

## Spin, Parity, and $j$ - $j$ Expansion Coefficients for States of Low Excitation in $O^{18}\dagger$

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A study of the angular distributions and of relative intensities of proton groups from the reaction  $O^{17}(d,p)O^{18}$  has established even parity and total angular momenta of 0, 2, and 4 for the lowest three known states in  $O^{18}$ . These results, as well as the  $d$ -wave stripping intensities, are in remarkable agreement with theoretical predictions of Redlich and of Elliott and Flowers. A mixed  $s$ - $d$  angular distribution for the first excited state is shown to determine sensitively the coefficients of the  $d_{\frac{3}{2}}$  and  $(d_{\frac{3}{2}})^2$  terms in a  $j$ - $j$  expansion of the wave function for this state.

### I. INTRODUCTION

THE scarcity in nature of the nuclides  $O^{17}$  and  $O^{18}$  has delayed the experimental study of the structure of  $O^{18}$  for a time wholly at odds with the significance of this nucleus as a test for the shell model. The theoretical work of Redlich<sup>1</sup> and of Elliott and Flowers<sup>2</sup> was carried out at about the time of the discovery in  $O^{18}$  of the first excited state.<sup>3</sup> Since then work by Bach and Hough<sup>4</sup> and by Jarmie<sup>5</sup> has established the experimental energy level diagram shown in Fig. 1. Flanking the experimental diagram are the theoretical level schemes.<sup>1,2</sup>

Only even states are to be compared with theory, but since odd states occur already at low excitation in  $O^{17}$ , it is important in  $O^{18}$  first to select the states of even parity. The reaction  $O^{17}(d,p)O^{18}$  is well adapted to do this. Moreover, the presence or absence of  $s$ -wave admixture in the  $d$ -wave neutron absorption, and the relative intensities of the proton groups, constitute sufficient additional information to make possible an identification of each of the three known states of lowest excitation with a particular state of the theory.

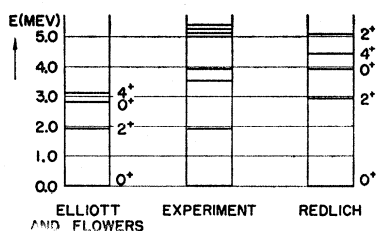


FIG. 1.  $O^{18}$  energy level schemes. Flanking the experimental diagram are the theoretical level schemes. The diagram on the left was extracted from Fig. 3 of reference 2, for  $V_c=50$  Mev. Dr. Elliott has also informed us that another  $J^\pi=2^+$  level has been calculated to lie around 4 Mev in their level scheme. Dr. Redlich's diagram has been constructed from energy eigenvalues associated with his wave function, references 1 and 12.

<sup>†</sup> This work was supported by the U. S. Atomic Energy Commission and by the Michigan Memorial Phoenix Project.

<sup>1</sup> M. G. Redlich, Phys. Rev. **95**, 448 (1954).

<sup>2</sup> J. P. Elliott and B. H. Flowers, Proc. Roy. Soc. (London) **A229**, 536 (1955).

<sup>3</sup> K. Ahnlund, Arkiv Fysik **8**, 489 (1954) and Phys. Rev. **96**, 999 (1954).

<sup>4</sup> D. R. Bach and P. V. C. Hough, Phys. Rev. **102**, 1341 (1956).

<sup>5</sup> N. Jarmie, Phys. Rev. **104**, 1683 (1956).

### II. EXPERIMENTAL PROCEDURE

The instrumentation for magnetic analysis of the external deuteron beam of the Michigan 42-in. cyclotron, and magnetic analysis of reaction products, is described in another paper.<sup>6</sup> In the present work a deuteron energy of  $7.772\pm 0.012$  Mev was used. Intensity was a primary consideration, dictating a target thickness of  $\sim 1$  mg/cm<sup>2</sup> and slits at the target admitting a wide range of incident-deuteron momenta. An over-all resolution of  $\sim 60$  kev was obtained.

The target was prepared by evaporation of Li metal, followed by a reaction with water vapor enriched<sup>7</sup> to

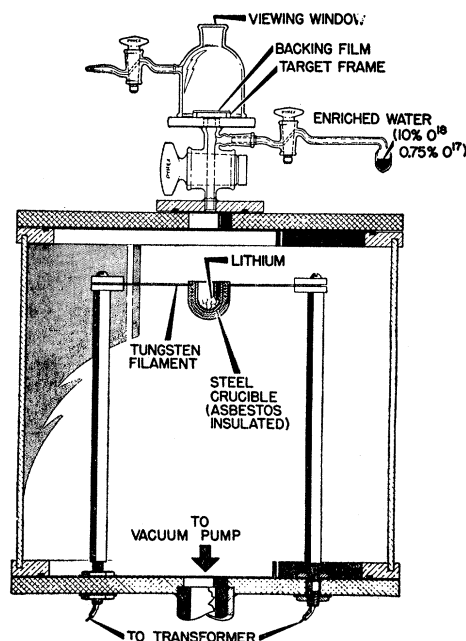


FIG. 2. Evaporation and reaction apparatus. Lithium is evaporated from a steel crucible onto a backing leaf of aluminum or Mylar and subsequently hydroxidized by the enriched water vapor.

