\times[(3+b^2)B_+ +4b(B_+ - D_+)-(1+3b^2)D_+],$$
$$\sigma_{RL}=\frac{2C}{3(1-b^2)^3}[(3+b^2)B_- -4b(B_- - D_-)-(1+3b^2)D_-]. \quad (10)$$

These cross sections depend on Z in two essential ways. They contain α^2m^2 in b, and the ionization potential I in the relation between k and p,

$$k=E-m+I.$$

In neither case is the Z dependence significant for photon energies above a few hundred kev, so to indicate roughly the trend of the polarization in a Z-independent fashion, we have plotted in Fig. 1 the asymmetry ratio $(\sigma_{RR}-\sigma_{RL})/(\sigma_{RR}+\sigma_{RL})$, setting both αm and I equal to zero. Although the validity of the curve for small k is questionable, it reliably predicts the rapid rise toward 100% polarization as the outgoing electron becomes relativistic.

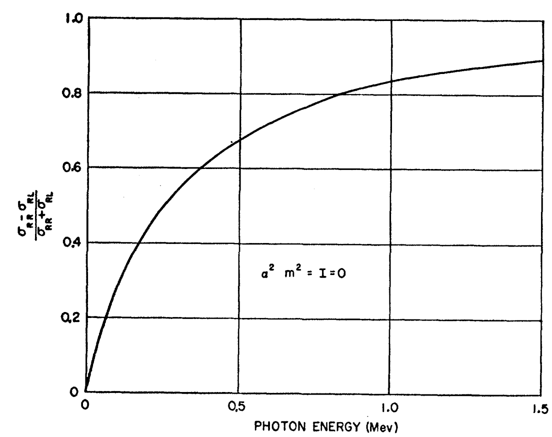

FIG. 1. Asymmetry ratio of the total cross sections for the photoelectric effect from K-shell electrons.

II. One-Quantum Annihilation of Positrons

The matrix element for one-quantum annihilation is related to that for the photoeffect by the substitution law; an outgoing electron becomes an incoming positron, and the photon changes from incoming to outgoing. An incoming positron can be described as an outgoing negative-energy electron, and the wave function which describes a right-positron of energy and momentum E and $\mathbf{p}$ is just $e^{-i\mathbf{p}\cdot\mathbf{r}}u_R(-E,-\mathbf{p})$, from Eq. (2). Remembering that the photon is now outgoing, we get for the matrix element in this case

$$M^A=\int d^3r e^{-i(\mathbf{p}-\mathbf{k})\cdot\mathbf{r}}[\varphi^*(\mathbf{r})(\boldsymbol{\alpha}\cdot\mathbf{e})u(-\mathbf{p},-E)]. \quad (11)$$

That for the photoeffect was

$$M^P=\int d^3r e^{i(\mathbf{p}-\mathbf{k})\cdot\mathbf{r}}[\varphi^*(\mathbf{r})(\boldsymbol{\alpha}\cdot\mathbf{e}^*)u(\mathbf{p},E)], \quad (4)$$

so M^A is evidently obtainable from M^P by the substitutions

$$\mathbf{k}\to-\mathbf{k},\quad \mathbf{p}\to-\mathbf{p},\quad E\to-E,$$

and right-photon $\to$ left-photon (but right-electron $\to$ right-positron). It is worth remarking that, in spite of this formal substitution, the photon still travels along the positive Z axis. Thus $p_Z\to-p_Z$, so $\cos\theta\to-\cos\theta$ in the numerator of Eq. (8), but since $q^2\to q^2$, the denominator remains unchanged.

Explicitly, the changes in the cross sections are as follows:

$$b\to b \text{ (but now } k=E+m-I)$$
$$A_\pm\to F_\pm=2mp\pm k(E-m),$$
$$B_\pm\to G_\pm=p(F_\pm)^2+p^3(E-m)^2, \quad (12)$$
$$D_\pm\to H_\pm=-2p^2(E-m)F_\pm.$$

