given in volts. If we accept for the Thomas-Fermi function

$$\Phi(x) = e^{-0.60x}, \tag{22}$$

and we calculate the total cross section Q_0 , we obtain the following formula for $Q_0 Z^{-\frac{2}{3}} / \pi a_0^{-2}$:

$$\frac{Q_0 Z^{-\frac{2}{3}}}{\pi a_0^2} = \frac{75.850}{1 + 0.7308 x^2}.$$
 (23)

This formula gives for x > 2 the same results as our

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Shock-Wave Compressions of Twenty-Seven Metals. Equations of State of Metals*

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An explosive system is used to drive a strong shock wave into a plate of 24ST aluminum. This shock wave propagates through the 24ST aluminum into small test specimens which are in contact with the front surface of the plate. A photographic technique is used to measure velocities associated with the 24ST aluminum shock wave and with the shock wave in each specimen.

The measured velocities are transformed, using the conservation relations, to pressure-compression points. Resulting pressure-compression curves are given for 27 metals. The range of data is different for each material but typically covers the pressure interval 150 to 400 kilobars; probable errors in reported experimental pressure-compression curves are 1 or 2% in compression for a given pressure.

The experimental curves, which consist thermodynamically of a known P, V, E locus for each material, are used to calculate a more complete high-pressure equation of state. This is done by means of a theoretical estimate of the volume variation of the Grüneisen ratio $\gamma(V) = V(\partial P/\partial E)_V$. Calculated P, V, T states are listed for the various materials. For 24ST aluminum, quantities of application in shock-wave hydrodynamics are also tabulated.

INTRODUCTION

IN Sec. I an experimental method to obtain dynamic pressure-compression curves for solids is described and resulting data, plotted in Figs. 3–29, are listed for the following 27 solids: beryllium, bismuth, cadmium, chromium, cobalt, copper, gold, iron, lead, magnesium, molybdenum, nickel, silver, thorium, tin, titanium, zinc, 24ST aluminum, brass, indium, niobium, palladium, platinum, rhodium, tantalum, thallium, and zirconium.

Section II is devoted to the problem of generalizing the experimental Hugoniot curves into a complete thermodynamic description of high pressure states neighboring the experimental curves.

Throughout the present discussion it is assumed that stresses behind the shock wave are isotropic, and that the compressed materials behind the shock wave are in thermodynamic equilibrium. Further discussion of basic principles underlying the present work is found in previous papers^{1,2} dealing with dynamic pressure-compression results.

formula (21), as is shown in Table II, and thus it may

be used for rapid calculation of the total cross section Q_0 . Equation (23) corresponds more to a WKB-model for the atom than a Thomas-Fermi model. For x in

the range $0 \le x \le 2$, the results for the total cross section

 Q_0 given by (21) and (23) differ considerably from each other, since Q_0 given by (21) is infinite for x=0 while

The authors are indebted to Professor F. J. Wiśniew-

 Q_0 given by (23) is finite for x=0.

ski for his interest on this paper.

I. DETERMINATION OF HUGONIOT CURVES

A. Measurement of Shock Wave and Free-Surface Velocities

The experimental method used to measure shock wave and free-surface velocities is illustrated in Fig. 1. The detonation of the explosive system, pictured at the top of the figure, causes a plane shock wave to be transmitted into the 24ST aluminum plate. This shock wave next interacts with the test specimen, Lucite, iron-shim assembly on the front surface of the plate. The essential features of this assembly are the two rows of test pellets and the 0.003-in. thick argon-filled flash gaps. The pellets consist of one row of five thick specimens (0.250 in. thick by 0.750 in. diameter) to provide for shock velocity measurements and one row of thin specimens (0.125 in. thick by 0.750 in. diameter) for

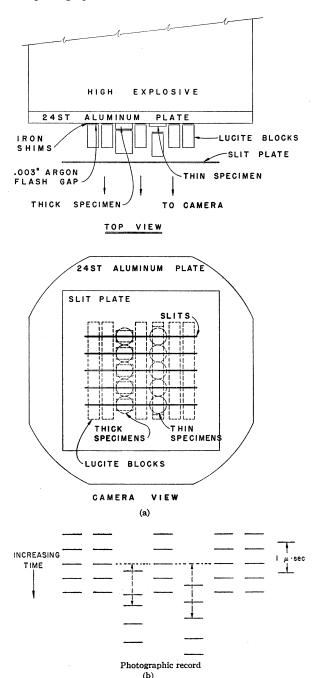
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^{*} Work done under the auspices of the U. S. Atomic Energy Commission.

¹ J. M. Walsh and R. H. Christian, Phys. Rev. 97, 1544 (1955). ² Goranson, Bancroft, Burton, Blechar, Houston, Gittings, and Landeen, J. Appl. Phys. 26, 1472 (1955).

free-surface velocity measurements of the same five materials. The flash gaps, when closed by the shock wave, provide light (due to multiple shock reflections in the argon) which is recorded by a moving image camera. The camera, in an underground bunker some 15 feet from the shot assembly, views the assembly through a slit system and sweeps the image in a direction normal to the slits.

A photographic record is also seen in Fig. 1, where





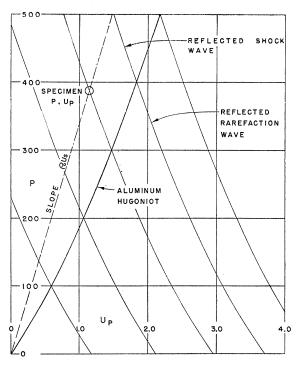


FIG. 2. Pressure versus particle velocity curves for 24ST aluminum, and a typical graphical solution to determine P, U_p for a test specimen.

pertinent features are identified. The record is analyzed by comparator reading, measured record offsets being converted to times by use of the known camera sweep speed. The measured time⁸ offset for the thick pellet is divided into pellet thickness to give the shock wave velocity for that material. The measured time for the thin pellet of the same material is the shock wave transit time through the thin pellet plus the time required for the free surface to traverse the free-run distance. When combined with the measured shock wave velocity and the known free-run distance, it gives the desired free-surface velocity. The measured shock wave and free-surface velocities are listed in Table I, second and third columns.

B. Transformation of Measured Velocities to Pressure-Volume Points

The transformation of measured velocities to pressurecompression points is done by two methods, both of

³ Small corrections are incorporated into the measured time offsets. These corrections arise from the fact that the 0.003-in. argon flash gaps over test specimen are closed at different velocities from the flash gaps over the main 24ST aluminum plate. The magnitude of each correction is determined by solution of the appropriate interface problems, using the graphical pressure particle velocity method discussed below. The applied corrections are taken to be the total difference in flash-gap closure times from the assumption that the light flash occurs in the final stages of closure of the flash gap. The corrections seldom affect measured velocities more than 0.5%, and more typically cause a change of about 0.2%.

| | Cheels mean | Ener auntress | Free surface : | approximation | | raphical soluti | on |
|--|---|--|---|---|---|---|---|
| Metal | Shock wave velocity Us (km/sec) | Free surface velocity $U_{f_{\bullet}}$ (km/sec) | Pressure P (kilobars) | Relative volume V/Vo | Shock particle velocity Up (km/sec) | Pressure P (kilobars) | Relative volume V/V₀ |
| Beryllium $\rho_0 = 1.845 \text{ g/cm}^3$ $C_p = 0.474 \text{ cal/g °C}$ $\frac{1}{V} \left(\frac{\partial V}{\partial T}\right)_p = 37 \times 10^{-6} / ^{\circ}\text{C}$ | 9.044 8.934 9.112 9.332 9.851 9.832 9.633 | 1.697 1.739 2.358 2.364 3.422 3.235 3.189 | 141.6 143.3 198.2 203.5 311.0 293.4 283.4 | 0.9062 0.9027 0.8706 0.8733 0.8263 0.8355 0.8345 | $\begin{array}{c} 0.847\\ 0.865\\ 1.189\\ 1.221\\ 1.730\\ 1.609\\ 1.592 \end{array}$ | 141.3 142.6 199.9 210.2 314.4 291.9 282.9 | 0.9063 0.9032 0.8695 0.8692 0.8244 0.8364 0.8364 |
| Bismuth | 2.696 2.585 3.075 3.084 3.682 3.659 | $1.401 \\ 1.318 \\ 1.793 \\ 1.800 \\ 2.476 \\ 2.564$ | 184.9 166.8 269.9 271.7 446.3 459.2 | $\begin{array}{c} 0.7402 \\ 0.7451 \\ 0.7085 \\ 0.7082 \\ 0.6638 \\ 0.6496 \end{array}$ | 0.718 0.676 0.914 0.922 1.212 1.222 | 189.5 171.1 275.2 278.4 436.9 437.7 | 0.7337 0.7385 0.7028 0.7010 0.6708 0.6660 |
| Cadmium $\rho_0 = 8.64 \text{ g/cm}^3$ $C_p = 0.055 \text{ cal/g °C}$ $\frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_p = 89.4 \times 10^{-6} / ^{\circ}\text{C}$ | $\begin{array}{c} 3.599 \\ 3.421 \\ 3.918 \\ 4.450 \\ 4.324 \end{array}$ | 1.464 1.277 1.757 2.496 2.400 | 227.5 188.6 297.2 479.6 448.1 | 0.7966 0.8134 0.7758 0.7196 0.7225 | 0.690 0.619 0.850 1.190 1.120 | 214.4 182.9 287.6 457.3 418.2 | 0.8083 0.8191 0.7830 0.7326 0.7410 |
| Chromium $\rho_0 = 7.13$ $C_p = 0.065$ $\frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_p = 18.6 \times 10^{-6} / {}^{\circ}\text{C}$ | $\begin{array}{c} 6.043\\ 5.923\\ 6.381\\ 6.370\\ 6.355\\ 6.357\\ 6.660\\ 6.674\end{array}$ | | , | | $\begin{array}{c} 0.5448 \\ 0.5395 \\ 0.7436 \\ 0.7449 \\ 0.7407 \\ 0.7403 \\ 1.007 \\ 1.008 \end{array}$ | 234.5 233 338 338 336 336 336 478 479 | $\begin{array}{c} 0.9098\\ 0.9089\\ 0.8835\\ 0.8831\\ 0.8834\\ 0.8835\\ 0.8488\\ 0.8490\end{array}$ |
| Cobalt $\rho_0 = 8.82$ $C_p = 0.099$ $\frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_p = 36.9 \times 10^{-6} / ^{\circ} \text{C}$ | 5.445 5.696 5.632 6.019 6.052 | 1.016 1.327 1.334 1.890 | 244.0 333.4 331.4 504.5 | 0.9067 0.8835 0.8816 0.8439 | 0.502 0.683 0.653 0.901 0.955 | 241.1 343.2 324.4 478.1 509.8 | $\begin{array}{c} 0.9078 \\ 0.8801 \\ 0.8841 \\ 0.8503 \\ 0.8422 \end{array}$ |
| Copper $\rho_0 = 8.90$ $C_p = 0.092$ $\frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_p = 49.5 \times 10^{-6} / {^{\circ}\text{C}}$ | 4.744 4.768 5.070 5.015 5.508 | 1.024 1.094 1.440 1.456 2.079 | 216.2 232.1 324.9 324.9 509.6 | 0.8921 0.8853 0.8580 0.8548 0.8113 | $\begin{array}{c} 0.511 \\ 0.570 \\ 0.711 \\ 0.731 \\ 1.032 \end{array}$ | 215.8 241.9 320.8 326.3 505.9 | 0.8923 0.8804 0.8598 0.8542 0.8126 |
| Gold $\rho_0 = 19.24$ $C_p = 0.312$ $\frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_p = 42.6 \times 10^{-6} / ^{\circ} \text{C}$ | 3.679 3.864 4.130 | 0.771 1.012 1.375 | 272.9 376.2 546.3 | 0.8952 0.8690 0.8335 | 0.380 0.505 0.666 | 269.0 375.4 529.2 | 0.8967 0.8693 0.8389 |
| Iron $\rho_0 = 7.84$ $C_p = 0.107$ $\frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_p = 35.1 \times 10^{-6} / ^{\circ} \text{C}$ | 5.652 5.474 5.458 5.438 5.229 5.231 5.206 5.209 | 2.163 2.037 1.988 2.005 1.789 1.773 1.755 1.755 | $\begin{array}{c} 479.2 \\ 437.1 \\ 425.3 \\ 427.4 \\ 366.7 \\ 363.6 \\ 358.2 \\ 358.4 \end{array}$ | $\begin{array}{c} 0.8087 \\ 0.8139 \\ 0.8179 \\ 0.8156 \\ 0.8289 \\ 0.8305 \\ 0.8314 \\ 0.8315 \end{array}$ | 1.085 1.013 0.993 0.994 | 480.8 434.7 424.9 423.8 | 0.8080 0.8149 0.8181 0.8172 |
| Lead $\rho_0 = 11.34$ $C_p = 0.030$ $\frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_p = 85.1 \times 10^{-6} / {}^{\circ}\text{C}$ | 2.914 3.268 3.250 3.724 | 1.230 1.745 1.731 2.420 | 203.0 323.1 318.7 510.5 | 0.7889 0.7330 0.7337 0.6751 | 0.590 0.819 0.802 1.118 | 194.8 303.2 295.3 471.7 | 0.7975 0.7494 0.7532 0.6998 |
| Magnesium $\rho_0 = 1.735$ $C_p = 0.250$ $\frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_p = 76.8 \times 10^{-6} / ^{\circ} \text{C}$ | 5.987 7.082 | 2.242 4.157 | 116.4 260.4 | 0.8128 0.7066 | | | |

