Radiation Widths of Levels in Nuclei near Closed Shells*†

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An experimental investigation has been made of the variations in the radiation widths of nuclear energy levels for isotopes in the regions of the neutron magic numbers, where fluctuations in level spacing and neutron binding energy are largest. Neutron transmission measurements were made using the Brookhaven fast chopper, and the Breit-Wigner level parameters were obtained for resonances in the following target isotopes: Sr^{87}, Sb^{121}, Sb^{123}, Ba^{135}, La^{139}, Nd^{145}, and Pt^{195}. Other elements around closed neutron shells, namely, Rb, Zr, Nb, Ce, and Tl, were examined but no accurate measurements could be made of the radiation widths for these elements. Results show that the fluctuations in the measured radiation widths are small compared to the large fluctuations in neutron scattering widths, and that they are related to the level spacing and the effective level excitation energy. An effect arising from the closed shells at 82 and 126 neutrons is observed. Analysis of all the available "good" data indicates that the essential features of the theory of Blatt and Weisskopf are generally valid; that is, radiation widths are strongly dependent on the effective level excitation energy and weakly dependent on the level spacing. Experimental results are compared with predictions from semiempirical formulas.

I. INTRODUCTION

THE improved resolution of neutron spectrometers has made it possible to obtain good measurements of the parameters of the highly excited states of compound nuclei formed by the interaction with neutrons of known energy. These measured values of the widths and spacings of levels have been compared with the predictions of nuclear theories. In particular, the ratio of the average reduced neutron width to level spacing $\bar{\Gamma}_n^0/D$, the radiation width Γ_γ, the fission width Γ_f, the size distribution of all these widths, and the level spacing distribution are of current interest. In this paper, we confine ourselves to a study of the dependence of Γ_γ on the properties of the compound nucleus. It is worthwhile to review briefly the increase of interest in radiation widths which has accompanied the accumulation of data.

An early estimate by Heidmann and Bethe[1] indicated a slow decrease of Γ_γ with increasing mass number of the target nucleus. The first comprehensive summary of measured radiation widths obtained from slow neutron resonances was given by Hughes and Harvey.[2] The outstanding feature which they observed was the relative constancy of radiation widths in spite of wide variations in mass number, level spacing and spin among the isotopes. This is in contrast to the observed behavior of neutron scattering widths, which show fluctuations by factors of at least several hundred. The most complete summary of measured radiation widths has been given by Levin and Hughes.[3] They have observed an increase in Γ_γ in levels of isotopes near $A=208$, and they indicate that the variations in Γ_γ can be correlated with variations in the neutron binding energy and spacing of the levels through the theoretical estimate of Blatt and Weisskopf.[4] In this theory, a modified independent-particle model is used to estimate the partial radiation widths which are then summed by using a statistical model for the level density. Measurements of radiation widths to a high degree of accuracy have been made by Landon and Igo.[5] By considering only measurements of Γ_γ made to a precision of 15% or better, Landon has indicated that deviations from the general trend expected on the basis of a statistical model occur. Recently, Cameron[6] has developed an expression for Γ_γ by using the new level-density formula of Newton.[7] Since this theory takes the effects of nuclear shell structure into account, it is directly applicable to the results of this investigation.

We have examined the neutron resonances in several isotopes which lie near the "magic" numbers 50, 82, and 126 neutrons, where the neutron binding energy and the level spacing show their widest variations. Good measurements of Γ_γ could be obtained only when radiative capture predominated over neutron scattering. We have also attempted in this report to bring up to date the rapidly accumulating data on radiation widths.

II. ANALYSIS OF DATA

The measurements were made with the Brookhaven fast chopper, which has been previously described.[8] A 20-meter neutron flight path was used. Transmission experiments were performed to observe low-energy

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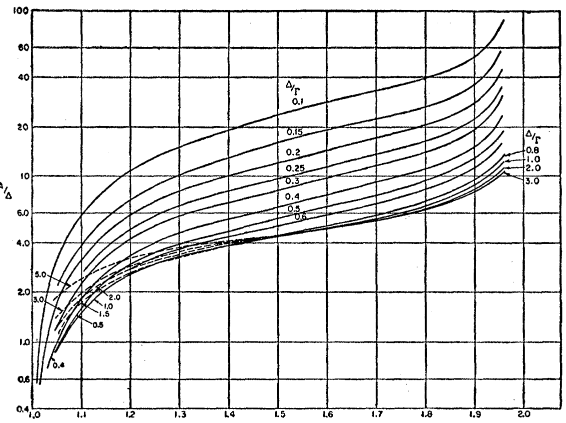

FIG. 1. Dependence of the power P in $\sigma_0\Gamma^P$ on the area above a transmission dip and the Doppler width.

resonances when samples are placed in the neutron beam. Most of the resonances were analyzed by measuring the area included between the transmission dip and the transmission due to potential scattering using samples of different thicknesses. This yields sufficient information to allow us to compute the Breit-Wigner level parameters. Due account was taken of the Doppler broadening of the resonances and of the area neglected in the resonance wings. This procedure gives values for the combination of parameters $\sigma_0\Gamma^P$, where the power P varies between 1 for a very thin sample ($n\sigma_0 \ll 1$) and 2 for a very thick sample ($n\sigma_0 \gg 1$). The symbols here have the usual meanings: Γ is the total width, σ_0 is the peak cross section in barns, and n is the sample thickness in atoms/barn.

It was felt that the new simplified method[9] of analysis was not appropriate when one wishes to combine the results obtained by using more than two sample thicknesses to obtain the best possible result. Instead, we have used a method essentially the same as previously described,[8] but we have made use of the family of curves shown in Fig. 1. These curves show how the power P in $\sigma_0\Gamma^P$ depends on the area A above the transmission dip and on the Doppler width Δ. The powers P plotted here were obtained from curves of the type given in Fig. 1 of the paper by Harvey *et al.*,[10] in which the quantity $\pi^{\frac{1}{2}}\Gamma/A$ is plotted against $n\sigma_0$ with Δ/Γ as the family parameter. The slope obtained from the latter curves for a given choice of A, Δ, and a small range of Γ gives the power of Γ associated with these quantities. The use of Fig. 1 is a time saver in the analysis of resonances. The simultaneous solution of a pair of equations involving $\sigma_0\Gamma^P$ obtained from thin and thick sample measurements then yields values for Γ and σ_0. In most of our measurements, more than two sample thicknesses were used so that we had sets of overdetermined systems of equations which were combined by a least-squares method[11] to obtain the best solutions and their associated errors.