<sup>6</sup> Bach, Childs, Hockney, Hough, and Parkinson, Rev. Sci. Instr. **27**, 516 (1956).

<sup>7</sup> The enriched water was obtained from the Chemical Division of the Borden Company, Philadelphia, Pennsylvania (Dr. K. C. Tsou).

0.75% in  $O^{17}$  to form a uniform layer of  $LiOH \cdot H_2O$ . The apparatus used is shown in Fig. 2. All spectra were recorded in Kodak NTB emulsion 100  $\mu$  thick. Proton peaks from  $O^{17}(d,p)O^{18}$  are identified positively by the measurement of the proton momentum as a function of reaction angle.

### III. EXPERIMENTAL RESULTS

#### Ground State Angular Distribution

The extreme weakness of this proton group may be appreciated by noting in Fig. 3(a) that the  $C^{13}(d,p)C_0^{14}$  group arising from the 1%  $C^{13}$  abundance in the slight carbon contamination of the target is comparable to the  $O^{17}(d,p)O_0^{18}$  group. Most data for this state were taken with a target backing of Al leaf. The measured angular distribution is shown in Fig. 4.

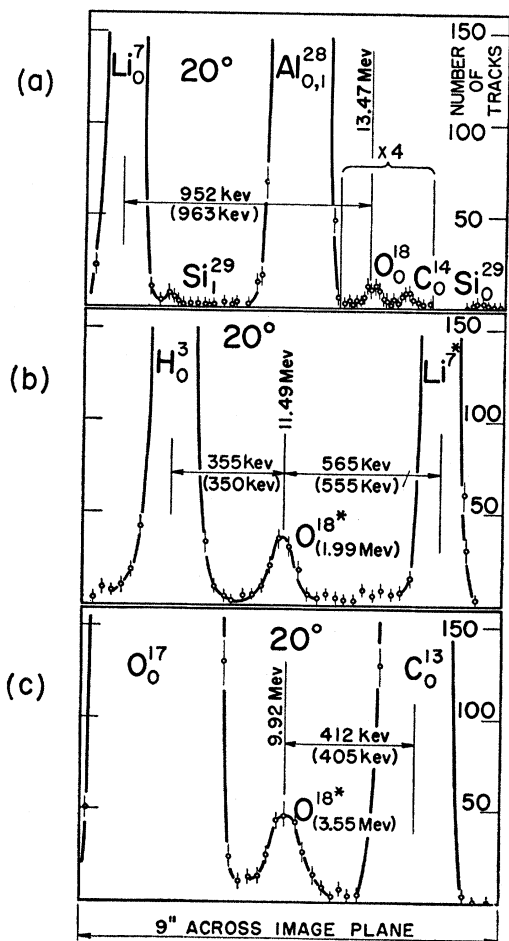


Fig. 3.  $O^{17}(d,p)O^{18}$  spectra at a reaction angle of  $20^\circ$  as recorded on 9 in.  $\times$  1 in. nuclear track plates (Kodak NTB, 100  $\mu$ ). All groups are labeled by the residual nucleus of the corresponding  $(d,p)$  reaction. The proton energy is given for the  $O_0^{18}$ ,  $O_{1.99}^{18}$  Mev, and the  $O_{3.55}^{18}$  Mev groups. Measured and calculated (in parentheses) separations of principal groups are indicated.

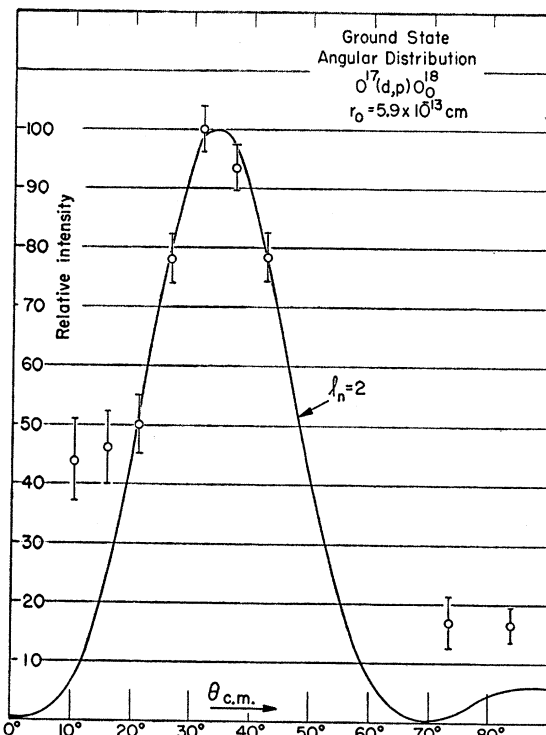


Fig. 4. Angular distribution of protons from  $O^{17}(d,p)O_0^{18}$ . The experimental data are best fitted with a theoretical Butler curve with  $r_0 = 5.9 \times 10^{-13}$  cm and  $l_n = 2$ .