TABLE I. Experimental data.

| | Shock wave | | | | | Graphical solution | | | |
|--|---|---|--|--|--|--|--|--|--|
| Metal | velocity Us (km/sec) | Free surface velocity U _{fs} (km/sec) | Pressure P (kilobars) | Relative volume V/Vo | Shock particle velocity Up (km/sec) | Pressure P (kilobars) | Relative volume V/V® | | |
| Molybdenum $\rho_0 = 10.20$ $C_p = 0.0612$ $\frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_p = 15 \times 10^{-6} ^{\circ} C$ | 5.699 5.647 5.955 5.861 6.210 6.124 | 0.874 0.888 1.176 1.200 1.724 1.636 | 254.0 255.7 357.2 358.7 546.0 511.0 | 0.9233 0.9214 0.9013 0.8976 0.8612 0.8664 | 0.437 0.444 0.591 0.606 0.850 0.792 | 254.0 255.2 359.0 362.3 538.4 494.7 | 0.9233 0.9214 0.9008 0.8966 0.8631 0.8707 | | |
| Nickel $\rho_0 = 8.86$ $C_p = 0.1050$ $\frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_p = 39 \times 10^{-6} / {^{\circ}\text{C}}$ | 5.417 5.653 5.620 6.031 5.969 5.952 | 0.981 1.350 1.390 1.955 1.981 1.835 | 235.3 337.8 345.8 522.0 523.5 483.5 | 0.9095 0.8806 0.8763 0.8379 0.8341 0.8459 | 0.490 0.678 0.687 0.957 0.982 0.887 | 235.0 339.4 341.8 511.0 519.0 467.4 | 0.9095 0.8801 0.8778 0.8413 0.8355 0.8510 | | |
| Silver $\rho_0 = 10.49$ $C_p = 0.056$ $\frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_p = 56.7 \times 10^{-6} / {^{\circ}\text{C}}$ | 4.065 4.113 4.378 4.846 4.848 | 1.015 1.049 1.448 2.041 2.074 | 216.4 226.3 332.5 518.8 527.4 | 0.8752 0.8725 0.8346 0.7894 0.7861 | 0.504 0.527 0.717 0.985 1.010 | 214.9 227.4 329.3 500.7 513.6 | 0.8760 0.8719 0.8362 0.7967 0.7917 | | |
| Thorium $\rho_0 = 11.68$ $C_p = 0.030$ $\frac{1}{V} \left(\frac{\partial V}{\partial T}\right)_p = 33.3 \times 10^{-6} / ^{\circ} \text{C}$ | 3.497 3.192 2.954 2.900 | 2.112 1.604 1.246 1.198 | 431.3 299.0 215.0 202.9 | 0.6980 0.7487 0.7891 0.7934 | 1.043 0.812 0.620 0.571 | 426.0 302.7 213.9 193.4 | 0.7017 0.7456 0.7901 0.8031 | | |
| Tin $p_0 = 7.28$ $C_p = 0.054$ $\frac{1}{V} \left(\frac{\partial V}{\partial T}\right)_p = 60 \times 10^{-6} / {^{\circ}\text{C}}$ | 4.555 4.435 4.004 3.557 3.524 | 2.704 2.539 1.912 1.486 1.364 | 448.3 409.9 278.7 192.4 175.0 | 0.7032 0.7138 0.7612 0.7911 0.8065 | 1.290 1.190 0.925 0.705 0.670 | 427.8 384.2 269.6 182.6 171.9 | 0.7168 0.7317 0.7690 0.8018 0.8098 | | |
| Titanium $\rho_0 = 4.51$ $C_p = 0.126$ $\frac{1}{V} \left(\frac{\partial V}{\partial T}\right)_p = 25.5 \times 10^{-6} / {^{\circ}\text{C}}$ | 6.329 5.790 5.501 5.469 | 2.723 1.926 1.437 1.364 | 388.1 251.3 178.1 168.1 | 0.7849 0.8337 0.8694 0.8753 | 1.370 0.980 0.723 0.684 | 390.8 255.7 179.3 168.6 | 0.7835 0.8307 0.8686 0.8749 | | |
| Zinc $\rho_0 = 7.135$ $C_p = 0.092$ $\frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_p = 100 \times 10^{-6} / {}^{\circ}\text{C}$ | 4.481 4.450 4.465 4.053 4.059 | 2.589 inc single-crystal 1.795 1.826 1.823 1.355 1.345 nc single-crystal e | 286.9 289.9 290.4 195.9 194.8 | 0.7997 0.7948 0.7959 0.8328 0.8343 | 1.250 1.190 0.88 0.894 0.650 0.673 | 447 414 281.4 283.9 188.0 198.3 | 0.7507 0.7556 0.8036 0.7991 0.8396 0.8370 | | |
| 24ST aluminum $\rho_0 = 2.785$ $C_p = 0.23$ $\frac{1}{V} \left(\frac{\partial V}{\partial T} \right) = 69.0 \times 10^{-6} / {}^{\circ}\text{C}$ | 4.022 4.029 7.531 6.927 6.500 Actual 2 | 1.295 1.310 3.230 2.319 1.700 4ST aluminum of ree are representa | 185.8 188.3 335.8 222.7 153.5 lata consists o | 0.8390 0.8374 0.7874 0.8333 0.8696 | 0.630 | 180.8 | 0.8434 | | |
| $V\left(\frac{\partial T}{\partial T}\right)_{p}$ Brass $c_{p}=0.09$ $\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_{p}=61.6\times10^{-6}/^{\circ}C$ | 4.446 4.440 4.731 4.726 5.236 5.220 | 1.181 1.143 1.553 1.569 2.200 | 220.9 213.5 309.1 311.9 484.6 | 0.8672 0.8713 0.8359 0.8340 0.7899 | 0.590 0.571 0.791 0.770 1.085 1.077 | 220.7 213.3 314.8 306.2 478.0 473.0 | 0.8673 0.8714 0.8328 0.8371 0.7928 0.7937 | | |
| Indium $\rho_0 = 7.27$ $C_p = 0.057$ $\frac{1}{V} \left(\frac{\partial V}{\partial T} \right) = 99 \times 10^{-6} / ^{\circ} C$ | 3.745 3.965 4.348 | | | | 0.7837 0.9812 1.281 | 213.5 283 405 | 0.7907 0.7525 0.7054 | | |

TABLE I.—Continued.

| | | | Free surface | approximation | | aphical soluti | on |
|--|---|--|-----------------------------|----------------------------|---|--|---|
| Metal | Shock wave velocity U. (km/sec) | Free surface velocity <i>Ufe</i> (km/sec) | Pressure P (kilobars) | Relative volume V/V₀ | Shock particle velocity Up (km/sec) | Pressure P (kilobars) | Relative volume V/Vo |
| Niobium $\rho_0 = 8.604$ $C_p = 0.065$ $\frac{1}{V} \left(\frac{\partial V}{\partial T}\right)_p = 23.3 \times 10^{-6} / {^{\circ}\text{C}}$ | 5.177 5.311 5.642 | | | | 0.5489 0.7434 0.9929 | 244.5 341 482 | 0.8940 0.8606 0.8240 |
| Palladium $\rho_0 = 11.95$ $C_p = 0.0583$ $\frac{1}{V} \left(\frac{\partial V}{\partial T}\right)_p = 35 \times 10^{-6} / {^{\circ}\text{C}}$ | 4.673 5.004 5.374 | | | | 0.4728 0.6200 0.8219 | 262.5 372 531 | 0.8988 0.8761 0.8471 |
| Platinum $\rho_0 = 21.37$ $C_p = 0.0322$ $\frac{1}{V} \left(\frac{\partial V}{\partial T}\right)_p = 26.7 \times 10^{-6}/^{\circ} \text{C}$ | 4.199 4.306 4.495 | | | | 0.329 0.4550 0.6102 | 295 416.5 586 | 0.9238 0.8943 0.8642 |
| Rhodium $\rho_0 = 12.42$ $C_p = 0.059$ $\frac{1}{V} \left(\frac{\partial V}{\partial T}\right)_p = 25 \times 10^{-6} / {^\circ}\text{C}$ | . 5.470 5.865 | | | | 0.4100 0.7566 | 278.5 551 | 0.9250 0.8710 |
| Tantalum $\rho_0 = 16.46$ $C_p = 0.034$ $\frac{1}{V} \left(\frac{\partial V}{\partial T}\right)_p = 19.5 \times 10^{-6} / {^{\circ}\text{C}}$ | 3.811 4.010 4.323 | | | | 0.4327 0.5800 0.7685 | 271.5 383 547 | 0.8865 0.8554 0.8222 |
| Thallium $\rho_0 = 11.84$ $C_p = 0.031$ $\frac{1}{V} \left(\frac{\partial V}{\partial T}\right)_p = 114 \times 10^{-6} ^{\circ} \text{C}$ | $\begin{array}{c} 2.804 \\ 2.817 \\ 3.120 \\ 3.145 \\ 3.538 \\ 3.541 \end{array}$ | | | | $\begin{array}{c} 0.6416 \\ 0.6386 \\ 0.8446 \\ 0.8406 \\ 1.090 \\ 1.089 \end{array}$ | 213 213 312 313 456.5 456.5 | $\begin{array}{c} 0.7712 \\ 0.7733 \\ 0.7293 \\ 0.7327 \\ 0.6919 \\ 0.6925 \end{array}$ |
| Zirconium $\rho_0 = 6.49$ $C_p = 0.068$ $\frac{1}{V} \left(\frac{\partial V}{\partial T}\right)_p = 15.6 \times 10^{-6} / ^{\circ} \text{C}$ | 4.494 4.674 4.920 | | | | 0.7117 0.9563 1.275 | 207.5 290 407 | 0.8416 0.7954 0.7408 |

TABLE I.—Continued.

which use the Rankine-Hugoniot relations

$$V/V_{0H} = (U_s - U_p)/U_s,$$
 (1)

$$P_{H} = \rho_{0} U_{s} U_{p} + P_{0H}, \qquad (2)$$

for the conservations of mass and momentum across a shock front. Here P_{0H} , V_{0H} refer to pressure and specific volume for the undisturbed state ahead of the shock front and P_H , V denote pressure and specific volume for the state behind the shock front. U_s and U_p are, respectively, the shock wave velocity, and the particle velocity for the state behind the shock front, each relative to the undisturbed state ahead of the shock front.

For the first transformation method, we note that the experimental free-surface velocity, U_{fs} , is the sum of the shock particle velocity and the particle velocity, U_r , due to the centered rarefaction wave which relieves the pressure, i.e.,

$$U_{fs} = U_p + U_r. \tag{3}$$

The approximate relation⁴ $U_r/U_p \approx 1$, or equivalently

$$U_p \approx U_{fs}/2, \tag{4}$$

is combined with Eqs. (1), (2), and measured values of U_s and U_{fs} to determine pressure-volume points. Resulting data are listed in Table I under *Free Surface* Approximation and are plotted in Figs 3-29 as \times 's.