In the isotopes which we have examined, radiative capture and neutron scattering are the only competitive processes. The radiation width is then found simply by subtracting the scattering width from the total width. The scattering width is found from the relation

$$\Gamma_n = \sigma_0\Gamma/(4\pi\lambda\!\!\!^{-}{}_0^2 g), \tag{1}$$

obtained from the Breit-Wigner formulas. In this relation $2\pi\lambda\!\!\!^{-}{}_0$ is the neutron wavelength at the resonance energy, and g is the statistical weight factor given by

$$g = \tfrac{1}{2}\left[1 \pm \frac{1}{2I+1}\right], \tag{2}$$

where I is the target-nucleus spin. Generally g is not known and is taken to be $\frac{1}{2}$, which usually introduces less uncertainty than the experimental errors involved. For nuclei with $I=0$, g is unity. Since the radiation width is determined as $\Gamma_\gamma = \Gamma - \Gamma_n$, the uncertainty in g is usually a small part of the error in Γ_γ when $\Gamma_\gamma \gg \Gamma_n$. Only when this latter condition is fulfilled is it possible to make good measurements of Γ_γ by means of transmission experiments alone.

III. RESULTS

The radiation widths of ten resonances in nuclei near the closed neutron shells were measured, and they are listed in Table I. The areas of measured transmission dips and their corresponding sample thicknesses are listed together with the values for Δ, P, Γ, Γ_n, and Γ_γ obtained from the analysis of the data. Three types of samples were used: powders, metal foils, and solutions. These provide the wide range of sample thicknesses which are necessary for the analysis of resonances by the area method. Powder samples were placed in special holders previously described.[12] The identification of the isotope responsible for each resonance was made by comparison of the size of the transmission dip produced with normal and isotopically enriched samples obtained

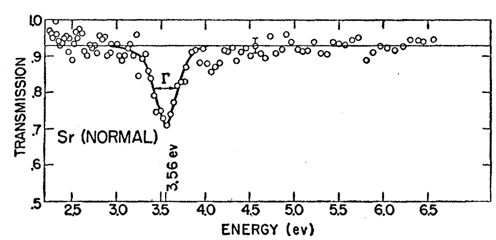

FIG. 2. The transmission dip for the 3.56-ev resonance due to the target nucleus Sr^{87} for a target with 3.49×10^{20} atoms/cm^2 of Sr^{87}.

[9] D. J. Hughes, J. Nuclear Energy 1, 237 (1955); Pilcher, Harvey, and Hughes, Phys. Rev. **103**, 1342 (1956).
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[12] F. G. P. Seidl, Brookhaven National Laboratory Report BNL-278, 1954 (unpublished).

TABLE I. Experimental data and results on 10 resonances. The areas include wing corrections. All widths are expressed in millielectron volts.

Target nucleus	E_0 (ev)	Δ (ev)	Area (ev)	n (atoms/barn)	P	$\Gamma(10^{-3}$ ev)	$\Gamma_n(10^{-3}$ ev)	$\Gamma_\gamma(10^{-3}$ ev)
${}_{38}Sr^{87}$	3.56	0.069	0.099±0.010	0.000349	1.06	206±20	0.60±0.09	205±20
			0.491±0.025	0.00162	1.46			
${}_{51}Sb^{121}$	6.24	0.073	0.149±0.015	0.000239	1.14	64±9	2.7 ±0.3	61±9
			0.294±0.018	0.000545	1.45			
			1.06 ±0.04	0.00965	1.98			
${}_{51}Sb^{121}$	15.5	0.115	0.148±0.016	0.000239	1.09	115±20	6.0 ±0.6	109±20
			0.330±0.026	0.000545	1.24			
			1.38 ±0.08	0.00965	1.96			
${}_{51}Sb^{121}$	29.7	0.158	0.159±0.024	0.000545	1.05	128±40	4.9 ±0.5	123±40
			1.00 ±0.08	0.00964	1.75			
${}_{51}Sb^{123}$	21.6	0.134	0.311±0.028	0.000179	1.17	111±20	25 ±3	86±20
			0.560±0.045	0.000407	1.45			
			2.01 ±0.14	0.00722	1.97			
${}_{56}Ba^{135}$	24.5	0.136	0.221±0.015	0.000316	1.11	124±17	9.8 ±0.5	114±17
			0.395±0.024	0.000653	1.25			
			0.83 ±0.12	0.00171	1.76			
			1.16 ±0.06	0.00582	1.89			
${}_{57}La^{139}$	73.5	0.23	0.57 ±0.06	0.00118	1.18	177±30	27 ±3	150±30
			0.77 ±0.09	0.00177	1.31			
			1.19 ±0.12	0.00645	1.65			
			2.63 ±0.21	0.0239	1.96			
${}_{60}Nd^{145}$	4.37	0.056	0.093±0.005	0.000243	1.11	49±5	1.28±0.04	48±5
			0.123±0.006	0.000265	1.16			
			0.145±0.012	0.000313	1.21			
			0.265±0.013	0.000974	1.58			
			0.685±0.027	0.00710	1.96			
${}_{78}Pt^{195}$	11.9	0.079	0.131±0.009	0.0000538	1.11	165±20	$J=1$, 10.8±0.7	$J=1$, 154±20
			0.702±0.028	0.000552	1.78		$J=0$, 32 ±2	$J=0$, 133±20
${}_{78}Pt^{195}$	19.6	0.101	0.197±0.022	0.000164	1.14	112±20	$J=1$, 9.4±0.8	$J=1$, 103±20
			0.924±0.037	0.00231	1.90		$J=0$, 28.2±2.4	$J=0$, 84±20

from Oak Ridge National Laboratory. Some data were also obtained for resonances not listed in Table I, and they will be described below. We have not included transmission curves for all the isotopes measured in this report, but cross-section plots for most of this work have been included in the neutron cross section compilation.[13]

A. Strontium

The transmission dip due to the 3.56-ev resonance in the target nucleus Sr^{87} is shown in Fig. 2. The sample used here was normal $SrCO_3$ powder, with 3.49×10^{20} atoms/cm² of Sr^{87}. This resonance occurs in a region where the instrumental resolution is sufficiently good for an analysis in which Γ is obtained directly from the observed width of the resonance. The ratio of the combined Doppler and resolution width to the true resonance width is 0.37. The correction curve given by Seidl *et al.*[8] then indicates that Γ is obtained by dividing the observed width by 1.3, giving a total width Γ of 0.206±0.020 ev. This result cannot be obtained from area analysis of the data given in Table I since these were both thin samples. The peak cross section was similarly obtained from the observed value, and gave $\sigma_0=1240\pm120$ barns. Another resonance in Sr^{87} was observed at about 620 ev.