#### 1.986-Mev State Angular Distribution

Since no proton group from carbon or  $O^{16}$  obscures the energy region studied, Mylar was chosen for the target backing. In the  $20^\circ$  spectrum shown in Fig. 3(b) there appear, besides the  $O^{18}$  group, peaks arising from  $H^2(d,p)H_0^3$  and  $Li^6(d,p)Li_1^7$ . (Deuterium was present in the target because the process used for the enrichment of water in  $H_2O^{17}$  also provided enrichment in  $H_2O$ .) The deuterium group overlaps the  $O^{18}$  group at angles less than  $15^\circ$ . Since the cross section near zero degrees is essential for the arguments establishing the nature of this state, an effort was made to reduce the deuterium concentration in the target. Through the courtesy of Professor R. W. Parry and Mr. C. W. Heitsch of the Department of Chemistry, University of Michigan, the relative concentration of deuterium and  $O^{17}$  was reduced a factor of seven by repeated ion exchange with  $NH_3Br$ . Figure 5 shows the composite peak then obtained at a reaction angle at  $2.5^\circ$ . The  $O^{18}$  intensity was determined by doubling the number of tracks in the left half of the peak. This procedure is justified by the symmetry of the peaks encountered in this investigation.

The resulting angular distribution is shown in Fig. 6. The points marked ( $\times$ ) at  $0^\circ$  and  $2.5^\circ$  were obtained by a less reliable technique, using data from a target of the original deuterium contamination. They serve only as confirmation of the substantial forward rise in the cross section.

## 3.555-Mev State Angular Distribution

Using Mylar as target backing, the cross section was measured from 0° to 55°. A sample spectrum is shown in Fig. 3(c). The angular distribution is shown in Fig. 7.

## Relative Intensities

Despite the competition between the groups O<sup>17</sup>(*d,p*)-O<sub>0</sub><sup>18</sup> and C<sup>13</sup>(*d,p*)C<sub>0</sub><sup>14</sup> mentioned above, careful preparation of a Mylar-backed target and precise beam alignment provided adequate separation of these peaks at angles 20°, 25°, and 30°. Therefore a single target could be used for intensity comparison of all three O<sup>18</sup> groups in exposures closely associated in time. Using the experimental angular distributions, the directly measured intensity ratios are expressed as ratios of peak cross sections. In the case of the first excited state the "peak" refers to the secondary maximum at ~30°. Calling the peak differential cross sections  $\sigma_d^{\max}$ , the measurements give

$$\sigma_d^{\max}(\text{ground}) : \sigma_d^{\max}(1.986 \text{ Mev}) : \sigma_d^{\max}(3.555 \text{ Mev}) \\ = 1 : (3.7 \pm 0.5) : (13.5 \pm 2.0).$$

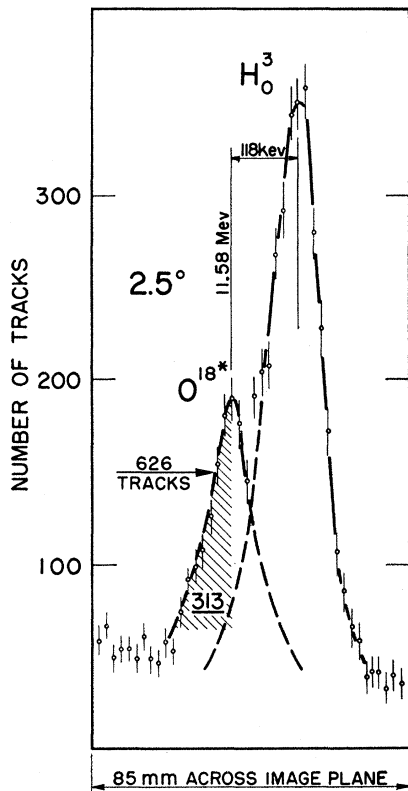


FIG. 5. O<sup>17</sup>(*d,p*)O<sup>18</sup><sub>1.99 Mev</sub> spectrum at 2.5° with the relative deuterium concentration in the target reduced by a factor of seven.