⁴ Calculated refinements of the free-surface velocity approximation are discussed in Sec. II-B and listed in Table VII. The approximation was also discussed in reference 1, where expressions for possible errors associated with its use were shown to be small.

The second transformation method makes use of certain equation of state data for 24ST aluminum. The necessary 24ST aluminum data (derived using experimental 24ST aluminum results in the next section of this paper) are the curves of pressure versus particle velocity shown in Fig. 2. The curve from the origin is the locus of all pressure-particle velocity states attainable by propagating a right-going shock wave into normal undisturbed 24ST aluminum at P=0, $U_p=0$. When such a shock wave interacts with the 24ST aluminum-specimen interface, a left-going disturbance is reflected into the aluminum. The locus of P, U_p states that can be reached by the reflected disturbance is given by the appropriate cross curve in Fig. 2. For pressures greater (and particle velocities smaller) than that corresponding to the initial right-going shock wave, the cross curve corresponds to reflected shock waves. For smaller pressures (and greater particle velocities) the cross curve corresponds to a rarefaction wave which is reflected leftward from the interface. The P=0 point on each cross curve, in particular, corresponds to the aluminum free-surface velocity for that shock strength.

For the procedure below, it is necessary to identify the cross curve corresponding to the shock wave in the 24ST aluminum plate. This information is obtained by

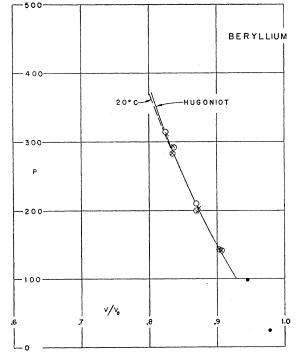


FIG. 3. Pressure-compression curves for beryllium. The solid curve in this figure and in Figs. 4 to 29 is an analytical fit (Table II) of Hugoniot curve experimental data obtained by the graphical solution method. Points plotted as circles in these figures are from the graphical solution method; points plotted as X's are from the free-surface approximation method. (See Sec. IB.) The dashed curve in each figure is a 20°C isotherm, computed using the Hugoniot curve and the methods given in Sec. II. Data points below 100 kilobars are from recent articles by P. W. Bridgman.

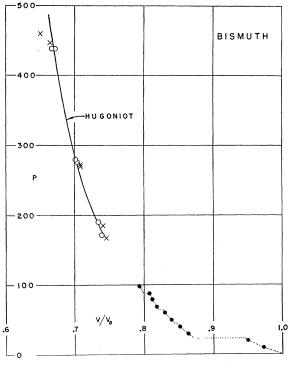


FIG. 4. Pressure-compression curves for bismuth. See caption to Fig. 3.

including a pair of 24ST pellets among the test specimens in each shot. The measured free-surface velocity then identifies the desired cross curve from its intercept with the particle velocity axis. Similarly, the measured shock wave velocity is used to construct a line from the origin in Fig. 2 of slope $\rho_0 U_s$ whose intercept with the above-described locus of right-going shocks determines [see Eq. (1) for $P_0 \approx 0$] the (P, U_p) point for the aluminum shock wave, and consequently determines the associated cross curve. The two determinations of the cross curve are averaged to give the value used in the succeeding analysis.

The boundary conditions that pressure and particle velocity must be continuous across the interface between the 24ST aluminum and the test specimen can now be used to construct a graphical solution. First, the known initial density ρ_0 and the measured shock velocity U_s for each specimen are used to construct a line from the origin (see Fig. 3) of slope $\rho_0 U_s$. From Eq. (1), for $P_0 \approx 0$, it is seen that the desired (P, U_p) point for the shock lies on this ray. The intersection of this ray with the aluminum cross curve then satisfies the boundary conditions and gives the desired pressure and particle velocity. The associated relative volume is then calculated by substituting this particle velocity and the measured shock velocity, Us, into Eq. (2). Pressurevolume points determined by this method are listed under Graphical Solutions in Table I and are plotted in Figs. 3-29 as circles.

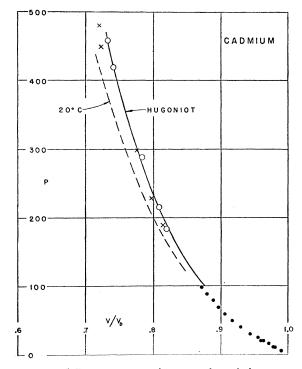
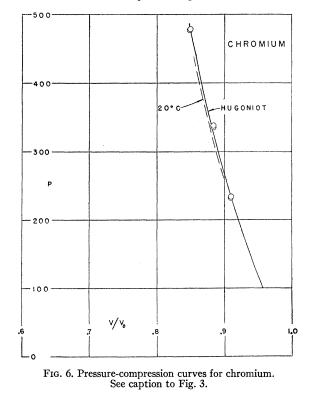


FIG. 5. Pressure-compression curves for cadmium. See caption to Fig. 3.



A few of the experiments reported in Table I were done without free-surface velocity measurements, so that only graphical solutions are listed. Iron and mag-

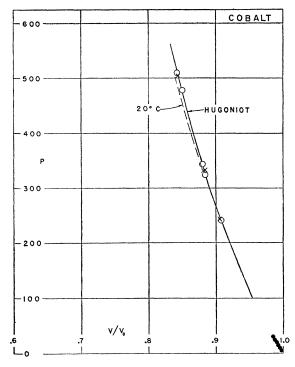


FIG. 7. Pressure-compression curves for cobalt. See caption to Fig. 3.

nesium experiments were performed without 24ST aluminum backer plates, so that only free-surface approximation solutions are reported.

C. Analytical Fits of Pressure-Volume Data

The Hugoniot curves which are drawn through the experimental points in Figs. 3-29 are reproduced by analytical fits of the form

$$P_H = A\mu + B\mu^2 + C\mu^3, \tag{5}$$

where

$$\mu \equiv (\rho/\rho_0) - 1 = (V_{0H}/V) - 1.$$

The values of A, B, and C for the various solids are listed in Table II. In reality this is a two-parameter fit, since the ratio B/A is determined theoretically by a method given in Sec. II. The two remaining parameters are then selected to fit the results of the present highpressure experiments. This procedure, as seen by inspection of the figures, gives a satisfactory fit of the present data, and it also forms a standardized extrapolation procedure from which rough comparisons will be made with the lower pressure data from static experimentation.

Finally, from the third mechanical conservation relation

$$E_{H} - E_{0H} = \frac{1}{2} (P_{H} + P_{0H}) (V_{0H} - V), \qquad (6)$$

it is seen that the specific internal energy E_H relative

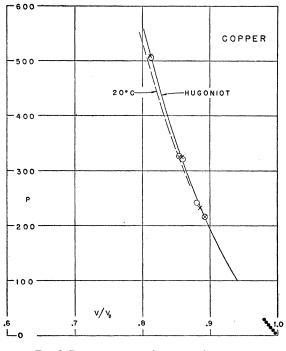


FIG. 8. Pressure-compression curves for copper. See caption to Fig. 3.

to the initial energy E_{0H} can be written (since $P_{0H} \doteq 0$)

$$E_H - E_{0H} = (A\mu^2 + B\mu^3 + C\mu^4) / [2\rho_0(\mu+1)].$$
(7)

Equations (5), (7), and Table II summarize all the experimental thermodynamic information which are available from the shock-wave measurements.

TABLE II. Analytical fits of Hugoniot curves, $P = A\mu + B\mu^2 + C\mu^3$, with pressure in kilobars.

| Metal | A | В | С |
|---------------|------|------|--------|
| Beryllium | 1182 | 1382 | C |
| Cadmium | 479 | 1087 | 2829 |
| Chromium | 2070 | 2236 | 7029 |
| Cobalt | 1954 | 3889 | 1728 |
| Copper | 1407 | 2871 | 2335 |
| Gold | 1727 | 5267 | 0 |
| Lead | 417 | 1159 | 1010 |
| Magnesium | 370 | 540 | 186 |
| Molybdenum | 2686 | 4243 | 733 |
| Nickel | 1963 | 3750 | 0 |
| Silver | 1088 | 2687 | 2520 |
| Thorium | 572 | 646 | 855 |
| Tin | 432 | 878 | 1935 |
| Titanium | 990 | 1168 | 1246 |
| Zinc | 662 | 1577 | 1242 |
| 24ST aluminum | 765 | 1659 | 428 |
| Brass | 1037 | 2177 | 3275 |
| Indium | 496 | 1163 | 02.00 |
| Niobium | 1658 | 2786 | ŏ |
| Palladium | 1744 | 3801 | 15 230 |
| Platinum | 2760 | 7260 | 0 |
| Rhodium | 2842 | 6452 | ő |
| Tantalum | 1790 | 3023 | Ő |
| Thallium | 317 | 938 | 1485 |
| Zirconium | 934 | 720 | 0 |

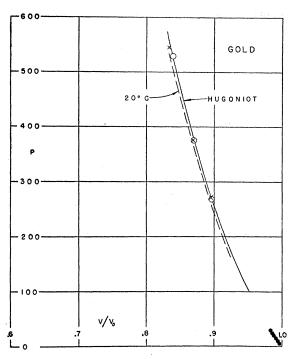
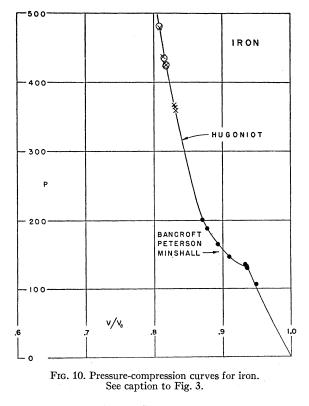


FIG. 9. Pressure-compression curves for gold. See caption to Fig. 3.



D. Zinc Single-Crystal Experiments

Two experiments with zinc single crystals were performed to determine whether observed compressions

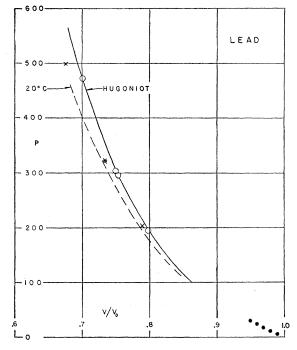


FIG. 11. Pressure-compression curves for lead. See caption to Fig. 3.

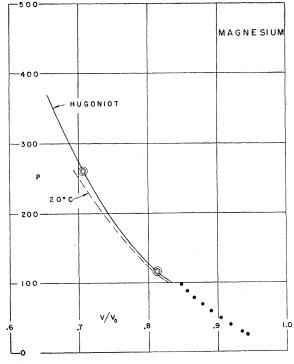


FIG. 12. Pressure-compression curves for magnesium. See caption to Fig. 3.

were dependent upon the crystal orientation. Three zinc crystals of known orientation (shock propagation directions along the C axis, along an A axis, and midway

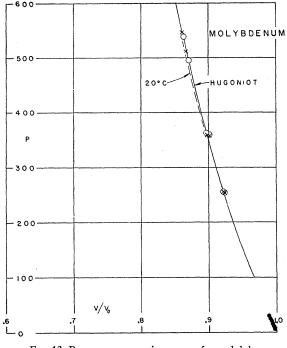


FIG. 13. Pressure-compression curves for molybdenum. See caption to Fig. 3.

between two A axes) were used in each shot. Shock velocity was measured for each crystal and shock strength was measured for the 24ST aluminum driver plate. The measured values were then used to determine pressure-volume points by the graphical solution method.

The total spread for the three zinc shock velocities in the high-pressure (414 kilobars) shot was 1%, and the corresponding spread for the low-pressure (200 kilobars) shot was 1.5%. Comparable scatters would be expected even if identical specimens were measured so that, within experimental error, one must conclude that results show no dependence of shock velocity (hence compressibility) upon crystallographic orientation. The measured shock velocities from each shot were averaged to obtain the value used in the graphical solutions. The data are listed under zinc in Table I and are also plotted in Fig. 19, where they exhibit good agreement with results obtained using ordinary cast polycrystalline zinc.

The shock wave results differ from the lower pressure static measurements⁵ which indicate a several-fold difference in compressibility depending upon whether the compression is parallel to the C axis or normal to it.