B. Antimony

The first four resonances which appear in the antimony spectrum have been analyzed. Sb_2O_3 powder was used for the thick samples. In order to obtain samples which were thin enough (i.e., P close to unity and hence $n\sigma_0\ll1$), a small amount of $SbCl_3$ was dissolved in HCl. These thin-sample runs required more chopper time since the combined effect of the hydrogen, chlorine, and quartz glass walls of the cell containing the solution resulted in the loss of $\frac{3}{4}$ of the neutrons. We have taken our isotopic assignments from the work of Palmer and Bollinger.[14] Our results for the 6.24-ev resonance in Sb^{121} are in agreement with results obtained by these authors. It is interesting to compare the three radiation widths due to the Sb^{121} isotope which are listed in Table I. The 15.5- and 29.7-ev levels have similar values of Γ_γ, while the 6.24-ev level has a somewhat smaller Γ_γ. This indicates that variations in Γ_γ which are larger than experimental errors are possible for levels in the same isotope.

C. Barium

The transmission dips for the 24.5-ev resonance in Ba^{135} are given for thin and thick target samples in Fig.

[13] D. J. Hughes and J. A. Harvey, *Neutron Cross Sections*, Brookhaven National Laboratory Report BNL-325 (Superintendent of Documents, U. S. Government Printing Office, Washington, D. C., 1955); and D. J. Hughes and R. B. Schwartz, Suppl. 1 to BNL 325, 1957.

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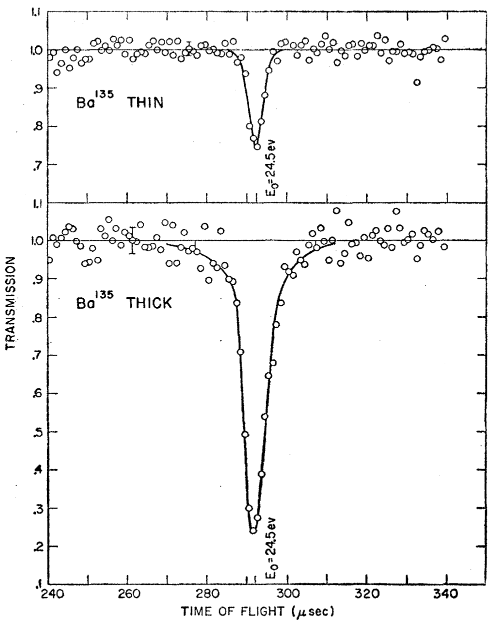

FIG. 3. Transmission dips for the 24.5-ev resonance due to the target nucleus Ba^{135} for targets with 3.16×10^{20} and 5.82×10^{21} atoms/cm^2 of Ba^{135}, respectively. The transmissions have been normalized to unity in the wings. The time of flight refers to a 20-meter flight path.

3 as an illustration of a typical set of experimental curves. These correspond to the thinnest and thickest samples listed in Table I for Ba^{135}. In these curves, the transmission in the wings has been normalized to unity. The samples used were $BaCO_3$ powders, both normal and isotopically enriched in Ba^{135} to 58.2%. Additional resonances due to Ba^{135} were observed at 82, 88, and 106 ev. A resonance at 103 ev is probably due to Ba^{136}. Data on these higher energy resonances and additional measurements will be published in a future paper.[15]

D. Lanthanum

A resonance at 73.5 ev was observed for the target isotope La^{139}. This was the only level observed for a range of neutron energies from 3 ev to 1000 ev. Since this nucleus is at a closed neutron shell, the large level spacing is not surprising. All the samples were normal La_2O_3 powder. This resonance cannot be due to the low abundant target isotope La^{138} since the observed peak cross section would then be larger than the maximum possible: $(\sigma_0)_{\max}=4\pi\lambda\!\!\!^{-}{}_0^2g$.

E. Neodymium

A large number of resonances are observed in the neodymium spectrum and many of them were isotopically identified. Levels due to Nd^{143} were found at 56 and 130 ev. Levels due to Nd^{145} were found at 4.37 and 43 ev. At higher energies the resonances overlap strongly, making analysis and identification difficult. All the samples, normal and isotopically enriched, were Nd_2O_3 powder. The 4.37-ev resonance in Nd^{145} was analyzed by the area method to yield the results listed in Table I. We did not attempt to obtain Γ from the observed width since the combined Doppler and resolution width was larger than Γ. A preliminary analysis of the 43-ev resonance indicated that $\Gamma_n>\Gamma_\gamma$.

[15] V. E. Pilcher *et al.* (to be published).

F. Platinum

The platinum spectrum also exhibits a large number of levels. Resonances associated with the Pt^{195} target isotope were found at 11.9, 19.6, 68, 120, and 153 ev. A resonance at 96 ev is probably due to Pt^{198}. Other overlapping resonances appear above 100 ev but were not identified. The samples were foils of normal platinum and isotopically enriched Pt^{195} powdered metal. A thick-thin area analysis of the 11.9- and 19.6-ev resonances gave the results in Table I. The target nucleus Pt^{195} has spin $I=\frac{1}{2}$, so that the compound nucleus spin $J=1$ or 0 (corresponding to $g=\frac{3}{4}$ or $\frac{1}{4}$, respectively). Since this produces uncertainties in Γ_n and Γ_γ which are comparable to the experimental errors in this case, we have listed the two alternative sets of parameters for these resonances in Table I. The uncertainty in the J's makes the Γ_γ's for these two levels compatible. Preliminary analysis of the 68- and 96-ev resonances indicated that they have large scattering widths, and they were not investigated further.