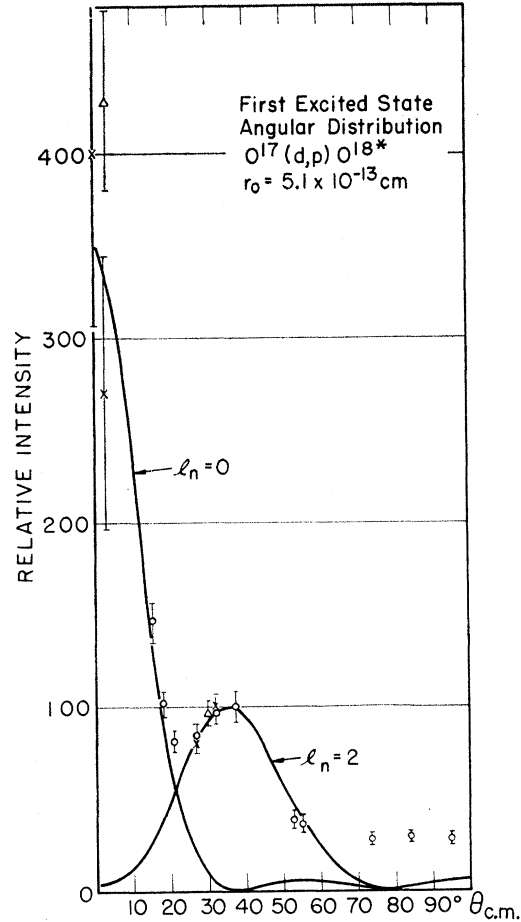


FIG. 6. Angular distribution of protons from O<sup>17</sup>(*d,p*)O<sup>18</sup><sub>1.99 Mev</sub>. The experimental data are best fitted with two Butler curves for  $l_n=0$  and  $l_n=2$ , both with  $r_0=5.1 \times 10^{-13}$  cm. Experimental points marked (x) were obtained from a target of high deuterium contamination, and for the forward angles are less reliable than the point marked ( $\Delta$ ), taken after the relative deuterium content was reduced by a factor of seven.

IV. RELATION BETWEEN STRIPPING INTENSITIES AND *j-j* EXPANSION COEFFICIENTS

Following the early work of Holt and Marsham,<sup>8</sup> others<sup>9</sup> have used relative stripping cross sections to draw conclusions concerning nuclear wave functions. As J. B. French and co-workers<sup>10</sup> have emphasized, the insecure basis for the Butler theory, and its disagreement with experiment as regards absolute cross section, encourages the development of methods for analysis of (*d,p*) reactions independent of the details of the theory.

The first such method consists simply in the restriction of the interpretation of relative intensities to

<sup>8</sup> See the review article by R. Huby, *Progress in Nuclear Physics* (Butterworths-Springer, London, 1953), Vol. 3, p. 177, especially Table I.

<sup>9</sup> J. B. French and B. J. Raz, *Phys. Rev.* **104**, 1411 (1956) and references therein.

<sup>10</sup> T. Auerbach and J. B. French, *Phys. Rev.* **98**, 1276 (1955), and reference 9.

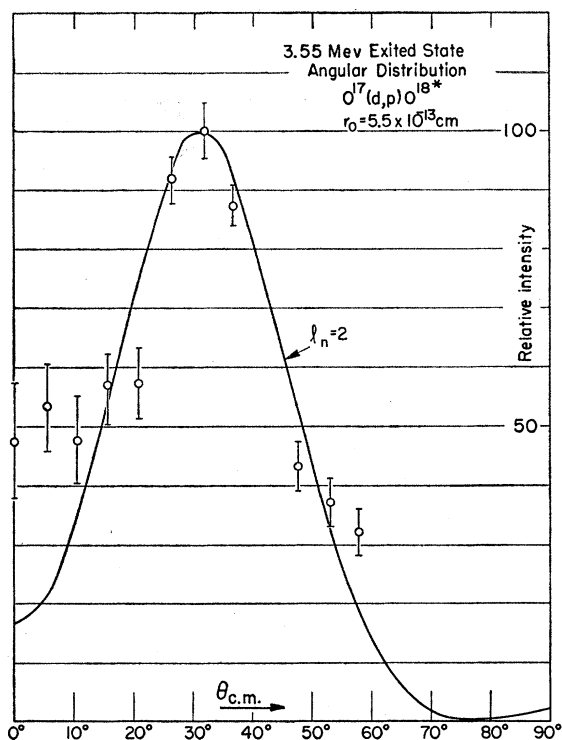


FIG. 7. Angular distribution of protons from  $O^{17}(d,p)O^{18}$ , 3.55 Mev. The experimental data are best fitted with a theoretical Butler curve with  $r_0 = 5.5 \times 10^{-13}$  cm and  $l_n = 2$ .

states in the same or neighboring nuclei reached by neutron capture with the *same* angular momentum  $l_n$ . In the case of  $O^{18}$  this means that the ground state and the 3.555-Mev state intensities may be compared directly. For the 1.986-Mev state, however, only the large-angle or  $d$ -part of the composite  $s$ - $d$  distribution is to be considered. In making the comparison, the detailed prediction of the Butler theory regarding the variation of the single-particle cross section with reaction  $Q$  may be taken into account or not, and it need only be assumed that the variation is slow. In the  $O^{18}$  case, the  $l_n = 2$  single-particle Butler cross sections<sup>11</sup> are in the ratio 1:1.2:1.4, for the ground, first, and second excited state transition. The much stronger variation observed experimentally arises then from variation in the statistical weight  $2J+1$  and  $j$ - $j$  composition of the nuclear wave function for the final state.