E. Discussion of Experimental Data

The probable error per data point, determined from the observed reproducibility, is 0.7% in shock velocity for a given free-surface velocity (about 1% in compression, $1-V/V_0$, at a given pressure). This estimate

⁵ P. W. Bridgman, Proc. Am. Acad. Arts Sci. 77, 189 (1949).

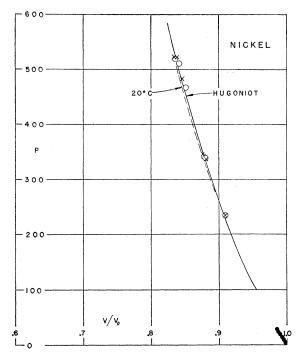


FIG. 14. Pressure-compression curves for nickel. See caption to Fig. 3.

does not apply, however, to the eight rare metals (In, Pd, Rh, Pt, Ta, Tl, Nb, Zr) at the bottom of Table I, the probable errors for which are approximately 3%in compression.

Previously reported¹ Hugoniot curves for 24ST aluminum, copper, and zinc are in substantial agreement with the present results. The present low-pressure (195-kilobar) zinc curve, however, indicates a 4%smaller compression than the previously drawn curve (Table III of reference 1). Similar comparisons of remaining data show agreement to 2% or better.

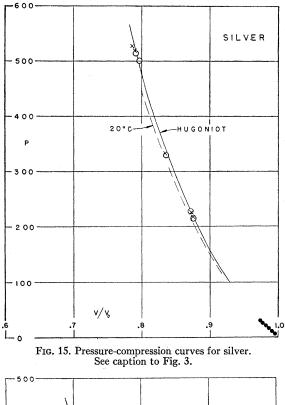
Present results may also be compared with shockwave data obtained by another Los Alamos group using an electrical pin-contactor method.² The agreement with their tabulated "recent data" (24ST aluminum data sufficient to determine the Hugoniot curve from 145 to 330 kilobars) is everywhere better than 1% in compression, and is sufficiently good to indicate freedom from sizable consistent error of either method.

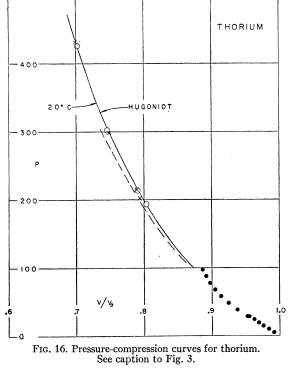
Impurities for the elements studied were determined by spectrochemical analysis. Specimens whose impurities exceeded 0.2% are: cobalt (0.5% Ni, 0.05% miscellaneous), nickel (0.1% Co, 0.05% each Mg, Si, Mn, Fe), titanium (0.05% each Al, Si, Cr, Mn, Sn).

The brass composition was 60.56% Cu, 39.31% zinc. The 24ST aluminum composition was 93.0% aluminum, 4.5% copper, 1.5% magnesium, 0.6% manganese.

F. Guide to Data Figures

The points below 100 kilobars, plotted as solid black disks in Figs. 3 to 29, are statically determined pressure-



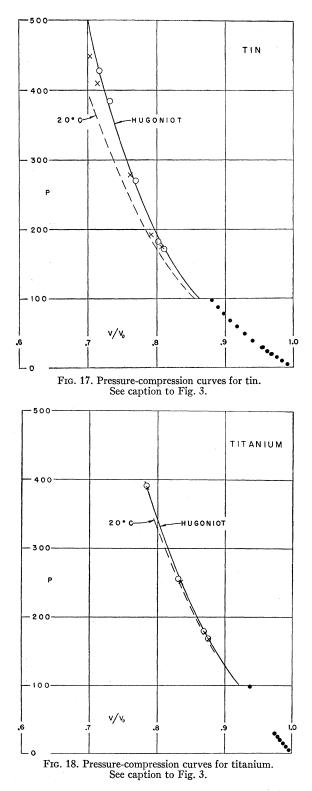


compression data taken from four articles by Bridgman.5-8

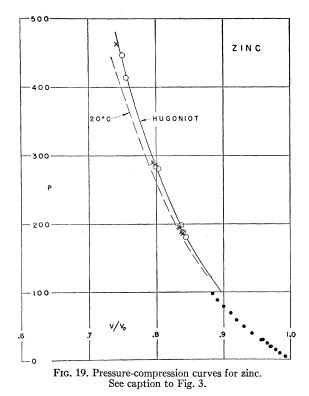
Except for the points around 400 kilobars, the iron

- ⁶ P. W. Bridgman, Proc. Am. Acad. Arts Sci. 76, 55 (1948).

⁷ P. W. Bridgman, Proc. Am. Acad. Arts Sci. **76**, 9 (1945).
 ***** P. W. Bridgman, Proc. Am. Acad. Arts Sci. **74**, 425 (1945).



data of Fig. 10 are reproduced from a recent article by Bancroft, Peterson, and Minshall.9



II. EQUATION OF STATE

The purpose of the present section is to provide a complete thermodynamic description of all states neighboring the experimental Hugoniot curves. To this end, the Mie-Grüneisen equation of state is employed, for which the volume dependence of the Grüneisen ratio is determined using the Dugdale-MacDonald relation. These considerations lead to a complete P, V, E equation of state. This equation of state and available zero-pressure data then permit the calculation of remaining thermodynamic data of interest, and numerical results are tabulated for the various metals.

A. Theory

Mie-Grüneisen Equation of State

For the thermodynamic states of interest here, we shall assume that the thermal energy of a metallic crystal can be adequately described by means of a set of simple harmonic oscillators (the normal modes of the dynamical system) whose frequencies, ν_{α} , are functions only of volume. The internal energy, E, is then given by¹⁰⁻¹²

$$E = \Phi + \frac{1}{2} \sum_{\alpha} h \nu_{\alpha} + \sum_{\alpha} \frac{h \nu_{\alpha}}{e^{h \nu_{\alpha}/kT} - 1}, \quad \alpha = 1, \cdots 3N, \quad (8)$$

¹⁰ See, for example, J. C. Slater, Introduction to Chemical Physics (McGraw-Hill Book Company, Inc., New York, 1939), Chap. ХШ.

⁹ Bancroft, Peterson, and Minshall, J. Appl. Phys. 27, 291 (1956).

 ¹¹ F. Seitz, Modern Theory of Solids (McGraw-Hill Book Company, Inc., New York and London, 1940), Chap. III.
 ¹² M. Born and K. Huang, Dynamical Theory of Crystal Lattices (Clarendon Press, Oxford, 1954), Chap. II.

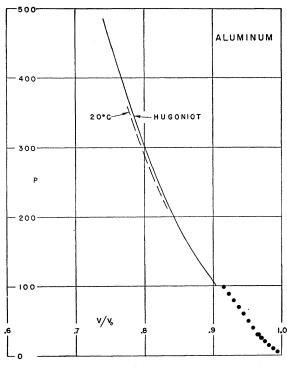


FIG. 20. Pressure-compression curves for 24ST aluminum. See caption to Fig. 3.

where Φ is the potential energy of the crystal with the particles at rest in their equilibrium positions, and the summation (\sum_{α}) is over the 3N normal modes of the crystal, N being the total number of atoms.

The pressure is given by

$$P = -\frac{d\Phi}{dV} + \frac{1}{V} \sum \gamma_{\alpha} \bigg\{ \frac{1}{2} h \nu_{\alpha} + \frac{h \nu_{\alpha}}{e^{h \nu_{\alpha}/kT} - 1} \bigg\}, \qquad (9)$$

where

$$\gamma_{\alpha} = -d \ln \nu_{\alpha}/d \ln V. \tag{10}$$

Equation (9) simplifies in two interesting cases. If all the γ_{α} are equal (the consequences of simplifying assumptions discussed below), these quantities may be factored from the summation as γ . Alternatively, in the classical limit, the energies of all oscillators are equal, so that these quantities may be factored and γ becomes the average of the γ_{α} . In either case Eq. (9) reduces to the equation of state of Mie and Grüneisen,¹³

$$P = -\frac{d\Phi}{dV} + \frac{\gamma}{V} E_{\rm vib},\tag{11}$$

where E_{vib} is the vibrational contribution to the internal energy. A rearrangement of the terms in Eq. (11) yields

$$P + \left\{ \frac{d\Phi}{dV} - \frac{\gamma}{V} \sum_{\alpha} \frac{1}{2} h \nu_{\alpha} \right\} = \frac{\gamma}{V} \sum_{\alpha} \frac{h \nu_{\alpha}}{e^{h \nu_{\alpha}/kT} - 1} \quad (12a)$$

¹³ E. Grüneisen, Handbuch der Physik (Verlag J. Springer, Berlin, 1926), Vol. 10, p. 22.

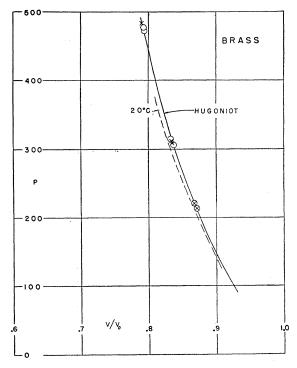


FIG. 21. Pressure-compression curves for brass, See caption to Fig. 3.

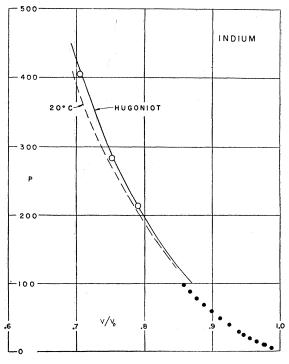


FIG. 22. Pressure-compression curves for indium. See caption to Fig. 3.

or $P - P_K = (\gamma/V)E_{\text{th}} = (\gamma/V)(E - E_K), \quad (12b)$

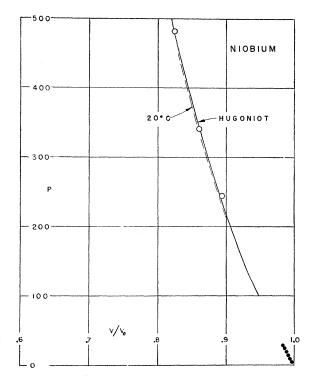


FIG. 23. Pressure-compression curves for niobium. See caption to Fig. 3.

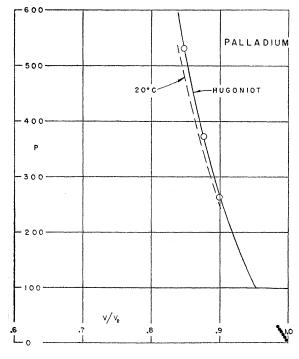
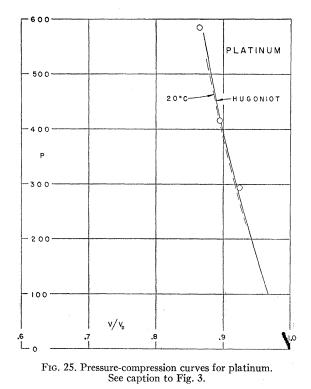


FIG. 24. Pressure-compression curves for palladium. See caption to Fig. 3.

where the subscript K refers to the quantities as a function of volume at 0°K. Grüneisen's ratio, γ , can be



expressed in terms of thermodynamic quantities by differentiating Eq. (12b). Since γ is a function only of volume,

$$\gamma = V \left(\frac{\partial P}{\partial E}\right)_{V} \equiv V \left(\frac{\partial P}{\partial T}\right)_{V} / C_{v}$$
$$\equiv -V \left(\frac{\partial P}{\partial V}\right)_{T} \left(\frac{\partial V}{\partial T}\right)_{P} / C_{v}$$
$$\equiv -V \left(\frac{\partial P}{\partial V}\right)_{S} \left(\frac{\partial V}{\partial T}\right)_{P} / C_{p}, \quad (13)$$

where C_v and C_p are the specific heats at constant volume and pressure, respectively. At zero pressure and room temperature γ can be evaluated from the measured values of the bulk modulus, thermal expansion coefficient, and specific heat; and for most metals the value so obtained lies between 1 and 3 (see Table III).

Equation (12b) can be rewritten in terms of any P, V, E curve, such as the experimental Hugoniot:

$$P - P_H = (\gamma/V)(E - E_H). \tag{14}$$

Determination below of the Grüneisen ratio $\gamma(V)$ then provides one with a complete P, V, E equation of state.