G. Zirconium

A single level, at 296 ev, was observed in the range of neutron energies 3 ev to 600 ev, and it was identified as due to the Zr^{91} isotope. Normal zirconium metal and isotopically enriched $Zr^{91}O_2$ powder samples were used. Assuming a reasonable $\Gamma_\gamma\sim0.2$ ev, we obtain a total width of 1.4 ev for this resonance.

H. Thallium

A large level was observed at 238 ev, and it was identified as belonging to the target isotope Tl^{203}. No other resonances were observed in the 3-ev to 600-ev range surveyed. An attempt was made to perform a thick-thin area analysis, using Tl_2O_3 powder for a thick sample and $TlNO_3$ dissolved in heavy water for a thin sample. The wide resolution width made the analysis difficult and good resonance parameters were not obtained. However, it was possible to establish the spin state of the level in this case. The maximum possible peak cross section is given by $4\pi\lambda\!\!\!^{-}{}_0^2g$. Since $I=\frac{1}{2}$ for the Tl^{203} nucleus, g is $\frac{1}{4}$ or $\frac{3}{4}$, which gives maximum peak cross sections of 2720 barns or 8170 barns, respectively. Using a thin sample, an observed peak cross section of 4100 barns was obtained without correcting for resolution and Doppler broadening. Therefore, $g=\frac{3}{4}$ and $J=1$ for this level. If we assume that $\Gamma_\gamma\sim0.4$ ev, we obtain a total width of 4.9 ev.

TABLE II. A summary of additions to the table of measured radiation widths in Levin and Hughes, reference 3, with experimental errors of 33% or less. Revisions have been included when they have amounted to changes of 33% or more. Other revisions can be found mostly in Harvey, Hughes, Carter, and Pilcher, reference 10, and Landon, reference 5. Radiation widths are given in millielectron volts.

Target isotope	Target spin	Resonance energy (ev)	Γ_γ (10^{-3} ev)	Reference
$_{17}Cl^{35}$	3/2	−140	480±20	a
$_{33}As^{75}$	3/2	47	270±20	b
		92	250±50	b
		254	230±50	b
		322	270±20	b
$_{34}Se^{74}$	0	27.0	350±80	c
$_{35}Br^{79}$	3/2	35.6	310±30	c
		53.4	430±70	c
$_{38}Sr^{87}$	9/2	3.56	205±20	d
$_{41}Nb^{93}$	9/2	35.9	229±50	e
		195	340±60	f
$_{42}Mo^{97}$	5/2	71.5	330±80	g
$_{47}Ag^{107}$	1/2	16.6	151±23	h
		52.2	112±24	h
$_{47}Ag^{109}$	1/2	30.8	121±13	h
$_{48}Cd^{111}$	1/2	27.7	90±20	i
$_{48}Cd^{112}$	0	67	90±30	i
$_{50}Sn^{117}$	1/2	39.4	106±25	g
$_{51}Sb^{121}$	5/2	6.24	61±9	d
		15.5	109±20	d
		29.7	123±40	d
$_{51}Sb^{123}$	7/2	21.6	86±20	d
$_{54}Xe^{135}$	3/2	0.085	94±3	j,k
$_{56}Ba^{135}$	3/2	24.5	114±17	d
$_{57}La^{139}$	7/2	73.5	150±30	d
$_{60}Nd^{145}$	7/2	4.37	48±5	d
$_{62}Sm^{147}$	7/2	32.0	55±15	l
$_{63}Eu^{151}$	5/2	−0.0006	67±5	m
		3.35	81±12	n
		3.72	69±7	n
$_{63}Eu^{153}$	5/2	2.46	89±2	n
		3.94	91±5	n
$_{64}Gd^{155}$	7/2	2.01	104±5	o
		2.57	111±4	o
		6.49	106±20	l
		20.2	91±25	l
		23.9	108±31	l
$_{64}Gd^{157}$	7/2	2.82	114±5	o
		17.1	85±16	l
$_{66}Dy^{161}$	7/2	2.72	119±10	p
		3.69	124±15	p
$_{66}Dy^{162}$	0	5.45	175±45	p
$_{66}Dy^{163}$	7/2	1.71	103±10	p
$_{70}Yb^{168}$	0	0.60	70±5	q
$_{71}Lu^{176}$	≥7	1.57	55±5	r
		2.60	50±1	r
$_{72}Hf^{177}$	≤3/2	1.10	67±2	s
$_{73}Ta^{180}$		0.43	30±5	t
$_{73}Ta^{181}$	7/2	14.0	55±13	g
		20.5	49±15	g
$_{74}W^{182}$	0	4.14	46±2	r
$_{74}W^{183}$	1/2	7.8	52±11	i
$_{75}Re^{185}$	5/2	2.16	56±1	u
$_{75}Re^{187}$	5/2	4.42	45±1	u
$_{77}Ir^{191}$	1/2	5.36	67±5	r
$_{78}Pt^{195}$	1/2	11.9	$J=0$, 133±20 $J=1$, 154±20	d
		19.6	$J=0$, 84±20 $J=1$, 103±20	d
$_{92}U^{233}$		1.78	48±13	v,w
		2.29	38±8	v,w
		3.61	43±9	v,w
$_{92}U^{235}$	5/2	0.29	39±6	x
		1.13	35±8	x
		2.04	31±5	x
$_{92}U^{238}$	0	36.8	25±4	y
		66.2	19±3	y
$_{93}Np^{237}$	5/2	0.49	32±3	z
$_{94}Pu^{239}$	1/2	0.30	40±5	aa,bb
		7.85	40±9	bb,cc
		10.95	39±12	bb,cc
		11.95	42±12	bb,cc
		22.3	41±15	bb,cc
$_{94}Pu^{240}$	0	1.053	39±3	cc,p

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I. Rubidium, Niobium, Cerium

No levels were observed in rubidium and cerium in the range of neutron energies from 3 ev to 600 ev. Several weak resonances were observed in niobium; these have been more recently studied at Harwell[16] and Argonne.[17]

[16] E. R. Rae, *Proceedings of the International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1955* (United Nations, New York, 1956), Vol. 4.
[17] Saplakoglu, Bollinger, and Coté, Bull. Am. Phys. Soc. Ser. II, **1**, 347 (1956).