The most general wave function for two nucleons in the  $2s$ ,  $1d$  shell is of the form

$$\psi = A_{\frac{1}{2}\frac{1}{2}}(d_{\frac{1}{2}})^2 + A_{\frac{1}{2}\frac{3}{2}}(d_{\frac{1}{2}}s_{\frac{1}{2}}) + A_{\frac{1}{2}\frac{3}{2}}(s_{\frac{1}{2}})^2 + A_{\frac{3}{2}\frac{1}{2}}(d_{\frac{3}{2}}d_{\frac{1}{2}}) + A_{\frac{3}{2}\frac{1}{2}}(d_{\frac{3}{2}}s_{\frac{1}{2}}) + A_{\frac{3}{2}\frac{3}{2}}(d_{\frac{3}{2}})^2. \quad (1)$$

With neglect of the internucleon force, each term is

<sup>11</sup> The calculation is greatly simplified by the use of the "Numerical Table of Butler-Born Approximation Stripping Cross Sections" by C. R. Lubitz, H. M. Randall Laboratory of Physics, University of Michigan, 1957 (unpublished). Copies are available on request.

separately an  $O^{18}$  eigenfunction, and the terms in (1) are in order of increasing excitation energy for that approximation. Table I lists, for reference, the coefficients calculated by Redlich<sup>12</sup> corresponding to the lowest  $J=0, 2, 4$  states shown on the right side of Fig. 1.

In the Butler-Born approximation as presented by Huby,<sup>13</sup> stripping in  $O^{18}$  can occur only to those parts of the wave function (1) for which at least one of the nucleons is in a state corresponding to the ground state of  $O^{17}$ , namely  $d_{\frac{1}{2}}$ . Use of Eq. (1) and of Huby's Eqs. (46) and (47) gives, with the aid of the orthogonality relations for the Wigner coefficients, a factor  $\bar{S}_{l_n}$  by which the single-particle Butler cross section for capture of a neutron with the angular momentum  $l_n$  is multiplied:

$$\bar{S}_2 = (2J+1)[(A_{\frac{1}{2}\frac{1}{2}})^2 + \frac{1}{2}(A_{\frac{1}{2}\frac{3}{2}})^2], \quad (2-a)$$

$$\bar{S}_0 = (2J+1)\frac{1}{2}(A_{\frac{1}{2}\frac{1}{2}})^2. \quad (2-b)$$

The quantity  $\bar{S}$  is analogous to the  $S$  of French and Raz,<sup>14</sup> except for the inclusion of the factor  $2J+1$ . The factor of two reduction in intensity which occurs for nonequivalent nucleons in (2-a) and (2-b) may be understood by noting that in this case stripping can occur only to half of the properly antisymmetrized  $O^{18}$  wave function.

Redlich's wave functions for the states  $J=2$  and  $J=4$  show only a 4% and 12% contribution to  $d$ -wave stripping via the  $d_{\frac{1}{2}}d_{\frac{1}{2}}$  component. If these wave functions are even qualitatively right,  $d$ -wave stripping intensities in  $O^{18}$  are determined primarily by the  $(A_{\frac{1}{2}\frac{1}{2}})^2$ .

As it stands, Eq. (2-b) is of no help for interpretation of  $s$ -wave stripping intensities in  $O^{18}$ , since the Butler theory is not relied on to give correctly the relative cross section for  $s$ -wave and  $d$ -wave stripping. However, the  $s$ - $d$  intensity ratios in  $O^{17}(d,p)O^{18}$  may be related to the corresponding measured ratios in the  $O^{16}(d,p)O^{17}$  reactions which lead to the  $d_{\frac{1}{2}}$  ground state and the  $s_{\frac{1}{2}}$  first excited state. The integral equation treatment of Gerjuoy<sup>15</sup> may be extended slightly to establish the connection. This theory gives for the amplitude of the outgoing proton of wave number  $k_p$ , at infinity,

$$A(\mathbf{k}_p) = -\frac{1}{4\pi} \left( \frac{2M}{\hbar^2} \right) \iiint d\mathbf{r}_n d\mathbf{r}_d d\mathbf{r}_p e^{-i\mathbf{k}_p \cdot \mathbf{r}_p} \times \Psi_J^M(\mathbf{r}_n, \mathbf{r}_d) (V_{np} + V_p + V_{pd}) \psi_0(\mathbf{r}_n, \mathbf{r}_p, \mathbf{r}_d). \quad (3)$$

In (3)  $\psi_0 = \psi_D(\mathbf{r}_n, \mathbf{r}_p) u_{\frac{1}{2}}^{M_0}(\mathbf{r}_d)$ , with  $u_{\frac{1}{2}}^{M_0}$  the wave function for the extra neutron in  $O^{17}$ .  $\psi_D$  may be Coulomb distorted and elastically scattered, so (3) is not a Born-approximation amplitude.  $\Psi_J^M$  is the  $O^{18}$  wave function, properly antisymmetrized. The function

<sup>12</sup> M. G. Redlich (private communication).

<sup>13</sup> Reference 8, especially Eqs. (19), (40), (45), (46), and (47).