It is interesting to note, with respect to C_v , that the thermodynamic identity

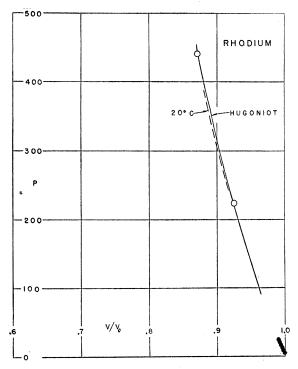


FIG. 26. Pressure-compression curves for rhodium. See caption to Fig. 3.

$$\left(\frac{\partial C_{v}}{\partial T}\right)_{s} = \frac{C_{v}}{T} + T\left(\frac{\partial P}{\partial T}\right)_{v} \left\{\frac{\partial}{\partial T}\left[\frac{C_{v}}{T(\partial P/\partial T)_{v}}\right]\right\}_{v}, \quad (15)$$

for γ a function only of volume, implies that C_v is a function only of entropy.

Grüneisen Ratio

Under the assumption that Poisson's ratio is independent of volume, Slater¹⁴ extended the Debye theory for an isotropic continuum to obtain

$$\gamma = -\frac{V}{2} \left(\frac{d^2 P/dV^2}{dP/dV} \right) - \frac{2}{3}.$$
 (16)

Dugdale and MacDonald¹⁵ proposed a modification of Slater's formula. Their result,

$$\gamma = -\frac{V}{2} \left(\frac{d^2 (PV^{\frac{2}{3}})/dV^2}{d(PV^{\frac{2}{3}})/dV} \right) - \frac{1}{3}, \tag{17}$$

follows for cubic lattices from the assumption that all of the interatomic force constants change the same (percentagewise) upon compression of the lattice.¹⁶

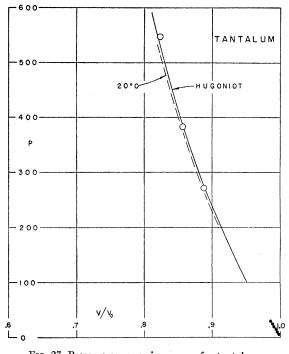


FIG. 27. Pressure-compression curves for tantalum. See caption to Fig. 3.

Equations (16) and (17) differ by 0.33 at P=0, with Eq. (16) indicating the larger value. For most metal P-V curves, the difference is less at higher pressure.

Quite recently, Barron^{17,18} extended the Born-von Kármán theory to an essentially exact calculation of the Grüneisen ratio for a few simple lattice models. His results show that the γ_{α} are far from equal, though equality of the γ_{α} is implied by the simplifying assumptions used to establish either Eq. (16) or Eq. (17). In the classical limit (the calculations indicate $T \gtrsim 0.3\Theta$, where Θ is the Debye temperature), the Mie-Grüneisen equation of state is again valid. Comparisons of the γ obtained from Eq. (16) or Eq. (17) with the hightemperature results by Barron indicate fair agreement, with the Dugdale-MacDonald formula, Eq. (17), more nearly reproducing Barron's results. [For example, a model of the NaCl lattice gives $\gamma = 2.3$ from Eq. (16), $\gamma = 2.0$ from Eq. (17), while Barron's calculations indicate $\gamma = 1.67.$]

At zero pressure, sufficient thermodynamic data exist to test the values of γ calculated from Eq. (16) or (17) against the thermodynamic values from Eq. (13). Slater^{19,20} and Gilvarry,²¹ using first and second derivatives of P obtained from Bridgman's data, have made extensive comparisons of the γ calculated from Eq. (16)

¹⁴ Reference 10, p. 239.

¹⁵ J. S. Dugdale and D. K. C. MacDonald, Phys. Rev. 89, 832 (1953).

¹⁶ Detailed in a forthcoming article "Compression of Solids by Strong Shock Waves" by Rice, McQueen, and Walsh, to appear in Solid State Physics, Advances in Research and Application (Academic Press, Inc., New York, 1957), Vol. VI. Zero-pressure tests of Eq. (17), results of which are summarized above, are also presented.

¹⁷ T. H. K. Barron, Ann. Phys. 1, 77 (1957). ¹⁸ T. H. K. Barron, Phil. Mag. 46, 720 (1955).

 ¹⁹ J. C. Slater, reference 10, Chap. XXVII.
 ²⁰ J. C. Slater, Phys. Rev. 57, 744 (1940).

²¹ J. J. Gilvarry, Phys. Rev. 102, 331 (1956).

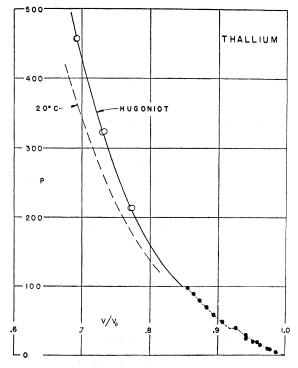


FIG. 28. Pressure-compression curves for thallium. See caption to Fig. 3.

with that from Eq. (13). For most metals the agreement is quite good. A similar comparison, using the present shock-wave data, has been carried out by the present authors.¹⁶ Such a test is of interest because the shock wave results limit properly at zero pressure to the desired adiabatic first and second derivatives; also the extended data region of the present experimentation should lead to better precision in the determination of second derivatives of P. The tests, which show less scatter than the previous work, indicate somewhat more success for the Dugdale-MacDonald formula, Eq. (17), than for Eq. (16), though both are in substantial agreement with the experimental results. The former reproduces the values of γ from Eq. (13) with an average error of 15%.

In the following calculations, an attempt is made to obtain the most accurate description of high-pressure states. Accordingly, the experimental thermodynamic properties, i.e., of specific heats, thermal expansions, and γ_0 , are employed along the zero-pressure isobars, and only the volume variation of γ is estimated by an empirical relation. The Dugdale-MacDonald formula, Eq. (17), is employed for this purpose. It is, of course, clear that the calculated values of γ , at high pressures, are not very accurate. In regard to calculated P-V curves, on the other hand, γ is used only to estimate the small offsets (typically 1% in volume) from the experimental Hugoniot and errors as large as 25% in the offsets (i.e., approximately 25% in γ) lead to uncer-

| Metal | γ 0 | A | В | С |
|---------------|------------|---------|---------|---------|
| Beryllium | 1.17 | -2.523 | 12.990 | -31.851 |
| Cadmium | 2.27 | 13.417 | -75.631 | 72.965 |
| Chromium | 1.08 | 10.965 | -54.874 | 49.000 |
| Cobalt | 1.99 | -5.906 | 26.354 | -48.076 |
| Copper | 2.04 | -3.296 | 10.493 | -19.264 |
| Gold | 3.05 | -21.876 | 115.18 | -213.17 |
| Lead | 2.78 | -8.406 | 22.791 | -22.648 |
| Magnesium | 1.46 | -2.078 | 4.621 | -4.840 |
| Molybdenum | 1.58 | -4.600 | 25.837 | -61.398 |
| Nickel | 1.91 | -8.007 | 35.275 | -59.812 |
| Silver | 2.47 | -5.670 | 19.334 | 32.891 |
| Thorium | 1.124 | 3.552 | -14.223 | 15.552 |
| Tin | 2.03 | 9.4186 | -52.133 | 66.016 |
| Titanium | 1.18 | 2.225 | -9.904 | 11.052 |
| Zinc | 2.38 | -6.087 | 18.626 | -23.535 |
| 24ST aluminum | 2.13 | -7.245 | 24.707 | -32.577 |
| Brass | 2.04 | 3.405 | -26.304 | 38.692 |
| Indium | 2.238 | -9.431 | 27.392 | -26.186 |
| Niobium | 1.679 | -5.882 | 26.261 | -49.145 |
| Palladium | 2.183 | 26.824 | -205.44 | 407.72 |
| Platinum | 2.627 | -16.911 | 100.10 | -216.84 |
| Rhodium | 2.265 | -11.228 | 55.898 | -109.85 |
| Tantalum | 1.689 | -5.166 | 15.925 | -18.991 |
| Thallium | 2.96 | -3.617 | 2.264 | -1.171 |
| Zirconium | 0.771 | -0.449 | 0.285 | -0.102 |
| | | | | |

TABLE III. Analytical fits of Grüneisen ratios. $\gamma = \gamma_0 + A \mu + B \mu^2 + C \mu^3$.

tainties which are only comparable to probable errors in the experimental curves. Temperature increases, calculated along constant-entropy curves, reflect an error which is roughly proportional to the volumeaverage error in γ . The use of the correct γ_0 at normal

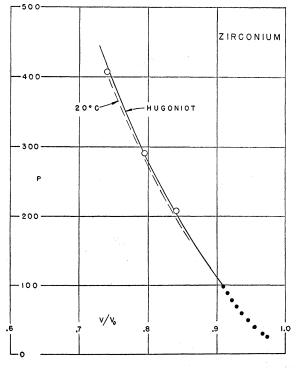


FIG. 29. Pressure-compression curves for zirconium. See caption to Fig. 3.

volume and an experimental additive term in the listed temperatures (T at P=0 on the adiabat through the point) then lead to temperatures which should be reliable to 10% or less of their centigrade values.

B. Calculations

Combining Eq. (17), which is assumed to be valid along the 0°K curve, with Eq. (14) yields the following equation for the pressure $P_K(V)$ at 0°K, in terms of the known Hugoniot curve, $P_H(V)$:

$$\frac{-\frac{1}{2}Vd^{2}(P_{K}V^{\frac{3}{2}})/dV^{2}}{d(P_{K}V^{\frac{3}{2}})/dV} - \frac{1}{3}}$$

$$= \frac{V(P_{H} - P_{K})}{\frac{1}{2}P_{H}(V_{0H} - V) + E_{0H} + \int_{V_{0K}}^{V} PdV}.$$
(18)

The initial conditions required for the integration of Eq. (18) are the specific volume, the compressibility at 0°K, $P_K=0$, and the specific internal energy, E_{0H} , at the foot of the Hugoniot (relative to an arbitrary zero energy at P=0, T=0°K). These quantities were obtained from rough extrapolations²² of available zero pressure data to 0°K. Zero-degree-Kelvin pressure volume curves, obtained by numerical integration of Eq. (18), are listed in Table IV. Analytical fits of the associated $\gamma(V)$ curves, which may now be obtained from either Eq. (14) or Eq. (17). are given in Table III.

Differentiating Eq. (14) gives

$$P_{A} = -\frac{dE_{H}}{dV} + (P_{H} - P_{A})\frac{d}{dV}\left(\frac{V}{\gamma}\right) + \frac{V}{\gamma}\left(\frac{dP_{H}}{dV} - \frac{dP_{A}}{dV}\right),\tag{19}$$

which is a first oddrer differential equation for an adiabat, $P_A(V)$, in terms of the known $P_H(V)$, $E_H(V)$,

 $[\gamma_{0H} + (d\gamma/dV)_{0H}(V_{0H} - V_{0K})][E_{0H} + \frac{1}{2}P_H(V_{0H} - V_{0K})] = V_{0K}P_H,$

where the first bracket is an approximate value for γ at V_{0K} . The compressibility at 0°K, $P_K=0$, was obtained from the derivative of Eq. (14) evaluated at $V=V_{0K}$:

$$V_{0K}(dP_K/dV)_{V=V_{0K}} = [P_H - E_H(d\gamma/dV) - \gamma(dE_H/dV)]_{V=V_{0K}}.$$

Approximate values of $d\gamma/dV$, necessary for the evaluation of each of the above relations, were obtained from Eq. (17), and the assumption that the right side of Eq. (17) can be determined using the adiabat through the foot of the Hugoniot curve. Consequent values of γ_0 and $d\gamma/dV$ are

$$\gamma_{0H} = B/A,$$

$$V_{0H} \left(\frac{d\gamma}{dV}\right)_{0H} = \frac{5}{2} \frac{B^2}{A^2} \frac{7}{6} \frac{B}{A} - \frac{3}{3} \frac{C}{-9}.$$

The first of these relations was also used in Sec. I to determine the values of B/A used in analytical fits of the experimental data.

and $\gamma(V)$. Equation (19) was integrated numerically to obtain serveral adiabats for each metal. Two of these, the adiabat which coincides with the foot of the Hugoniot and an adiabat which intersects the Hugoniot curve at a pressure near the upper limit of the experimental data, are listed in Table IV.