The differential cross sections for the two K-shell elec-

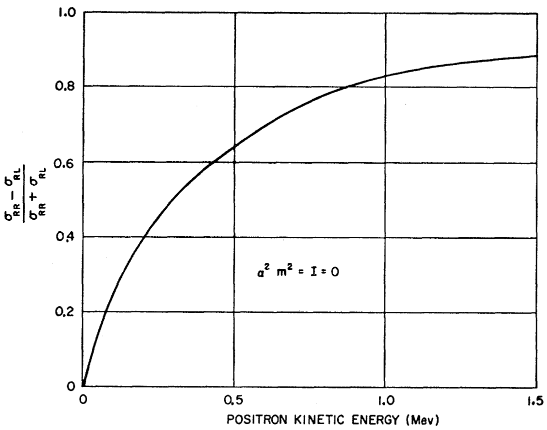

FIG. 2. Asymmetry ratio of the total cross sections for one-quantum annihilation with K-shell electrons.

trons are

$$\frac{d\sigma_{RR}}{d\Omega}=\frac{C(k^2/p^2)}{(1-b\cos\theta)^4}\times[G_-+(G_--H_-)\cos\theta-H_-\cos^2\theta),$$
$$\frac{d\sigma_{RL}}{d\Omega}=\frac{C(k^2/p^2)}{(1-b\cos\theta)^4}\times[G_+-(G_+-H_+)\cos\theta-H_+\cos^2\theta], \tag{13}$$

where the subscripts refer to photon and positron, and their order is immaterial. The total cross sections are

$$\sigma_{RR}=\frac{2C(k^2/p^2)}{3(1-b^2)^3}\times[(3+b^2)G_-+4b(G_--H_-)-(1+3b^2)H_-],$$
$$\sigma_{RL}=\frac{2C(k^2/p^2)}{3(1-b^2)^3}\times[(3+b^2)G_--4b(G_+-H_+)-(1+3b^2)H_+]. \tag{14}$$

Again these expressions have the same Z dependence as Eq. (10); it is insignificant for energies above a few hundred kev for the same reason, and Fig. 2 gives the asymmetry ratio for the annihilation-radiation from polarized positrons, with $\alpha^2m^2=I=0$. The asymptotic form of this ratio, for high-energy positrons, is given by

$$(\sigma_{RR}-\sigma_{RL})/(\sigma_{RR}+\sigma_{RL})\approx 1-(11/6)(m/p)^2.$$

In the same way as before, the photons have predominantly the same helicity as the positrons when the positrons are relativistic.

If the positrons are themselves not completely polarized, they will be described by a spinor which we may write as

$$u=au_R+be^{i\phi}u_L, \tag{15}$$

with a and b real. Instead of the simple matrix elements given above, we would then get, e.g., for the emission of right-photons,

$$M=aM_{RR}+be^{i\phi}M_{LR}. \tag{16}$$

If the beam of positrons is such that ϕ (which is actually the azimuthal angle of the plane defined by $\mathbf{p}$ and $\langle\boldsymbol{\sigma}\rangle$) is fixed, the cross section will contain a cross-term proportional to $e^{i\phi}$, but if ϕ is random, this cross-term averages to zero. This latter situation describes the positrons coming from β decay, if no other direction (i.e., recoil momentum or spin direction of the parent nucleus) is measured. Consequently for this case the expression

$$|M|^2=a^2|M_{RR}|^2+b^2|M_{RL}|^2 \tag{17}$$

gives the cross section for right-photons from positrons in an arbitrary state of longitudinal polarization.

ACKNOWLEDGMENTS

I wish to express my appreciation to Professor F. J. Dyson, and to Dr. Adam Bincer, Dr. Feza Gürsey, and Dr. Michael Moravcsik for many stimulating discussions.

APPENDIX

The exact matrix elements, defined by Eq. (4), can be expressed as follows. Let $\alpha m=l$, and define the functions

$$S(a,l,q)=\frac{4\pi\Gamma(a+1)(1+a)^{\frac{1}{2}}}{q(l^2+q^2)^{(a+1)/2}}\times\sin[(1+a)\tan^{-1}(q/l)],$$
$$T(a,l,q)=-\frac{4\pi\Gamma(a)(1-a)^{\frac{1}{2}}}{q^2(l^2+q^2)^{(a+2)/2}}\{alq\cos[a\tan^{-1}(q/l)]-[l^2+(1+a)q^2]\sin[a\tan^{-1}(q/l)]\}, \tag{A-1}$$
$$v=N/[pE(p+p_z)(E+m)]^{\frac{1}{2}},$$

where N is given in Eq. (3). The $\mathbf{r}$ integration then gives the matrix elements as

$$M_{RR}^{(1)}=M_{LL}^{(2)*}=ivT(E+m)(p+p_z)q_-/q,$$
$$M_{LR}^{(1)}=M_{RL}^{(2)*}=ivp_+[pS+(E+m)Tq_z/q], \tag{A-2}$$

$$M_{LL}^{(1)}=-M_{RR}^{(2)*}=-iv(p+p_z)\times[pS-(E+m)Tq_z/q], \tag{A-3}$$
$$M_{RL}^{(1)}=-M_{LR}^{(2)*}=-ivT(E+m)p_-q_-/q.$$