<sup>14</sup> Reference 9. Our Eq. (2-a) is in conflict with their Eq. (2). Professor French has informed us that their sum on  $j$  should be taken after squaring. Such a change brings the two formulas into agreement.

<sup>15</sup> E. Gerjuoy, Phys. Rev. **91**, 645 (1953).

$\psi_0$  is not antisymmetrized, so exchange effects of the type considered by French<sup>16</sup> are neglected. Spin variables are not explicitly written in (3), although they have been included in the calculation.  $V_{np}$  is the neutron-proton interaction in the deuteron, and gives the usual Butler formula for stripping.  $V_p$  is the interaction of the stripped proton with the magic O<sup>16</sup> nucleus, including the Coulomb interaction.  $V_{p\xi}$  contains the interaction of the stripped proton with the one extra magic nucleon in O<sup>17</sup>. With the assumption that  $V_p + V_{p\xi}$  may be collapsed into a single smooth potential representing the interaction of the stripped proton with O<sup>17</sup>, the amplitude (3) breaks into an integral over  $\xi$ , which expresses the overlap of the O<sup>18</sup> wave function with that of the O<sup>17</sup> ground state and an integral which summed over magnetic quantum numbers gives an expression for the O<sup>16</sup>( $d,p$ )O<sup>17</sup> cross section in much higher approximation than the Butler formula. The result, expressed in terms specifically applicable to the  $s$ - $d$  peak ratio for the first excited state in O<sup>18</sup>, is

$$\begin{aligned} & (\sigma_s^{\max}/\sigma_d^{\max})_{O^{18}} \\ &= \frac{\frac{1}{2}(A_{\frac{1}{2}\frac{1}{2}})^2[\sigma_1/(2 \times \frac{1}{2} + 1)k_{p1}]'}{[(A_{\frac{1}{2}\frac{1}{2}})^2 + \frac{1}{2}(A_{\frac{1}{2}\frac{1}{2}})^2][\sigma_0/(2 \times \frac{1}{2} + 1)k_{p0}]'}, \quad (4-a) \end{aligned}$$

$\sigma_0$  and  $\sigma_1$  are the ground and first excited state peak cross sections in O<sup>16</sup>( $d,p$ )O<sup>17</sup>, and have a measured ratio  $\sigma_1:\sigma_0=100:12$ .<sup>17</sup> The factors  $2j+1$  and outgoing proton wave number  $k_p$  in the O<sup>16</sup> intensities are extracted, according to Eq. (4-a), before use in deduction of the O<sup>18</sup> ratio. The primes express the fact that these parentheses are to be evaluated for a binding energy of the captured neutron equal to that which occurs in O<sup>18</sup> (1.986-Mev state), while experimentally the cross sections are observed for bindings peculiar to the O<sup>17</sup> nucleus and different for  $\sigma_0$  and  $\sigma_1$ . Only the Butler cross section is available to correct the measured values; however, it is reassuring to find that the corrections are only  $-15\%$  and  $-5\%$  for numerator and denominator, respectively, so that not much uncertainty is introduced.

Substitution of numerical values into (4-a) gives

$$\left(\frac{\sigma_s^{\max}}{\sigma_d^{\max}}\right)_{O^{18}} = \frac{\frac{1}{2}(A_{\frac{1}{2}\frac{1}{2}})^2}{(A_{\frac{1}{2}\frac{1}{2}})^2 + \frac{1}{2}(A_{\frac{1}{2}\frac{1}{2}})^2} \times 29. \quad (4-b)$$

The number 29, which follows from the O<sup>16</sup>( $d,p$ )O<sup>17</sup> experiment, is to be compared with a number 20, predicted by the straightforward application of the Butler formula.<sup>11</sup>

<sup>16</sup> A. P. French, Phys. Rev. **107**, 1655 (1957).

<sup>17</sup> W. J. Childs, Ph.D. dissertation, University of Michigan, 1956 (unpublished).

TABLE I. Expansion coefficients  $A_{jj'}$ , as calculated by Redlich for the lowest  $J=0, 2, 4$  states in O<sup>18</sup> (references 1 and 12).

$J \setminus jj'$	$\frac{5}{2} \frac{5}{2}$	$\frac{5}{2} \frac{1}{2}$	$\frac{1}{2} \frac{1}{2}$	$\frac{5}{2} \frac{3}{2}$	$\frac{1}{2} \frac{3}{2}$	$\frac{3}{2} \frac{3}{2}$
0	0.86	...	0.40	...	...	0.31
2	0.71	0.63	...	-0.20	0.20	0.14
4	0.90	...	...	-0.44	...	...