At constant entropy the thermodynamic identity

$$TdS = C_v dT + T(\partial P/\partial T)_V dV = 0$$

can be integrated to give temperature as a function of volume:

$$T = T_i \exp\left[-\int_{V_i}^{V} \frac{\gamma}{V} dV\right].$$
 (20)

Here the relation $\gamma = V(\partial P/\partial T)_V/C_v$ was used, and T_i is an initial temperature at some volume V_i on the adiabat. Values of T_i , V_i were obtained along P=0from available thermal expansion data. Equation (20) can then be used together with the adiabatic P-V curves determined above to obtain the temperature at any P-V point neighboring the Hugoniot curve. Resulting temperatures for pressure-volume points along two adiabats are listed in Table IV. Temperatures along the Hugoniot curve for each metal are also listed in Table IV and the 20°C isotherms are plotted in Figs. 3–29.

Calculated values of the ratio

$$\frac{U_{r}}{U_{p}} = \int_{0}^{P_{H}} (-dV/dP_{A})^{\frac{1}{2}} dP_{A} / [P_{H}(V_{0H} - V)]^{\frac{1}{2}}$$
(21)

(see Eq. 4 and associated discussion) are listed in Table V for the various solids. The denominator in Eq. (21) is the expression for the shock wave particle velocity corresponding to shock pressure P_H and is a consequence of Eqs. (1) and (2). The numerator²³ is the Reimann integral for the particle velocity due to the centered, simple rarefaction wave which relieves the pressure from P_H to zero, and is evaluated using the adiabat which intersects the Hugoniot curve at P_H . The refinements indicated by the present calculations cause a slight shift of the P, V points, plotted as \times 's in the figures, which were obtained by using Eq. (4). The corrections reduce the compression offsets (between the \times 's and the curves drawn through the graphical solutions) from an average magnitude of 1.1% to 0.7%.

It should be noted that three metals, lead, tin and cadmium, exhibit free-surface-approximation solutions which are in sizable disagreement with results obtained from the graphical solutions. The three are all lowmelting-point metals and are, indeed, the only metals (among those for which free-surface velocity measure-

²² The values of E_{0H} were obtained by integrating the Debye specific heat curve. Values of the initial volume, V_{0K} , were those which satisfied Eq. (14) evaluated at $V = V_{0K}$:

²³ See, for example, R. Courant and K. O. Friedricks, *Supersonic Flow and Shock Waves* (Interscience Publishers, Inc., New York, 1948). The present expression can be obtained from their Eq. (34.05).

| $\sum P(\text{kilobars})$ | 0 | 100 | 150 | 200 | 250 | 300 | 350 | 400 | 450 | 500 |
|--|---|--|--|---|---|--|---|--|--|---|
| Beryllium 0°K Hugoniot T_H Adiabat T_A | 0.9982 1.000 20°C 1.000 20°C | 0.9263 0.9277 50° 0.9276 45 | 0.8972 0.8990 70° 0.8987 55 | 0.8716 0.8725 97° 0.8728 64 | 0.8487 0.8503 127° 0.8493 74 | 0.8275 0.8299 168° 0.8284 82 | 0.8082 0.8110 213° | | | |
| Adiabat (309 kilobars) T _A | 1.003 97°C | 0.9301 129 | 0.9010 143 | 0.8742 155 | 0.8506 164 | 0.8299 181 | 0.8103 187 | | | |
| Cadmium $0^{\circ}K$ Hugoniot T_H Adiabat T_A Adiabat | 0.9764 1.000 20°C 1.000 20°C | 0.8524 0.8742 210 0.8708 157 | 0.8188 0.8403 349 0.8339 213 | 0.7932 0.8141 515 0.8050 257 | 0.7716 0.7932 697 0.7819 297 | 0.7527 0.7752 895 0.7620 331 | 0.7364 0.7597 1111 0.7450 359 | 384 | 1563 0.7164 408 | 0.6972 0.723 1800 0.704 431 |
| (318 kilobars) T_A | 1.028 310°C | 0.8976 559 | $\begin{array}{c} 0.8568\\674\end{array}$ | 0.8244 784 | 0.7982 866 | 0.7763 945 | 0.7573 1014 | 0.7414 1072 | 0.7265 1125 | 0.713 1177 |
| Chromium $0^{\circ}K$ Hugoniot T_H Adiabat T_A Adiabat (522 kilobars) | 0.9969 1.000 20°C 1.000 20°C 1.003 | 0.9528 0.9561 39 0.9561 37 0.9595 | 0.9336 0.9373 54 0.9372 46 0.9410 | 0.9160 0.9202 73 0.9201 56 0.9238 | 0.9006 0.9050 96 0.9044 66 0.9081 | 0.8860 0.8908 126 0.8899 76 0.8933 | 0.8730 0.8779 161 0.8717 86 0.8798 | 0.8607 0.8658 199 0.8642 95 0.8674 | 0.8492 0.8548 242 0.8529 103 0.8560 | 0.8387 0.8447 291 0.8421 112 0.8449 |
| | 170°C | 194 | 209 | 223 | 238 | 253 | 267 | 281 | 295 | 307 |
| Cobalt 0°K Hugoniot T_H Adiabat T_A Adiabat (523 kilobars) | 0.9931 1.000 20°C 1.000 20°C 1.005 | $0.9500 \\ 0.9552 \\ 48 \\ 0.9552 \\ 45 \\ 0.9581$ | 0.9318 0.9368 65 0.9366 57 0.9392 | 0.9150 0.9200 86 0.9197 67 0.9221 | 0.9000 0.9050 109 0.9045 76 0.9065 | 0.8863 0.8915 134 0.8905 86 0.8926 | 0.8734 0.8782 166 0.8770 96 0.8792 | 0.8112 0.8160 202 0.8145 104 0.8165 | 0.8500 0.8551 241 0.8531 111 0.8551 | $\begin{array}{r} 0.8395\\ 0.8450\\ 283\\ 0.8422\\ 117\\ 0.8447\end{array}$ |
| T_A | 120°C | 155 | 171 | 185 | 199 | 211 | 224 | 235 | 245 | 254 |
| Copper 0°K Hugoniot T_H Adiabat T_A Adiabat (504 kilobars) | 0.9903 1.000 20°C 1.000 20°C 1.010 | 0.9337 0.9412 63 0.9408 57 0.9490 | 0.9109 0.9186 89 0.9180 73 0.9255 | 0.8908 0.8980 121 0.8974 85 0.9038 | 0.8732 0.8803 158 0.8781 99 0.8852 | 0.8571 0.8643 201 0.8625 111 0.8683 | $\begin{array}{c} 0.8424\\ 0.8500\\ 255\\ 0.8472\\ 123\\ 0.8528\end{array}$ | 0.8290 0.8370 311 0.8334 134 0.8387 | $0.8165 \\ 0.8249 \\ 373 \\ 0.8209 \\ 145 \\ 0.8255$ | $0.8049 \\ 0.8137 \\ 444 \\ 0.8092 \\ 155 \\ 0.8143$ |
| T_A | 215°C | 279 | 306 | 331 | 353 | 375 | 395 395 | 415 | 432 | 448 448 |
| Gold $0^{\circ}K$ Hugoniot T_H Adiabat T_A | 0.9900 1.000 20°C 1.000 20°C | 0.9458 0.9521 65 0.9520 59 | 0.9277 0.9334 96 0.9331 74 | 0.9113 0.9167 121 0.9161 86 | 0.8966 0.9018 153 0.9014 97 | 0.8829 0.8882 200 0.8874 108 | 0.8707 0.8759 253 0.8744 117 | 0.8589 0.8643 311 0.8626 125 | 0.8480 0.8537 372 0.8514 133 | 0.8377 0.8438 443 0.8412 141 |
| $\begin{array}{l} \text{Adiabat} \\ \text{(518 kilobars)} \\ T_A \end{array}$ | 1.009 235°C | 0.9583 311 | 0.9382 338 | 0.9210 361 | 0.9053 383 | 0.8910 401 | 0.8779 419 | 0.8660 434 | $0.8546 \\ 449$ | $\begin{array}{c} 0.8441\\ 464 \end{array}$ |
| Lead $0^{\circ}K$ Hugoniot T_H Adiabat T_A Adiabat | 0.9762 1.000 20°C 1.000 20°C | 0.8492 0.8623 228 0.8600 139 | 0.8113 0.8253 375 0.8203 179 | 0.7808 0.7958 609 0.7888 210 | 0.7558 0.7722 861 0.7628 241 | $0.7340 \\ 0.7523 \\ 1150 \\ 0.7403 \\ 267$ | 0.7150 0.7348 1459 0.7206 289 | 0.6979 0.7191 1812 0.7034 310 | 0.6829 0.7051 2192 0.6878 329 | 0.6690 0.6930 2575 0.6736 360 |
| (204 kilobars) T_A | 1.020 249°C | 0.8695 485 | 0.8288 563 | 0.7959 624 | 0.7696 682 | 0.7465 730 | 0.7266 773 | 0 7585 814 | 0.7426 854 | 0.7280 891 |

TABLE IV. Pressure-volume loci and associated temperatures (degrees centigrade). The first adiabat listed for each material coincides with the Hugoniot curve at 20°C, zero pressure. The second adiabat intersects the Hugoniot curve near the high-pressure limit of the experimental data. The pressure at which the second adiabat crosses the Hugoniot curve is given in parentheses.