IV. SUMMARY OF RECENT MEASUREMENTS OF Γ_γ

Recent measurements of radiation widths, including those of this investigation are summarized in Table II, which is a supplement to the summary of radiation widths given by Levin and Hughes.[3] We have not included measurements with experimental errors in excess of 33%; some of the measurements listed have errors as small as 2%. The values in this table were obtained either by area analysis of the resonances or by fitting the shapes to the Breit-Wigner single-level

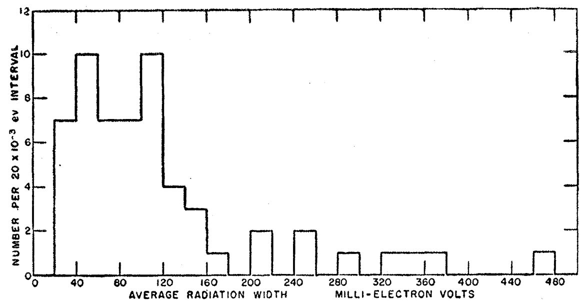

FIG. 4. Size distribution of measured average radiation widths.

formula. The table includes revisions only when they have amounted to changes of 33% or more from the values listed by Levin and Hughes. Other revisions, including some made to a higher degree of accuracy, can be found mostly in articles by Harvey *et al.*[10] and Landon.[5] In the following discussion we have used the best available data for all the isotopes in which radiation widths have been measured.

V. DISCUSSION

Under the column labeled $\bar{\Gamma}_\gamma$ in Table III, we have listed the average measured radiation widths for 58 isotopes. We have taken arithmetic averages for the radiation widths and their errors when measurements have been made on more than one resonance in an isotope. By excluding measurements with errors greater than 33%, we have introduced some weighting in $\bar{\Gamma}_\gamma$. However, we did not use a weighted average since there is evidence that Γ_γ may differ by ~25% from one level to another,[5,18] including the results on Sb^{121} reported in this paper. It is possible that this effect is due to a dependence of Γ_γ on the level spin, but this point will not be settled until the level spins in question are measured. Programs for the measurement of these compound nucleus level spins are now being pursued at several laboratories.

The size distribution of these radiation widths is shown in Fig. 4. With only one exception, all of the widths which are larger than 180 millielectron volts are for levels in nuclei with $A<100$. If we exclude these, we see that the distribution is quite narrow in spite of the effect of the closed shells, in contrast to the wide distribution of neutron scattering widths.[10] It seems likely therefore that a statistical model of the nucleus in which an excited state can decay to a large number[19] of lower states with the emission of gamma radiation is essentially correct.

In Fig. 5, $\bar{\Gamma}_\gamma$ is plotted *versus* the atomic weight of the target nucleus. Experimental errors are indicated for all points except those whose limits of error are smaller than the size of the symbol. The straight line is the one which Hughes and Harvey[2] drew through the experimental points which were available in 1954. The newer points indicate that several peaks and valleys are superimposed on this straight line. The effect on $\bar{\Gamma}_\gamma$ of the closed shells at 82 and 126 neutrons is apparent. Below $A=100$, the level spacing is large and $\bar{\Gamma}_\gamma$ increases with decreasing A. The behavior of $\bar{\Gamma}_\gamma$ follows in a general way the behavior of the neutron binding energy[20] and the level spacing[21] which also show anomalies at the closed shells. The fluctuations of $\bar{\Gamma}_\gamma$ at the shells can be correlated with the variations in level excitation energy and level spacing through the theoretical expression obtained by Blatt and Weisskopf,[4] which is of the form

$$\Gamma_\gamma = \text{constant } A^{\frac{2}{3}} D(E_B) \int_0^{E_B} \frac{E^3 dE}{D(E_B - E)}, \tag{3}$$

if one assumes electric dipole radiation[22] to be dominant. In this expression, E_B is the neutron binding energy (which is the level excitation energy since the kinetic energy of the neutron is negligible), $D(E_B)$ is the spacing of levels near E_B of the same spin and parity as the radiating level, and $D(E_B-E)$ is the spacing of levels to which dipole transitions are permitted. The integration of this expression depends on the functional form assumed for the level spacing. Levin and Hughes[3] used the form given by the statistical gas model of the nucleus to obtain an expression in which Γ_γ depends on $(E_B)^4$ and is weakly dependent on $D(E_B)$. Recently, Cameron[6] has used a level-spacing formula given by Newton[7] to obtain an expression for Γ_γ. Newton's

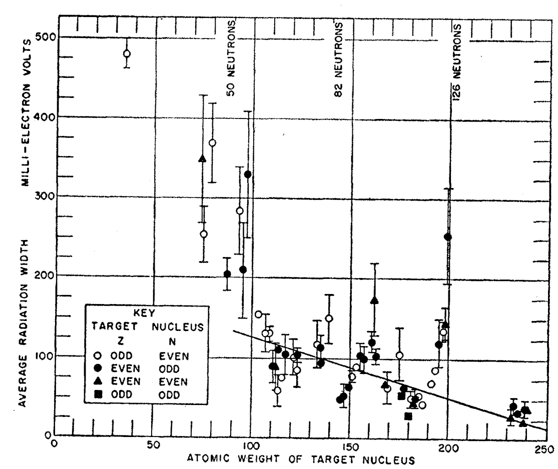

FIG. 5. Average radiation width *versus* atomic weight of the target nucleus. The limits of error are not indicated when they are smaller than the symbol. The straight line is the one drawn by Hughes and Harvey in 1954.

[18] H. H. Landon and V. L. Sailor, Phys. Rev. **98**, 1267 (1955).
[19] C. E. Porter and R. G. Thomas, Phys. Rev. **104**, 483 (1956).
[20] J. A. Harvey, Phys. Rev. **81**, 353 (1951); N. S. Wall, Phys. Rev. **96**, 664 (1954).
[21] J. A. Harvey, Phys. Rev. **98**, 1162 (1955); H. W. Newson and R. H. Rohrer, Phys. Rev. **94**, 654 (1954); Hughes, Garth, and Levin, Phys. Rev. **91**, 6 (1953).
[22] B. B. Kinsey and G. A. Bartholomew, Phys. Rev. **93**, 1260 (1954).