## V. SPIN AND PARITY ASSIGNMENTS

### Even Parity for the Ground State

The ground-state angular distribution (Fig. 4) can be fitted<sup>11</sup> reasonably only with a theoretical stripping curve corresponding to neutron absorption with orbital angular momentum  $l_n=2$ . Thus, the parity of the O<sup>18</sup> ground state is identical with that of O<sup>17</sup>, i.e., even. The spin of O<sup>18</sup> is known to be zero.<sup>18</sup>

### $J^\pi=2^+$ for the 1.986-Mev State

The sharp forward rise in the angular distribution of Fig. 6 indicates clearly the ( $d,p$ ) reaction to this state proceeds at least in part by  $s$ -wave neutron absorption. Therefore, its parity is even, and its spin 2 or 3.

The lowest  $3^+$  state in O<sup>18</sup> is predicted to occur at considerably higher excitation energy. Furthermore, the  $3^+$  wave function is sure to be mostly  $d_{\frac{1}{2}}s_{\frac{1}{2}}$ , with a lesser component of  $d_{\frac{3}{2}}s_{\frac{1}{2}}$ . Under this circumstance the ratio of forward maximum to secondary peak would be greater than 15:1 instead of the observed 4:1. Such an observed ratio is quite consistent with reasonable  $J^\pi=2^+$  wave functions, as discussed below. We may say then that  $J^\pi=2^+$  for the 1.986-Mev state.

### $J^\pi=4^+$ for the 3.555-Mev State

As in the case of the ground state, the angular distribution of Fig. 7 can be fitted only with  $l_n=2$ . Therefore, the parity of the state is even, and its spin 0, 1, 2, 3, 4, or 5. No  $J=5^+$  antisymmetric state can be constructed from  $s,d$  configurations, and in fact the lowest configurations which could furnish such a state are  $f_{7/2}f_{5/2}$  and  $f_{7/2}p_{3/2}$ . However, the strength of the observed stripping transition excludes all such unlikely configurations, and  $J=5$  may be dismissed as spin for this state. The absence of a forward maximum in the angular distribution implies that no  $s$  neutrons are captured to form the 3.555-Mev state of O<sup>18</sup>; therefore its spin is not 2 or 3. More precisely, the data indicate that the wave function for this state contains a  $d_{\frac{1}{2}}s_{\frac{1}{2}}$  component whose amplitude squared is less than 0.05. All low  $2^+$  and  $3^+$  states are expected to have  $d_{\frac{1}{2}}s_{\frac{1}{2}}$  square amplitudes  $\sim 0.5$ , and no mechanism is known which would reduce them to the experimental limit.

The remaining possibilities,  $J=0, 1$  and  $4$  would lead to cross sections about  $\frac{1}{5}, 3$ , and  $12$  times the

<sup>18</sup> Miller, Javan, and Townes, Phys. Rev. **82**, 454 (1951).

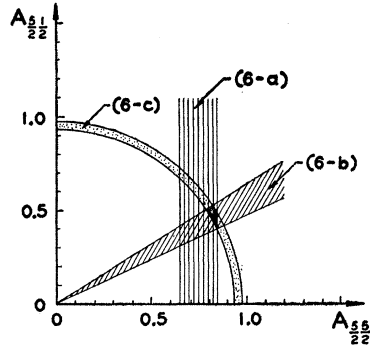


FIG. 8. Semi-empirical graphical determinations of  $A_{\frac{5}{2}}^{\frac{5}{2}}$  and  $A_{\frac{3}{2}}^{\frac{3}{2}}$   $j$ - $j$  expansion coefficients. Numbers in parentheses indicate corresponding equations in the text.

ground state cross section, respectively. The observed ratio of  $13.5 \pm 2.0$  thus prefers  $J=4$ . In confirmation of the rejection of  $J=1$ , such a state cannot arise from the lowest configurations  $(d_{\frac{5}{2}})^2$ ,  $d_{\frac{5}{2}}s_{\frac{3}{2}}$ , and  $(s_{\frac{3}{2}})^2$  and therefore is expected at an excitation energy of 10 Mev or more.

#### VI. WAVE FUNCTIONS

The work of Sec. IV has shown that the  $d$ -wave intensities reported in Sec. III determine mainly the  $A_{\frac{5}{2}}^{\frac{5}{2}}$  coefficient in the wave functions for the three states under investigation. If the theoretical coefficient  $A_{\frac{5}{2}}^{\frac{5}{2}}=0.90$  for  $J=4$  is adopted as standard, then Eq. (2-a), the Butler kinematic corrections (1:1.2:1.4 of Sec. IV), and the experimental intensities give for  $J=0$ ,

$$\frac{(2 \times 0 + 1) \times 1 \times (A_{\frac{5}{2}}^{\frac{5}{2} J=0})^2}{(2 \times 4 + 1) \times 1.4 \times (0.90)^2} = \frac{1}{13.5 \pm 2},$$

or

$$A_{\frac{5}{2}}^{\frac{5}{2} J=0} = 0.88 \pm 0.08, \quad (5)$$

a result in agreement with Redlich's coefficient 0.86.