To first order in α,

$$S\approx[4\sqrt{2}\pi\alpha/(\alpha^2m^2+q^2)^2]2m,$$
$$T\approx[4\sqrt{2}\pi\alpha/(\alpha^2m^2+q^2)^2]q. \tag{A-4}$$

Using these approximations, and remembering that $q_\pm=p_\pm$ in the coordinate system being used, we get the approximate expressions given in Eqs. (5) and (6).

NOTE ADDED IN PROOF.—ELLIPTICALLY POLARIZED RADIATION AND UNPOLARIZED ELECTRONS (POSITRONS)

The above cross sections were calculated for circularly-polarized photons. It was subsequently suggested[5] that there might be considerable interest in the form they would take for elliptically polarized photons. In the case of linearly polarized photons the differential cross sections given by this "plane wave approximation" are found to differ quite markedly from the results given by Sauter,[6] who included lowest order corrections from the continuum Coulomb function. Since the actual physical situation lies somewhere between the extremes represented by these two approaches, it seemed worthwhile to include a brief description of the ways in which they differ.

In order to generalize our expressions to the case of photons of an arbitrary degree of elliptical polarization, we shall describe the photon's polarization by the three real and positive numbers a_R, a_L, and δ, by writing the polarization vector as

$$\mathbf{e}=a_R e^{2i\delta}\mathbf{e}_R+a_L\mathbf{e}_L, \tag{B-1}$$

with $\mathbf{e}_{R,L}=(\mathbf{e}_x\pm i\mathbf{e}_y)/\sqrt{2}$ if the photon travels up the z axis. The photon's polarization state is described by only two independent constants because of the normalization, $a_R^2+a_L^2=1$; in terms of the polarization ellipse, they are $(a_R^2-a_L^2)$, the eccentricity, and δ, its orientation angle about the propagation direction.

A. Photoelectric Effect

Since it is most convenient experimentally not to distinguish between R and L electrons, we shall sum the cross section over electron spin states. For circularly-polarized photons, as we saw above,

$$d\sigma/d\Omega=d\sigma_{RR}/d\Omega+d\sigma_{RL}/d\Omega=d\sigma_{LR}/d\Omega+d\sigma_{LL}/d\Omega, \tag{B-2}$$

and the result is independent of the sense of the photon's polarization. In the general case, keeping only the lowest order in $(Z/137)$, the cross section for the two K-shell electrons is (setting $k=E-m$, $p_x=p\sin\theta\cos\varphi$)

$$\frac{d\sigma_K}{d\Omega}=r_0^2\frac{Z^5}{(137)^4}\left(\frac{m^5p}{E^4k^4}\right)(1-\beta\cos\theta)^{-4}$$
$$\times\{2(1-2a_Ra_L)m(E+m)\sin^2\theta+E(E-m)(1-\beta\cos\theta)$$
$$+8a_Ra_Lm(E+m)\sin^2\theta\cos^2(\varphi-\delta)\}. \tag{B-3}$$

Two interesting conclusions can be drawn from this expression. (1) As in the case of pure circular polarization, the cross section is independent of the sense of polarization. From a measurement of the symmetric product (a_Ra_L) we can determine only $|a_R^2-a_L^2|$, i.e., the *degree* of circular polarization, but not its sign. (2) Even the measurement of (a_Ra_L) depends on the existence of the azimuthal term. If the beam comes from an unpolarized source so that δ, the orientation of the polarization ellipse, is averaged over, the (a_Ra_L) terms disappear, and we are left with just Eq. (B-2), which contains no information about the photon polarization. If, however, δ *is* a constant of the beam (meaning that a plane of polarization exists, in the limiting case of plane polarization), a measurement of the azimuthal asymmetry of the photoelectron intensity enables one to determine it. This could conceivably be of some use in experiments involving polarized γ emitters (e.g., a β-decay daughter nucleus if the parent was polarized), for it provides a method of measuring the degree of polarization of the source.