TABLE III. Measured, adjusted, and computed radiation widths. The meanings of the symbols are given in the text. Also listed are the neutron binding energy in the compound nucleus E_B, the effective level excitation energy U, and the average observed level spacing per spin state $D(U)$. All widths are expressed in millielectron volts.

Target nucleus	E_B (Mev)	U (Mev)	$D(U)$ (ev)	$\bar{\Gamma}_\gamma$	Γ_γ' $\alpha=2$	Γ_γ' $\alpha=4$	Γ_γ' $\alpha=6$	Γ_γ''	Γ_γ (Cameron)	Γ_γ [Eq. (8)]
$_{17}Cl^{35}$	8.56[a]	8.56	~1000	480±20	748	432	248	461	1790	326
$_{33}As^{75}$	7.30[a]	7.30	180	255±35	332	263	209	228	243	178
$_{34}Se^{74}$	8.0[a]	6.6	~200	350±80	557	534	513	309	121	122
$_{35}Br^{79}$	7.8[a]	7.8	90	370±50	409	283	197	381	252	206
$_{38}Sr^{87}$	11.07[a]	8.45	~800	205±20	181	107	63	115	412	537
$_{41}Nb^{93}$	7.19[a]	7.19	85	285±55	333	272	222	268	176	159
$_{42}Mo^{95}$	9.15	6.60	~400	210±60	287	278	270	132	158	166
$_{42}Mo^{97}$	8.29[a]	5.75	~400	330±80	584	746	952	205	99	94
$_{45}Rh^{103}$	6.79[a]	6.79	~150	155±5	189	174	159	118	96	154
$_{47}Ag^{107}$	7.27[a]	7.27	55	132±24	137	110	88	126	105	163
$_{47}Ag^{109}$	6.7	6.7	35	131±9	158	149	140	138	92	106
$_{48}Cd^{111}$	9.4	7.0	68	90±20	99	85	74	79	109	151
$_{48}Cd^{112}$	6.40[a]	5.19	~230	90±30	177	278	434	58	61	58
$_{48}Cd^{113}$	9.05[a]	6.65	61	112±5	134	128	123	108	97	111
$_{49}In^{113}$	7.0	7.0	14	60±20	65	56	48	78	104	103
$_{49}In^{115}$	6.6[a]	6.6	14	77±3	93	90	87	99	91	83
$_{50}Sn^{117}$	9.3[a]	6.9	140	106±25	114	100	88	76	114	182
$_{51}Sb^{121}$	6.80[a]	6.80	30	102±23	112	102	93	104	102	116
$_{51}Sb^{123}$	6.6[a]	6.6	70	86±20	100	99	97	71	102	128
$_{52}Te^{123}$	9.0	6.7	30	104±9	117	110	105	105	114	108
$_{54}Xe^{135}$	7.9[a]	5.7	~1000	94±3	137	178	233	37	162	141
$_{55}Cs^{133}$	6.73[a]	6.73	42	118±30	124	116	108	104	220	128
$_{56}Ba^{135}$	9.1[a]	6.9	70	114±17	113	101	90	88	265	161
$_{57}La^{139}$	5.10[a]	5.10	~1000	150±30	266	433	703	58	107	89
$_{60}Nd^{145}$	7.5[a]	5.4	50	48±5	75	110	161	38	77	54
$_{62}Sm^{147}$	8.1[a]	6.0	14	55±15	68	81	96	60	96	63
$_{62}Sm^{149}$	8.0[a]	5.9	6.6	65±2	88	115	149	85	89	50
$_{63}Eu^{151}$	6.5[a]	6.5	1.2	79±7	82	82	82	156	113	50
$_{63}Eu^{153}$	6.3	6.3	2.4	90±4	98	105	111	148	81	52
$_{64}Gd^{155}$	7.3	5.3	4.2	104±17	161	245	373	148	74	28
$_{64}Gd^{157}$	7.0	4.9	29	100±17	174	302	522	87	64	35
$_{66}Dy^{161}$	7.3	5.3	2.3	122±13	179	266	394	196	64	26
$_{66}Dy^{162}$	5.7	4.7	~200	175±45	326	619	1172	92	51	48
$_{66}Dy^{163}$	7.0	5.0	10	103±10	169	282	470	113	57	30
$_{69}Tm^{169}$	6.1	6.1	15	65±19	71	80	91	63	76	77
$_{70}Yb^{168}$	6.8	5.8	~30	70±5	84	104	130	58	67	75
$_{71}Lu^{175}$	6.1	6.1	7	107±33	114	129	147	123	72	65
$_{71}Lu^{176}$	7.3	6.4	~3	55±3	55	57	60	80	81	63
$_{72}Hf^{177}$	7.6	5.7	9	64±2	77	98	126	69	64	53
$_{73}Ta^{180}$	7.55[a]	6.63	~3	30±5	27	26	25	42	81	76
$_{73}Ta^{181}$	6.07[a]	6.07	9	51±11	54	62	71	54	68	69
$_{74}W^{182}$	6.18[a]	5.27	50	46±2	64	98	149	32	54	57
$_{74}W^{183}$	7.42[a]	5.61	30	52±11	64	86	115	41	62	67
$_{75}Re^{185}$	6.3	6.3	6	56±1	54	58	61	65	78	75
$_{75}Re^{187}$	6.0	6.0	~15	45±1	47	56	65	41	69	76
$_{77}Ir^{191}$	6.3	6.3	~7	70±3	66	70	75	76	84	79
$_{77}Ir^{193}$	6.1	6.1	~7	87±1	87	99	112	94	74	70
$_{78}Pt^{195}$	7.92[a]	6.21	70	120±30	115	126	138	73	81	134
$_{79}Au^{197}$	6.49[a]	6.49	60	135±12	118	118	118	84	124	153
$_{80}Hg^{198}$	6.5	5.7	~100	145±20	166	219	289	79	135	97
$_{80}Hg^{199}$	7.8	6.1	~100	255±60	248	280	315	139	121	139
$_{90}Th^{232}$	5.16[a]	4.46	20	30±10	50	106	224	22	24	26
$_{92}U^{233}$	6.74[a]	5.35	1.1	43±10	49	73	108	65	38	28
$_{92}U^{235}$	6.29[a]	4.91	1.3	35±6	47	83	145	51	31	20
$_{92}U^{238}$	4.87[a]	4.19	20	23±3	42	102	244	17	20	20
$_{93}Np^{237}$	5.19[a]	5.19	1.3	32±3	39	61	95	46	34	26
$_{94}Pu^{239}$	6.28[a]	4.94	6	40±11	53	92	159	39	30	30
$_{94}Pu^{240}$	5.55[a]	4.88	20	39±3	50	89	157	27	30	39

