Coulomb Corrections to Delbrück Scattering*

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Observation of Delbrück scattering is most feasible on heavy elements and in the 1–3 Mev region, where the Born approximation is expected to be very poor. The Coulomb corrections to the known forward scattering amplitude in Born approximation can be computed with the aid of the dispersion relations, making use of theoretical and experimental knowledge of the total cross section for pair production. Explicit calculations are carried out for a lead target. The results show that the corrections are effectively much smaller than anticipated. In the energy region of interest the corrections to the dispersive amplitude are only a few percent. The corrections to the absorptive amplitude are largest (a factor of two near 1.33 Mev) where this amplitude is negligible compared to the dispersive scattering amplitude.

THE scattering of photons by a Coulomb potential, known as Delbrück scattering, is known for all energies only in the Born approximation and only for forward scattering.[1] In the high-energy limit the angular distribution is also known.[2] Whereas the effect of screening is only a small correction to these results,[3] the use of the Born approximation is expected to lead to large errors at low energies (1.33 Mev and 2.62 Mev) where the relevant experiments are performed. The reason for this expectation lies in the fact that the Born approximation is the first term in an expansion of the S-matrix in powers of the external field. The next term is therefore a correction of order $(\alpha Z)^2$ in the amplitude (Furry's theorem does not permit odd powers). For heavy nuclei this implies a 30% correction. As a further argument the connection of the absorptive Delbrück scattering amplitude with the pair production cross section is cited, and it is noted that the Born approximation for the latter breaks down in the 1-Mev region.

It is the purpose of this note to show that these arguments are fallacious, and that the Coulomb corrections are *effectively* at most of the order of 10%, and usually much smaller. By "effectively" is meant that the large Coulomb corrections occur only in the absorptive part of the amplitude and only at those energies where this part is small compared to the dispersive part (example: 1.33 Mev).

Consider first the total cross section for pair production,

$$\sigma_{\text{pair}}(\omega)=\alpha Z^2 r_0^2 \Gamma(\omega). \tag{1}$$

The function $\Gamma(\omega)$ is well known in Born approximation (Bethe-Heitler formula). At high energies Coulomb corrections were computed analytically.[4] At lower energies the Coulomb corrections were obtained numerically for several elements and energies.[5] $\Gamma(\omega)$ is conveniently separated into Born approximation and Coulomb corrections,

$$\Gamma(\omega)=\Gamma_B(\omega)+\Gamma_C(\omega). \tag{2}$$

Davies, Bethe, and Maximon obtain for Pb (we use natural units)

$$\Gamma_C{}^{\text{Pb}}(\omega)=-1.03+11.8/\omega, \tag{3}$$

the first term being the theoretical high-energy limit, the second one an empirical correction. Equation (3) agrees with experiments very well above 15 Mev.

A semiempirical formula between threshold and 15 Mev can easily be found using the few computed values[5] and recent experimental results.[6] The complete semiempirical curve $\Gamma_C(\omega)$ for Pb is shown in Fig. 1.

The Delbrück scattering amplitude in the forward direction, $(\alpha Z)^2 r_0(a_1+ia_2)$, is also conveniently sepa-

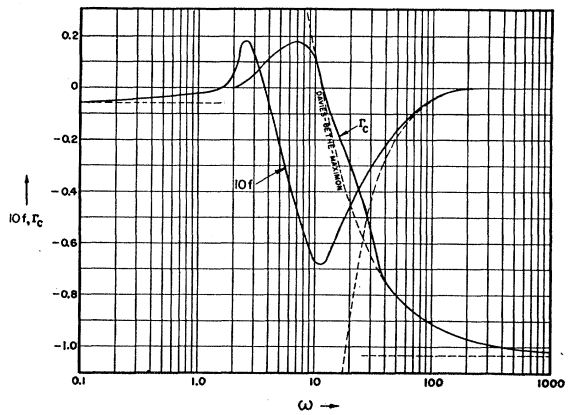

FIG. 1. The functions $10f(\omega)$ and $\Gamma_C(\omega)$ which determine the dispersive and absorptive parts of the Delbrück forward scattering amplitude for Pb. The semiempirical description of pair production by Davies, Bethe, and Maximon is indicated. The asymptotic forms are drawn as dashed lines.

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[1] F. Rohrlich and R. Gluckstern, Phys. Rev. **86**, 1 (1952).
[2] H. A. Bethe and F. Rohrlich, Phys. Rev. **86**, 10 (1952); J. S. Toll, Doctoral thesis, Princeton, 1952 (unpublished).
[3] J. S. Toll, reference 2.
[4] Davies, Bethe, and Maximon, Phys. Rev. **93**, 788 (1954); Olsen, Maximon, and Wergeland, Phys. Rev. **106**, 27 (1957).
[5] J. C. Jaeger and H. R. Hulme, Proc. Roy. Soc. (London) **A153**, 443 (1946); J. C. Jaeger, Nature **137**, 781 (1936) and **148**, 86 (1941).
[6] I. E. Dayton, Phys. Rev. **89**, 544 (1953); Rosenblum, Schrader, and Warner, Phys. Rev. **88**, 612 (1952); G. D. Adams, Phys. Rev. **78**, 1707 (1948); R. S. Paul, Phys. Rev. **96**, 1563 (1954).