For the special case $a_R=a_L=1/\sqrt{2}$, we get the cross section for plane polarized photons:

$$\frac{d\sigma_K}{d\Omega}=r_0^2\frac{Z^5}{(137)^4}\left(\frac{m^5p}{E^4k^4}\right)(1-\beta\cos\theta)^{-4}$$
$$\times\{E(E-m)(1-\beta\cos\theta)+4m(E+m)\sin^2\theta\cos^2(\varphi-\delta)\}. \tag{B-4}$$

Although Archibald[7] has apparently discussed Eq. (B-4) previously, it appears that the only other published calculation of the relativistic photoeffect is that given by Sauter.[6] As Sommerfeld[8] showed explicitly, Sauter's calculation differs from our "plane wave approximation" by assuming that the outgoing electron sees a pure Coulomb field and including to first order in $(Z/137)$ the corresponding correction to its wave function. It should be noted that this modifies our calculation by terms of the *same order* in $(Z/137)$ as we have already kept, so that, insofar as the electron sees a pure Coulomb field, Sauter's calculation provides the more consistent expansion in $(Z/137)$. Since the actual field seen however, is a *screened* Coulomb field, it is not clear offhand whether the plane wave approximation or the Sauter approximation is the more realistic one. Although they agree in the low-energy limit, the differential cross sections are very different at high energies. In particular, Sauter's is zero at $\theta=0$ and $\theta=\pi$, while the plane wave approximation is not, and Sauter's predicts that at high energies the maximum photoelectron intensity is *normal* to the photon polarization plane, while the plane wave approximation predicts it to be *in* the polarization plane at all energies. These differences have been noticed by other workers, and although recent experiments seem to indicate that the electron intensity is not zero in the forward and backward directions,[9] they give conflicting results on the azimuthal distribution.[10]

The total cross section is

$$\sigma_K=\frac{4\pi}{3}r_0^2\frac{Z^5}{(137)^4}\left(\frac{mp}{k^4}\right)(3E^2+mE+4m^2). \tag{B-5}$$

Peculiarly enough, this agrees with Sauter in *both* the low and high-energy limits, but is larger at intermediate energies; its maximum ratio to Sauter's cross section is 1.5 at $k=m$.

B. One-Quantum Annihilation of Positrons

An exactly analogous argument gives the annihilation cross section for an *unpolarized* positron beam

$$\frac{d\sigma_K}{d\Omega}=r_0^2\frac{Z^5}{(137)^4}\left(\frac{m^5}{E^4k^2p}\right)(1-\beta\cos\theta)^{-4}\{E(E+m)(1-\beta\cos\theta)$$
$$-8a_Ra_Lm(E-m)\sin^2\theta\cos^2(\varphi-\delta)$$
$$-2(1-2a_Ra_L)m(E-m)\sin^2\theta\}. \tag{B-6}$$

For linearly-polarized light, $(\varphi-\delta)$ is the angle between the polarization vector and the $(\mathbf{pk})$ plane; because of the negative coefficient of $\cos^2(\varphi-\delta)$, the cross section is largest when the polarization vector is *normal* to the $(\mathbf{pk})$ plane. In both the high- and low-energy limits the dominating term is $E(E+m)(1-\beta\cos\theta)$, so that there is no φ dependence in either limit.

The total cross section is

$$\sigma_K=\frac{4\pi}{3}r_0^2\frac{Z^5}{(137)^4}\left(\frac{m}{k^2p}\right)(3E^2-mE+4m^2). \tag{B-7}$$

Again this is in disagreement with Heitler's[11] nonrelativistic expression, and Bhabha and Hulme's[12] relativistic one, which was based on Sauter's calculation.

It is clear from these comparisons that screening may have a very marked effect on these calculations, and would seem to be worthy of a more detailed investigation.

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[5] I am indebted to Professor V. L. Telegdi for a stimulating discussion on this point.

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[7] W. J. Archibald (unpublished); see reference in W. McMaster and F. Hereford, Phys. Rev. **95**, 723 (1954).

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[12] H. R. Hulme and H. J. Bhabha, Proc. Roy. Soc. (London) **146**, 723 (1934).