| $\searrow P(\text{kilobars})$ | 0 | 100 | 150 | 200 | 250 | 300 | 350 | 400 | 450 | 500 |
|--|-------------------------|-------------------------|-------------------------|-------------------------|---------------------------|---------------------------|---------------------------|--------------------------|--------------------------|--------------------------|
| Magnesium 0°K Hugoniot | 0.9985 1.000 | 0.8200 0.8300 | 0.7604 0.7712 | 0.7304 0.7432 | 0.6980 0.7123 | 0.6699 0.6861 | | | | - |
| T_H Adiabat T_A | 20°C 1.000 20°C | $174 \\ 0.8278 \\ 74$ | 313 0.7670 131 | 487 0.7362 155 | 691 0.7032 176 | 923 0.6737 196 | | | | |
| Adiabat (229 kilobars) T _A | 1.023 296°C | 0.8395 447 | 0.7769 526 | 0.7450 576 | 0.7110 620 | 0.6816 660 | | | | |
| Molybdenum 0°K | 0.9971 | 0.9636 | 0.9486 | 0.9347 | 0.9219 | 0.9096 | 0.8979 | 0.8870 | 0.8768 | 0.8670 |
| $\begin{array}{l} \operatorname{Hugoniot} \\ T_H \end{array}$ | 1.000 20°C | 0.9660 37 | 0.9510 49 | 0.9347 0.9369 62 | 0.9240 79 | 0.9090 0.9119 101 | 0.9002 125 | 0.8895 154 | 0.8794 188 | 0.8698 |
| Adiabat T _A Adiabat | 1.000 20°C | 0.9659 35 | 0.9508 43 | 0.9368 49 | 0.9239 55 | 0.9117 61 | 0.8999 67 | 0.8890 72 | 0.8786 77 | 0.8685 83 |
| (500 kilobars) T_A | 1.002 139°C | 0.9674 161 | 0.9522 171 | 0.9382 180 | 0.9250 189 | 0.9128 197 | 0.9011 205 | 0.8900 213 | 0.8799 221 | 0.8696 228 |
| Nickel 0°K | 0.9930 | 0.9551 | 0.9320 | 0.9151 | 0.8999 | 0.8855 | 0.8720 | 0.8599 | 0.8480 | 0.8370 |
| Hugoniot T_H | 1.000 20°C | 0.9501 0.9501 43 | 0.9320 0.9367 61 | 0.9131 0.9197 79 | 0.8999 0.9042 101 | 0.8855 0.8900 125 | 0.8720 0.8769 150 | 0.8399 0.8643 181 | 0.8480 0.8530 217 | 0.8422 |
| $\begin{array}{c} \text{Adiabat} \\ T_{A} \\ \text{Adiabat} \end{array}$ | 1.000 20°C | 0.9500 43 | 0.9364 54 | 0.9195 63 | 0.9038 71 | 0.8890 79 | 0.8758 86 | 0.8633 93 | 0.8517 100 | 0.8401 106 |
| Adiabat (508 kilobars) T _A | 1.005 136°C | 0.9588 171 | 0.9397 186 | 0.9220 199 | 0.9062 210 | 0.8918 221 | 0.8781 231 | 0.8653 241 | 0.8536 251 | 0.8423 259 |
| Silver | 0.0070 | 0.0000 | 0.0051 | 0.0724 | 0.0545 | 0.0100 | 0.0000 | 0.0000 | 0 50 40 | 0 5000 |
| 0°K Hugoniot <i>T_H</i> | 0.9878 1.000 20°C | 0.9202 0.9291 88 | 0.8951 0.9037 130 | 0.8734 0.8818 186 | $0.8547 \\ 0.8632 \\ 255$ | $0.8380 \\ 0.8470 \\ 326$ | $0.8229 \\ 0.8322 \\ 417$ | 0.8090 0.8190 520 | 0.7960 0.8066 627 | 0.7833 0.7953 747 |
| $\begin{array}{c} \text{Adiabat} \\ T_{\boldsymbol{A}} \end{array}$ | 1.000 20°C | 0.9286 74 | 0.9028 96 | 0.8804 114 | 0.8612 131 | 0.8437 148 | 0.8279 168 | 0.8138 176 | 0.8006 189 | 0.7886 201 |
| Adiabat (479 kilobars) T _A | 1.017 314°C | 0.9406 433 | 0.9133 483 | 0.8900 524 | 0.8697 560 | 0.8520 594 | 0.8353 627 | 0.8207 656 | 0.8074 682 | 0. 7980 708 |
| Thorium | | | | | | | | | | |
| 0°K Hugoniot T _H | 0.9920 1.000 20°C | 0.8643 0.8727 122 | 0.8222 0.8310 227 | 0.7878 0.7979 377 | $0.7594 \\ 0.7705 \\ 552$ | 0.7348 0.7473 752 | 0.7128 0.7274 969 | 0.6939 0.7099 1197 | 0.6767 0.6943 1419 | 0.6610 0.6806 1631 |
| Adiabat T_A | 1.000 20°C | 0.8716 76 | 0.8283 101 | 0.7938 124 | 0.7647 145 | 0.7396 164 | 0.7176 181 | 0.6979 197 | 0.6802 212 | 0.6634 |
| Adiabat (483 kilobars) T _A | 1.035 802°C | 0.8989 1003 | 0.8540 1095 | 0.8163 1183 | 0.7856 1263 | 0.7586 1339 | 0.7352 1405 | 0.7146 1467 | 0.6960 1525 | 0.6784 1581 |
| Tin | | | | | | | | | | |
| 0°K Hugoniot T | 0.9806 1.000 20°C | 0.8439 0.8614 219 | 0.8068 0.8248 377 | 0.7778 0.7964 556 | 0.7540 0.7735 | 0.7334 0.7544 953 | 0.7159 0.7382 1139 | 0.7002 0.7240 1318 | | |
| T_H Adiabat T_A | 1.000 20°C | 0.8580 149 | 0.8183 198 | 0.7877 239 | 752 0.7622 275 | 0.7414 305 | 0.7229 332 | 0.7062 358 | | |
| Adiabat (205 kilobars) T _A | 1.012 208°C | 0.8710 415 | 0.8297 497 | 0.7970 569 | 0.7703 631 | 0.7479 683 | 0.7289 731 | 0.7120 773 | | |
| Titanium | | | | | | | | | | |
| 0°K Hugoniot | 0.9944 1.000 20°C | 0.9119 0.9170 | 0.8801 0.8857 | 0.8527 0.8587 | 0.8290 0.8354 | 0.8077 0.8148 | 0.7888 0.7964 394 | 0.7713 0.7809 | | • • • |
| T _H Adiabat T _A | 1.000 20°C | 65 0.9168 54 | 105 0.8851 70 | 155 0.8576 84 | 223 0.8334 97 | 302 0.8118 111 | 0.7921 123 | 491 0.7730 135 | | |
| Adiabat (340 kilobars) T _A | 1.005 210°C | 0.7222 266 | 0,8898 290 | 0.8618 314 | 0.8375 337 | 0.8155 359 | 0.7963 380 | 0.7776 400 | | 2122 14 |

TABLE IV.—Continued.

| ∑ P(kilobars) | 0 | 100 | 150 | 200 | 250 | 300 | 350 | 400 | 450 | 500 |
|---|-------------------|--------------------|------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| Zinc | | | | | | | | | | |
| 0°K Hugoniot | 0.9800 1.000 | 0.8834 0.8960 | 0.8507 0.8633 | 0.8234 0.8363 | 0.8004 0.8140 | 0.7802 0.7942 | 0.7622 0.7767 | 0.7458 0.7615 | 0.7310 0.7482 | 0.7180 0.7360 |
| T_H | 20°C | 119 | 187 | 272 | 369 | 482 | 609 | 747 | 900 | 1061 |
| $\begin{array}{c} \text{Adiabat} \\ T_A \end{array}$ | 1.000 20°C | 0.8949 96 | 0.8608 123 | 0.8328 147 | 0.8082 | 0.7875 189 | 0.7690 207 | 0.7522 224 | 0.7371 239 | 0.7232 254 |
| Adiabat | | | | | | | | | | |
| (404 kilobars) T_A | 1.030 302°C | 0.9126 479 | 0.8760 540 | 0.8456 593 | 0.8208 641 | 0.7985 681 | 0.7792 720 | 0.7619 756 | 0.7460 791 | 0.7318 821 |
| Aluminum | | | | | | | | | | |
| 0°K Hugoniot | $0.9874 \\ 1.000$ | 0.8966 0.9045 | 0.8641 0.8716 | $0.8362 \\ 0.8441$ | 0.8126 0.8210 | 0.7915 0.8008 | 0.7724 0.7824 | $0.7553 \\ 0.7661$ | $0.7400 \\ 0.7513$ | 0.7255 0.7380 |
| T_H | 20°C | 94 | 153 | 223 | 308 | 405 | 518 | 637 | 770 | 909 |
| Adiabat T_{A} | 1.000 20°C | 0.9036 78 | 0.8701 100 | 0.8422 119 | 0.8180 135 | 0.7961 150 | 0.7770 163 | 0.7594 177 | 0.7435 189 | 0.7288 201 |
| Adiabat (513 kilobars) | 1.034 | 0.9250 | 0.8886 | 0.8578 | 0.8319 | 0.8092 | 0.7892 | 0.7710 | 0.7543 | 0.7392 |
| T_A | 454°C | 633 | 703 | 755 | 799 | 841 | 881 | 915 | 949 | 981 |
| Brass | | | | | | | | ÷ ===== | | |
| 0°K Hugoniot | 0.9869 1.000 | 0.9140 0.9250 | 0.8876 0.8984 | 0.8649 0.8758 | 0.8453 0.8564 | 0.8283 0.8395 | $0.8130 \\ 0.8250$ | 0.7990 0.8115 | 0.7864 0.7992 | $0.7745 \\ 0.7882$ |
| T_H | 20°C | 89 | 129 | 175 | 235 | 305 | 382 | 467 | 557 | 651 |
| $\begin{array}{c} \text{Adiabat} \\ T_{A} \end{array}$ | 1.000 20°C | 0.9244 73 | 0.8971 96 | 0.8739 118 | 0.8538 137 | 0.8360 155 | 0.8200 171 | 0.8056 187 | 0.7922 201 | 0.7803 215 |
| Adiabat (446 kilobars) | 1.014 | 0.9363 | 0.9081 | 0.8839 | 0.8620 | 0.8442 | 0.8277 | 0.8129 | 0.7991 | 0.7860 |
| T_A | 230°C | 323 | 363 | 401 | 436 | 468 | 499 | 527 | 552 | 577 |
| Indium | | | | | | | | | | |
| 0°K Hugoniot | $0.9801 \\ 1.000$ | 0.8604 0.8701 | 0.8210 0.8302 | 0.7880 0.7979 | 0.7600 0.7710 | $0.7351 \\ 0.7478$ | 0.7135 0.7270 | $0.6943 \\ 0.7087$ | 0.6769 0.6922 | $0.6610 \\ 0.6774$ |
| T_H | 20°C | 153 | 260 | 397 | 561 | 745 | 950 | 1179 | 1439 | 1710 |
| Adiabat T _A | 1.000 20°C | 0.8687 99 | 0.8276 124 | 0.7939 144 | 0.7650 163 | 0.7400 181 | 0.7180 195 | 0.6983 210 | 0.6800 223 | 0.6624 233 |
| Adiabat (474 kilobars) | 1.002 | 0.8969 | 0.8507 | 0.8142 | 0.7833 | 0.7566 | 0.7336 | 0.7124 | 0.6936 | 0.6760 |
| T_A | 1.002 | 1055°C | 1157 | 1242 | 1314 | 1381 | 1439 | 1495 | 1547 | 1593 |
| Niobium | | | | | | | | | | |
| 0°K Hugoniot | $0.9951 \\ 1.000$ | $0.9440 \\ 0.9476$ | 0.9226 0.9260 | 0.9032 0.9067 | $0.8856 \\ 0.8894$ | $0.8694 \\ 0.8730$ | $0.8544 \\ 0.8582$ | $0.8404 \\ 0.8449$ | 0.8271 0.8321 | 0.8148 0.8197 |
| T_H | 20°C | 49 | 73 | 97 | 133 | 177 | 227 | 284 | 351 | 427 |
| $\begin{array}{c} \text{Adiabat} \\ T_{\boldsymbol{A}} \end{array}$ | 1.000 20°C | $0.9475 \\ 45$ | 0.9256 55 | 0.9061 65 | 0.8885 73 | 0.8721 81 | 0.8566 89 | 0.8427 96 | 0.8296 103 | 0.8171 110 |
| Adiabat | | 0.9526 | 0.9302 | 0.9104 | 0.8923 | 0.8756 | 0.8601 | 0.8459 | 0.8326 | 0.8200 |
| (528 kilobars) T _A | 1.006 287°C | 335 | 358 358 | 377 | 393 | 409 | 424 | 439 | 452 | 465 |
| Palladium | | | | | | | | | | |
| 0°K Hugoniot | 0.9918 1.000 | 0.9422 0.9520 | 0.9233 0.9330 | 0.9071 0.9170 | 0.8930 0.9029 | 0.8808 0.8903 | 0.8696 0.8792 | 0.8596 0.8692 | $0.8500 \\ 0.8600$ | 0.8410 0.8513 |
| T_H | 20°C | 65 | 97 | 135 | 180 | 231 | 289 | 353 | 423 | 497 |
| Adiabat T _A | 1.000 20°C | 0.9517 61 | 0.9326 79 | 0.9159 104 | 0.9011 125 | 0.8882 143 | 0.8766 159 | 0.8658 177 | 0.8556 192 | 0.8466 |
| Adiabat (481 kilobars) | 1.006 | 0.9587 | 0.9399 | 0.9230 | 0.9080 | 0.8941 | 0.8820 | 0.8710 | 0.8609 | 0.8512 |
| T_A | 1.000 187°C | 24 6 | 278 | 311 | 343 | 375 | 403 | 429 | 454 | 477 |
| Platinum | | | | | 0.00 | | 0.0075 | 0.0075 | | |
| 0°K Hugoniot | 0.9940 1.000 | 0.9632 0.9679 | 0.9500 0.9540 | 0.9377 0.9412 | 0.9260 0.9298 | $0.9154 \\ 0.9190$ | 0.9052 0.9087 | 0.8959 0.8993 | 0.8868 0.8903 | 0.8779 0.8819 |
| T_H | 20°C | 46 | 60 | 77 | 95 | 117 | 144 | 174 | 207 | 244 |
| Adiabat T_{A} | 1.000 20°C | 0.9678 44 | 0.9539 54 | 0.9410 63 | 0.9292 71 | 0.9183 78 | 0.9081 85 | 0.8985 91 | 0.8891 97 | 0.8806 102 |
| Adiabat (481 kilobars) | 1.003 | 0.9697 | 0.9567 | 0.9429 | 0.9310 | 0.9199 | 0.9096 | 0.8999 | 0.8905 | 0.8819 |
| (HOL KHODATS) | 119°C | 151 | 165 | 177 | 187 | 197 | 207 | 216 | 224 | 231 |

TABLE IV.—Continued.