[a] Experimental values obtained from D. J. Hughes and J. A. Harvey, *American Institute of Physics Handbook* (McGraw-Hill Book Company, Inc., New York, 1957), Sec. 8; W. H. Johnson and A. O. Nier, Phys. Rev. **105**, 1014 (1957); D. M. Van Patter and W. Whaling, Revs. Modern Phys. **26**, 402 (1954); A. H. Wapstra, Physica **21**, 367 (1955); J. A. Harvey, Phys. Rev. **81**, 353 (1951); N. S. Wall, Phys. Rev. **96**, 664 (1954). Other binding energies were obtained from semiempirical mass formula values tabulated by J. Riddell, Chalk River Report CRP-654, 1956 (unpublished) and N. Metropolis and G. Reitwiesner, Atomic Energy Commission Report NP-1980, 1950 (unpublished).

formula takes into account the effect of the closed shells on the level spacing, and also includes a correction for the depression of the excitation energy due to pairing of protons or neutrons. The latter correction is introduced by using the "effective" excitation energy U instead of the neutron binding energy E_B, where

$$U=E_B-\delta, \tag{4}$$

$$\delta=\begin{cases} 0 \text{ for compound nuclei with both } Z \text{ and } N \text{ odd} \\ 1.68-0.0042A \text{ for odd-}A\text{ compound nuclei} \\ 2(1.68-0.0042A) \text{ for compound nuclei with both} \\ \quad Z \text{ and } N \text{ even.} \end{cases}$$

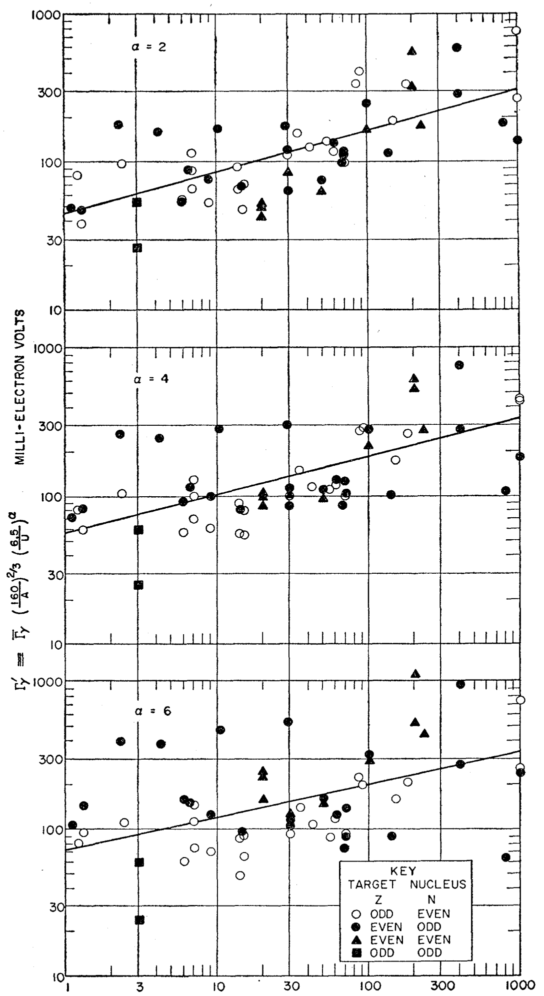

FIG. 6. Radiation widths compensated for the effects of level excitation energy and mass number *versus* observed level spacings. The curves correspond to three assumed power dependences on the effective excitation energy.

We will adopt a semiempirical approach, and attempt to fit measured radiation widths to an expression of the type

$$\Gamma_\gamma = K A^{\frac{2}{3}} U^{\alpha} [D(U)]^{\beta}. \qquad (5)$$

The powers α and β and the constant K are to be determined from the experimental results. We do not include any statistical factors[23] since they do not obviously improve the agreement with measured widths. We first attempt to determine the dependence of Γ_γ on $D(U)$ by defining a quantity Γ_γ' in which the dependence on A and U are compensated for:

$$\Gamma_\gamma' \equiv \bar{\Gamma}_\gamma \left(\frac{160}{A}\right)^{\frac{2}{3}} \left(\frac{6.5}{U}\right)^{\alpha}, \qquad (6)$$

where U is expressed in Mev. Values of Γ_γ' for $\alpha = 2$, 4 and 6 are listed in Table III, and they are plotted against level spacings in log-log plots in Fig. 6. The level spacings $D(U)$ are also listed in Table III; these are the experimentally observed spacings for spin zero target nuclei and twice this amount for all other nuclei, since we require the level spacing per spin state.[24] About 80% of the points lie within a factor of 2 of the straight line in each case. For the cases $\alpha = 2$, 4, and 6, these straight lines have slopes of 0.28, 0.25, and 0.22, respectively. Although these lines are simply visual fits to the data, it is clear that the slope (which gives the value of β) does not depend strongly on the choice of α. Since in many cases the level spacing is poorly known, we take $\beta = 0.25$ as a reasonable value. The use of the effective level excitation energy U instead of the neutron binding energy E_B in computing values of Γ_γ' eliminates troublesome even-odd effects[3]; one curve fits nuclei of all types. It is worth noting that the dependence of the total radiation width on level spacing found here is much weaker than has been indicated by Kinsey[25] for the partial radiation widths.