For  $J=2$ ,

$$\frac{(2 \times 2 + 1) \times 1.2 \times (A_{\frac{5}{2}}^{\frac{5}{2} J=2})^2}{(2 \times 4 + 1) \times 1.4 \times (0.90)^2} = \frac{3.7 \pm 0.5}{13.5 \pm 2.0},$$

or

$$A_{\frac{5}{2}}^{\frac{5}{2} J=2} = 0.68 \pm 0.10, \quad (6-a)$$

in agreement with Redlich's coefficient 0.71. Assuming  $\frac{1}{2}(A_{\frac{5}{2}}^{\frac{5}{2}})^2 \ll (A_{\frac{3}{2}}^{\frac{3}{2}})^2$  in Eq. (4-b), the measured  $s$ - $d$  peak ratio of  $4.3 \pm 1.0$  for the first excited state gives

$$(A_{\frac{3}{2}}^{\frac{3}{2}})^2 / (A_{\frac{5}{2}}^{\frac{5}{2}})^2 = (4.3 \pm 1.0) / (\frac{1}{2} \times 29) = 0.30 \pm 0.07.$$

If one takes like signs for the coefficients, as given by theory, then

$$A_{\frac{3}{2}}^{\frac{3}{2}} / A_{\frac{5}{2}}^{\frac{5}{2}} = 0.55 \pm 0.06. \quad (6-b)$$

It may perhaps be justified to assume, as in Table I,

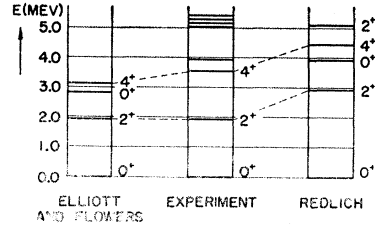


FIG. 9.  $O^{18}$  energy level schemes showing the proposed correlation of theoretical and experimental states.

that the aggregate percentage of the three higher configurations in the  $J=2$  state is 10%, with an uncertainty of  $\pm 5\%$ . In this case the normalization of the wave function requires

$$(A_{\frac{5}{2}}^{\frac{5}{2}})^2 + (A_{\frac{3}{2}}^{\frac{3}{2}})^2 = 0.90 \pm 0.05. \quad (6-c)$$

In Fig. 8 are plotted, in the  $A_{\frac{5}{2}}^{\frac{5}{2}} - A_{\frac{3}{2}}^{\frac{3}{2}}$  plane, the regions delineated by Eqs. (6-a), (6-b), and (6-c). The graphical information may be summarized, with some loss of accuracy, by saying

$$A_{\frac{5}{2}}^{\frac{5}{2}} = 0.81 \pm 0.05, \quad A_{\frac{3}{2}}^{\frac{3}{2}} = 0.48 \pm 0.05. \quad (7)$$

These results are to be compared with Redlich's predicted values of 0.71 and 0.63. It should be noted that the errors quoted in (6) and (7) are of very limited significance, and are to be considered in the light of the considerable uncertainties which have been pointed out at each stage of the analysis. At the same time the degree of correspondence between theory and experiment is remarkable and encourages the most detailed study of the reliability of stripping intensities for wave-function determination. In this connection it would be of great value to deduce from the integral equation (or otherwise), reliable estimates for the relative variation in stripping intensity with neutron binding energy.

#### VII. DISCUSSION

Figure 9 shows the proposed correlation of theoretical and experimental states. The second  $O^+$  level of the theory has not yet been identified. It would have escaped detection in the present work if the corresponding  $(d, p)$  cross section were  $\lesssim \frac{1}{4}$  of the ground state cross section. But the predicted intensity ratio is 0.22, and therefore its absence is not in conflict with theory.

The availability of fairly direct information concerning nuclear wave functions suggests a modification in the usual theoretical approach so as to make use of such data at an early stage of the calculation.

#### VIII. ACKNOWLEDGMENT

It is a pleasure to thank M. G. Redlich for providing us with his unpublished  $O^{18}$  wave functions.

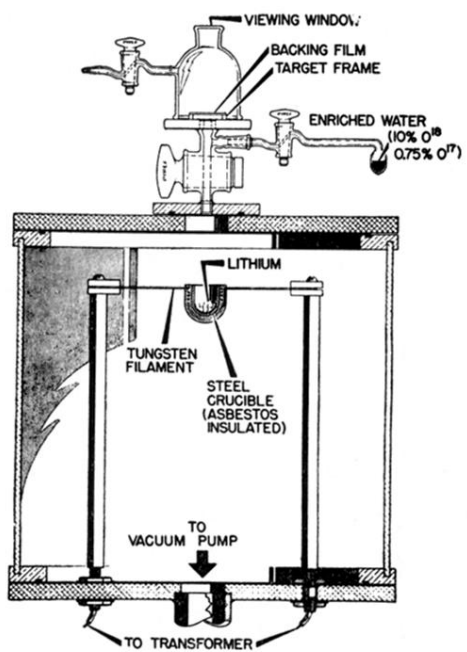


FIG. 2. Evaporation and reaction apparatus. Lithium is evaporated from a steel crucible onto a backing leaf of aluminum or Mylar and subsequently hydroxidized by the enriched water vapor.