| $\searrow P(kilobars)$ | 0 | 100 | 150 | 200 | 250 | 300 | 350 | 400 | 450 | 500 |
|------------------------|--------|--------|--------|--------|--------|--------|--------|-----------|--------|--------|
| Rhodium | | | | | | | | · · · · · | | |
| 0°K | 0.9946 | 0.9642 | 0.9509 | 0.9380 | 0.9264 | 0.9156 | 0.9053 | 0.8958 | 0.8864 | 0.8778 |
| Hugoniot | 1.000 | 0.9683 | 0.9548 | 0.9419 | 0.9301 | 0.9191 | 0.9090 | 0.8992 | 0.8899 | 0.8812 |
| T_H | 20°C | 42 | 54 | 69 | 85 | 104 | 127 | 153 | 181 | 218 |
| Adiabat | 1.000 | 0.9683 | 0.9545 | 0.9417 | 0.9299 | 0.9188 | 0.9081 | 0.8982 | 0.8889 | 0.8802 |
| T_{A} | 20°C | 41 | 48 | 56 | 64 | 71 | 77 | 83 | 89 | 97 |
| Adiabat | | | | | | | | | | |
| (478 kilobars) | 1.002 | 0.9698 | 0.9561 | 0.9432 | 0.9311 | 0.9200 | 0.9095 | 0.8998 | 0.8901 | 0.8811 |
| T_{A} | 106°C | 133 | 144 | 155 | 165 | 173 | 181 | 189 | 197 | 204 |
| Tantalum | | | | | | | | | | |
| 0°K | 0.9952 | 0.9463 | 0.9273 | 0.9089 | 0.8921 | 0.8768 | 0.8622 | 0.8489 | 0.8363 | 0.8244 |
| Hugoniot | 1.000 | 0.9510 | 0.9307 | 0.9122 | 0.8955 | 0.8803 | 0.8657 | 0.8524 | 0.8400 | 0.8284 |
| T_H | 20°C | 47 | 69 | 92 | 121 | 160 | 207 | 260 | 315 | 379 |
| Adiabat | 1.000 | 0.9510 | 0.9304 | 0.9119 | 0.8951 | 0.8795 | 0.8649 | 0.8510 | 0.8383 | 0.8264 |
| $T_{\boldsymbol{A}}$ | 20°C | 45 | 55 | 61 | 70 | 79 | 86 | 93 | 99 | 106 |
| Adiabat | | | | | | | | | | 200 |
| (540 kilobars) | 1.005 | 0.9555 | 0.9345 | 0.9156 | 0.8983 | 0.8827 | 0.8680 | 0.8539 | 0.8411 | 0.8290 |
| T_{A} | 272°C | 314 | 336 | 354 | 369 | 383 | 397 | 410 | 423 | 435 |
| Thallium | | | | | | | | | | |
| 0°K | 0.8446 | 0.8229 | 0.7850 | 0.7558 | 0.7314 | 0.7117 | 0.6937 | 0.6782 | 0.6642 | 0.6510 |
| Hugoniot | 1.000 | 0.8440 | 0.8063 | 0.7785 | 0.7558 | 0.7368 | 0.7203 | 0.7063 | 0.6940 | 0.6822 |
| T_H | 20°C | 315 | 531 | 791 | 1079 | 1392 | 1719 | 2105 | 2447 | 2831 |
| Adiabat | 1.000 | 0.8387 | 0.7987 | 0.7672 | 0.7416 | 0.7200 | 0.7016 | 0.6849 | 0.6708 | 0.6578 |
| T_{A} | 20°C | 192 | 248 | 293 | 333 | 367 | 398 | 425 | 450 | 473 |
| Adiabat | | | | | | | | | 100 | 110 |
| (489 kilobars) | 1.097 | 0.9072 | 0.8542 | 0.8142 | 0.7822 | 0.7557 | 0.7335 | 0.7141 | 0.6969 | 0.6817 |
| TA | 671°C | 1383 | 1656 | 1181 | 2076 | 2247 | 2401 | 2534 | 2657 | 2769 |
| Zironium | | | | | | | | | | |
| 0°K | 0.9968 | 0.9068 | 0.8709 | 0.8394 | 0.8112 | 0.7860 | 0.7629 | 0.7420 | 0.7227 | 0.7049 |
| Hugoniot | 1.000 | 0.9098 | 0.8739 | 0.8421 | 0.8144 | 0.7894 | 0.7670 | 0.7420 | 0.7227 | 0.7049 |
| T_H | 20°C | 55 | 92 | 143 | 214 | 298 | 395 | 503 | 616 | 737 |
| Adiabat | 1.000 | 0.9090 | 0.8733 | 0.8414 | 0.8131 | 0.7879 | 0.7646 | 0.7435 | 0.7244 | 0.7062 |
| T_A | 20°C | 41 | 50 | 59 | 67 | 75 | 81 | 88 | 94 | 99 |
| Âdiabat | | | | | 0. | | 01 | 00 | 27 | 77 |
| (459 kilobars) | 1.008 | 0.9159 | 0.8792 | 0.8470 | 0.8182 | 0.7925 | 0.7690 | 0.7480 | 0.7281 | 0.7100 |
| T_A | 447°C | 502 | 526 | 547 | 565 | 584 | 601 | 618 | 633 | 647 |
| - A | TT/ U | JU2 | 520 | J#1 | 505 | 304 | 001 | 018 | 033 | 647 |

TABLE IV.—Continued.

ments were made) for which the calculations indicate melting in the present experimental range. Shock waves just strong enough for incipient melting of lead, tin, and cadmium (initially at 20°C) are 245 kilobars, 225 kilobars, and 325 kilobars, respectively. For stronger shock waves, at least partial melting occurs as the material is relieved to zero pressure. Melting phenomena are not included in any of the calculations of the present paper.

24ST aluminum data for hydrodynamic applications are listed in Tables VI and VII. The calculations were carried out by the methods outlined above and did incorporate the refinement of the free-surface velocity approximation.

SUMMARIZING REMARKS

Shock-wave experiments were performed to determine Hugoniot curves to pressures of several hundred kilobars. The Hugoniot curves, the Mie-Grüneisen equation of state, and the Dugdale-MacDonald formula were then employed to calculate complete thermodynamic descriptions of the various metals, for states neighboring the experimental curves. The calculated offsets between the Hugoniot curves and neighboring P-V TABLE V. Ratio of the Riemann integral to the shock wave particle velocity, as a function of shock pressure.

| Metal | 100 kilobars | 300 kilobars | 500 kilobars |
|---------------|-----------------|-----------------|-----------------|
| Beryllium | 1.000 | 1.003 | |
| Cadmium | 1.005 | 1.031 | melting |
| Chromium | 1.000 | 1.000 | 1.001 |
| Cobalt | 1.000 | 1.003 | 1.008 |
| Copper | 1.001 | 1.005 | 1.012 |
| Gold | 1.000 | 1.006 | 1.016 |
| Lead | 1.009 | melting | melting |
| Magnesium | 1.005 | 1.027 | montang |
| Molybdenum | 1.000 | 1.001 | 1.002 |
| Nickel | 1.000 | 1.003 | 1.007 |
| Silver | 1.002 | 1.011 | 1.024 |
| Thorium | 1.001 | 1.010 | 1.022 |
| Tin | 1.003 | melting | melting |
| Titanium | 1.000 | 1.002 | 1.006 |
| Zinc | 1.004 | 1.022 | 1.042 |
| 24ST aluminum | 1.003 | 1.015 | 1.030 |
| Brass | 1.001 | 1.009 | 1.019 |
| Indium | 1.008 | melting | melting |
| Niobium | 1.000 | 1.003 | 1.007 |
| Palladium | 1.000 | 1.004 | 1.009 |
| Platinum | 1.001 | 1.003 | 1.006 |
| Rhodium | 1.000 | 1.002 | 1.005 |
| Tantalum | 1.000 | 1.002 | 1.005 |
| Thallium | 1.014 | melting | melting |
| Zirconium | 1.000 | 1.001 | 1.003 |

| Shock wave pressure P (kilobars) | Relative volume V/V0 | Shock wave velocity U. km/sec | Shock particle velocity U_p km/sec | Sound speed C km/sec | Temper- ature T °C |
|--|----------------------------|---|--|-------------------------------|-----------------------------|
| 100 | 0.9043 | 6.125 | 0.571 | 6.307 | 99 |
| 125 | 0.8873 | 6.305 | 0.712 | 6.497 | 125 |
| 150 | 0.8716 | 6.475 | 0.831 | 6.667 | 154 |
| 175 | 0.8573 | 6.640 | 0.947 | 6.825 | 187 |
| 200 | 0.8441 | 6.793 | 1.057 | 6.970 | 223 |
| 225 | 0.8322 | 6.940 | 1.165 | 7.106 | 264 |
| 250 | 0.8210 | 7.082 | 1.267 | 7.233 | 308 |
| 275 | 0.8104 | 7.220 | 1.368 | 7.348 | 356 |
| 300 | 0.8008 | 7.350 | 1.465 | 7.465 | 406 |
| 325 | 0.7912 | 7.476 | 1.561 | 7.624 | 460 |
| 350 | 0.7824 | 7.598 | 1.654 | 7.675 | 516 |
| 375 | 0.7740 | 7.718 | 1.744 | 7.771 | 576 |
| 400 | 0.7661 | 7.836 | 1.832 | 7.862 | 637 |
| 425 | 0.7585 | 7.950 | 1.920 | 7.948 | 702 |
| 450 | 0.7513 | 8.062 | 2.003 | 8.032 | 768 |
| 475 | 0.7445 | 8.171 | 2.082 | 8.112 | 837 |
| 500 | 0.7380 | 8.276 | 2.170 | 8.190 | 907 |

TABLE VI. Shock wave parameters for 24ST aluminum.

curves of interest are generally small, only a few percent in compression. Hence, despite the approximations inherent in the Mie-Grüneisen and Dugdale-MacDonald equations, errors arising in the P-V curves due to the calculations are probably only comparable to uncertainties in the experimental data.

The important question of equivalence for the shockwave results and laboratory pressure-volume data is perhaps best evaluated by examination of the data plots, Figs. 3 to 29. For most metals, the compatibility, if judged by downward extrapolation of the analytical fits, is quite good. This is especially true of comparisons TABLE VII. Pressure versus particle velocity curves for 24ST aluminum. Each number in parentheses is a particle velocity (km/sec) for the corresponding shock pressure (kilobars). Remaining numbers in a given column then trace out the associated cross curve (see Fig. 2 and Section IB).

| P | Particle velocity | | | | | | | | | |
|-----|-------------------|---------|---------|---------|---------|---------|---------|---------|--------|--|
| 0 | 1.165 | 1.655 | 2.098 | 2.576 | 2.945 | 3.380 | 3.680 | | | |
| 100 | (0.571) | 1.075 | 1.775 | 1,960 | 2.345 | 2.750 | 3.082 | 3.445 | | |
| 150 | 0.342 | (0.831) | 1.285 | 1.707 | 2.100 | 2.493 | 2.833 | 3.188 | 3.613 | |
| 200 | 0.118 | 0.606 | (1.057) | 1.480 | 1.875 | 2.260 | 2.605 | 2.958 | 3.375 | |
| 250 | | 0.400 | 0.848 | (1.267) | 1.662 | 2.042 | 2.394 | 2.744 | 3.130 | |
| 300 | | 0.203 | 0.652 | 1.066 | (1.465) | | 2.195 | 2.543 | 2.950 | |
| 350 | | 0.008 | 0.465 | 0.880 | 1.280 | (1.654) | 2.010 | 2.352 | 2.688 | |
| 400 | | | 0.290 | 0.700 | 1.107 | 1.473 | (1.832) | 2.177 | 2.507 | |
| 450 | | | 0.120 | 0.527 | 0.935 | 1.302 | 1.662 | (2.003) | | |
| 500 | • • • | | | 0.364 | 0.765 | 1.136 | 1.500 | 1.838 | (2.170 | |

with the recent measurements to 30 kilobars by Professor Bridgman. Several of the static measurements to 100 kilobars, however, indicate compressions which are a few percent smaller than the corresponding shock wave results. In regard to the latter comparisons, it should be noted that the approximate nature of either of the present basic assumptions (thermal equilibrium and isotropy) would cause the shock wave results to indicate too little compression, and hence is not in the desired direction to account for the small observed offsets.

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