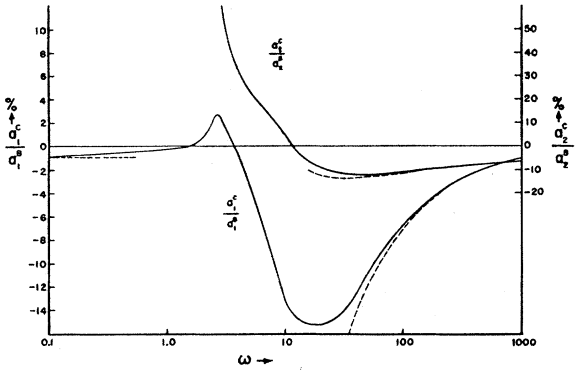

FIG. 2. Coulomb corrections for Pb to the dispersive part (a_1^B) and absorptive part (a_2^B) of the forward scattering amplitude in Born approximation. Note that different scales apply in the two cases. The dashed lines are the asymptotic values.

rated into Born approximation and Coulomb correction,

$$a_1(\omega)=a_1^B(\omega)+a_1^C(\omega), \quad a_2(\omega)=a_2^B(\omega)+a_2^C(\omega). \tag{4}$$

The amplitude $a_1^B+ia_2^B$ was given previously.[1]

The Bohr-Peierls-Placzek relation[7] gives

$$a_2^C(\omega)=\frac{\omega}{4\pi}\Gamma_C(\omega). \tag{5}$$

Thus, $a_2^C(\omega)$ for Pb is known from $\Gamma_C(\omega)$ in Fig. 1.

The calculation of a_1^C proceeds exactly as in Born approximation. The dispersion relation yields

$$a_1^C(\omega)=\frac{\omega^2}{2\pi^2}P\int_2^\infty \frac{\Gamma_C(\omega')d\omega'}{\omega'^2-\omega^2}\equiv\frac{\omega^2}{2\pi^2}f(\omega). \tag{6}$$

The function $\Gamma_C(\omega)$ was so chosen that it not only gives a fair representation of the experimental results, but also can be expressed in terms of simple analytical expressions, so that (6) can be integrated in closed form. The result, expressed by $f(\omega)$, is also given in Fig. 1.

The shape of $f(\omega)$ is not very surprising. Since Γ_C has the general appearance of a distorted absorption curve, f, as computed via the dispersion integral (6), has the typical appearance of a distorted dispersion curve. We further note that $\Gamma_C=0$ at about 6 Mev (Born approximation incidentally exact at that energy). Near that zero f has a minimum. The maximum of f occurs near the production threshold (very close to 1.33 Mev).

[7] Bohr, Peierls, and Placzek, Nature **144**, 200 (1939). For further details see J. M. Jauch and F. Rohrlich, *Theory of Photons and Electrons* (Addison-Wesley, Cambridge, 1955), especially Appendix 7.

For very small and very large ω, asymptotic expressions can be derived. One finds, for small ω,

$$a_1^B=\frac{73}{72}\frac{\omega^2}{32}, \quad a_1^C=-0.0059\omega^2, \tag{7}$$
$$a_1^C/a_1^B=-0.94\% \quad (\omega\ll 1),$$

and for very large ω,

$$a_1^B=\frac{7}{18}\omega-\frac{9}{4}+(2\ln 2-1)\frac{\ln\omega}{\omega}, \quad a_1^C=-\frac{11.8}{2\pi^2}\ln\omega, \tag{8a}$$
$$a_1^C/a_1^B=-154(\ln\omega/\omega)\% \quad (\omega\gg 1),$$

and

$$a_2^B=\frac{7}{9\pi}\omega\left(\ln 2\omega-\frac{109}{42}\right), \quad a_2^C=\frac{\omega}{4\pi}\left(-1.03+\frac{11.8}{\omega}\right), \tag{8b}$$
$$\frac{a_2^C}{a_2^B}=-33.1\left(\frac{1-11.46/\omega}{\ln 2\omega-109/42}\right)\%.$$

The asymptotic values are indicated in Fig. 1.

Of special interest are the relative Coulomb corrections. These are given in percent in Fig. 2. The corrections to the dispersive part are seen to be small. They are largest (-10 to -15%) in the region from 4 to 30 Mev. The largest positive correction is 2.5% and occurs near 1.3 Mev.

The absorptive part has a correction of $\sim -10\%$ above 10 Mev, no correction at 6 Mev, and a positive correction below this energy. At 2.62 Mev, $a_1^B+ia_2^B=0.912+i0.265$ is changed to $a_1+ia_2=0.88+i0.33$. At 1.33 Mev, $a_1^B+ia_2^B=0.241+i0.0058$ is changed to $a_1+ia_2=0.247+i0.0122$. Thus, although the percentage correction to a_2^B is very large, it makes relatively little difference in the cross section, since the dispersive part is dominant.

The results just presented are for Pb and for the forward scattering amplitude only. However, they permit estimates of the Coulomb corrections for other elements and for finite angles. In particular, the Born approximation can now be expected to be fairly satisfactory for all Z and for small enough impact parameters; this reasoning is based on the validity of the impact parameter method at high energies,[2] on the results for the related process of Rayleigh scattering,[8] and on the fact that for low energies and small angles the dispersive part is dominant.

[8] G. E. Brown and D. F. Mayers, Proc. Roy. Soc. (London) **A234**, 387 (1955) and later work (in press).