We can now use this empirical result for β to compensate for the level-spacing effect in order to more closely study the dependence of Γ_γ on U. We define another quantity,

$$\Gamma_\gamma'' \equiv \bar{\Gamma}_\gamma \left(\frac{160}{A}\right)^{\frac{2}{3}} \frac{2}{[D(U)]^{0.25}}, \qquad (7)$$

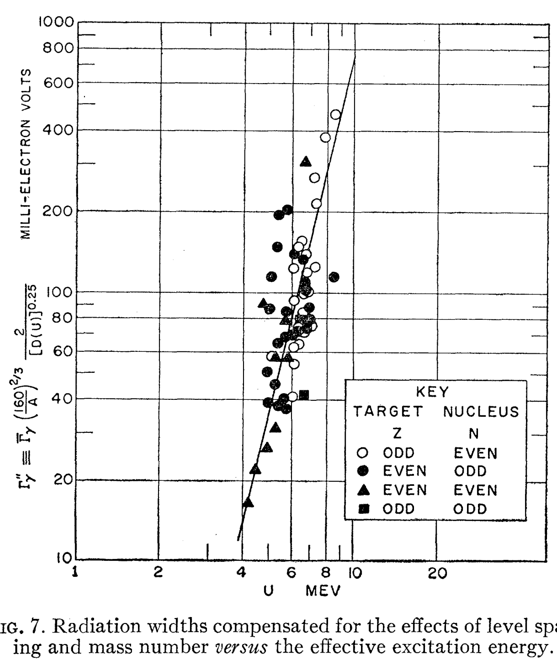

FIG. 7. Radiation widths compensated for the effects of level spacing and mass number *versus* the effective excitation energy.

[23] D. J. Hughes, Phys. Rev. **94**, 740 (1954).

[24] The assumption that the level spacing is the same for both possible spin states may not be valid; see V. L. Sailor, Phys. Rev. **104**, 736 (1956).

[25] B. B. Kinsey, in *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (Interscience Publishers, Inc., New York, 1955), p. 818.

where $D(U)$ is expressed in ev. Computed values of Γ_γ'' are given in Table III, and they are plotted *versus* U in a log-log plot in Fig. 7. The slope of this curve, which gives α, is 4.3. About 85% of the points fall within a factor of 2 of the straight line. There is no apparent dependence on the type of nucleus. When treated in this manner, the data clearly demonstrates the strong dependence of Γ_γ on the level excitation energy. We thus obtain the following empirical formula:

$$\Gamma_\gamma = (5.3\times10^{-4})A^{\frac{2}{3}}[D(U)]^{0.25}U^{4.3}, \tag{8}$$

where $D(U)$ is in ev, U is in Mev, and the result is in millielectron volts (10^{-3} ev). The numerical factor was chosen to yield the best distribution of deviations from experimental values. This formula is very similar to the theoretical expression obtained by Levin and Hughes.[3] Radiation widths computed from this formula and from Cameron's formula[26] are listed in Table III. We have computed the root-mean-square values of the percent deviations of theoretical radiation widths from experimental values, and these are listed in Table IV. When

TABLE IV. Root-mean-square values of the percent deviations of theoretical radiation widths from experimental values.

	All 58 nuclei	Nuclei with $A<100$	Nuclei with $A>100$	Nuclei near closed neutron shells
Hughes and Harvey	40%	52%	37%	53%
Cameron	60%	110%	47%	63%
Equation (8)	49%	72%	44%	31%

the deviations for all of the 58 nuclei are considered, it is seen that the line drawn by Hughes and Harvey (Fig. 5) is a better fit to the data then either of the other formulas. When the nuclei with $A<100$ are considered alone, all of the theoretical formulas are poor fits to the experimental values. When nuclei with $A<100$ are excluded, all of the theoretical formulas yield better fits to the data. It is clear that none of these formulas are valid for the lighter nuclei. If we now consider only those compound nuclei which contain a number of neutrons which is within ±8 of the magic numbers 82 and 126 (i.e., the following target nuclei: Xe^{135}, Cs^{133}, Ba^{135}, La^{139}, Nd^{145}, Sm^{147}, Sm^{149}, Eu^{151}, Pt^{195}, Au^{197}, Hg^{198}, Hg^{199}), then Eq. (8) yields the best fit. This is probably because we were able to use experimentally determined values for both U and $D(U)$ to compute radiation widths, whereas Cameron's formula makes use of a theoretical expression for the level spacing.[27]

[26] A recent communication from Cameron indicates that some of these calculated values should be slightly revised because values for the binding energies have been revised.

VI. CONCLUSION

We have shown that it is possible to determine empirically the dependence of the radiation width on the level excitation energy and level spacing. It is well to point out that although the dependence on U is strong and the dependence on $D(U)$ is weak, the fluctuations of $D(U)$ at the closed neutron shells are much larger than the fluctuations in U, and this could be the predominant effect. For the target nucleus La^{139}, the large spacing makes Γ_γ larger for this nucleus than for its neighbors in spite of the low binding energy. For nuclei near the closed neutron shells, a formula like Eq. (8) which attempts to correlate Γ_γ with measured values of U and $D(U)$ gives a better fit to the experimental data than a formula which does not.

The size distribution of the radiation widths for $A>100$ is narrow compared to the distributions of neutron scattering widths, in agreement with a statistical model of the nucleus in which the excited state can decay to a large number of lower states. The widening of the distribution when we include lighter nuclei can then be interpreted as a breakdown of the statistical model when the level spacing becomes large, and the transitions go preferentially to the low-lying levels. This would explain the absence of an observed effect on Γ_γ of the closed shell at 50 neutrons.

VII ACKNOWLEDGMENTS

We would like to thank D. J. Hughes, R. S. Carter, H. Palevsky, and V. E. Pilcher, all of the Brookhaven National Laboratory, who were very helpful in various phases of this work. Our thanks are also due to M. H. Shamos of New York University whose cooperation made this investigation possible.

[27] *Note added in proof.*—At the recent International Conference on Neutron Interactions with the Nucleus held in New York (September, 1957), Cameron has shown that a plot of the ratio of observed to calculated radiation widths (using his formula) *versus* mass number shows peaks in the same mass number regions as the *s*- and *p*-wave neutron strength functions. He indicates that this is evidence for large admixtures in these regions of single-particle wave functions in the initial or final states connected by the radiative transitions.