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Long Lifetime of Positronium in Liquid Helium*

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The long orthopositronium lifetime observed by Paul and Graham and Wackerle and Stump in liquid helium is evidence of a pickoff annihilation rate smaller by more than one order of magnitude than the rate to be expected on the basis of the average electron density. This discrepancy is removed by taking into account the repulsive positronium-helium exchange force already derived in previous work. By repelling the helium atoms and creating a cavity, or bubble, in the liquid, a positronium atom is able to avoid contact with the liquid and thereby greatly increase its lifetime. The small pickoff rate observed is attributed to the saturated vapor in the bubble. The repulsive exchange force further decreases this rate, and good agreement is obtained with experiment. It is predicted that increasing the temperature one degree Kelvin, from the boiling point to the critical point, will decrease the lifetime by a factor of three. Positron lifetime measurements in helium gas at liquid-nitrogen temperature and 0–100 atmospheres pressure would also provide a check on the theory.

THE purpose of this note is to point out that the recently discovered[1,2] very long lifetime of orthopositronium in liquid helium can be explained in a simple manner by extending some work already reported.[3] We also want to suggest some crucial measurements to test our proposed explanation of the long lifetime. Before concentrating on the long lifetime (mean life), τ_2, it should be mentioned that the shorter lifetime,[4] τ_1, due to free positrons, is satisfactorily explained, at least qualitatively, by Ore's calculation.[5] He found that the increase in the density of the positron at the electron shells of the helium atoms, caused by the attractive polarization force, is simulated by increasing the number of electrons per atom from two to $Z_1 = 2.8$. This would give a mean life of 3.42×10^{-9} sec/1.4, or $\tau_1 = 2.44\times10^{-9}$ sec. (The value 3.42×10^{-9} sec is the mean life expected for plane-wave positrons in liquid helium at 4.2°K and one atmosphere.) The experimental value is[1] $(1.83\pm0.15)\times10^{-9}$ sec. Thus, there is some evidence for additional enhancement of the annihilation by electron-positron correlation, which was not included by Ore (he mentions that it is not strictly correct to use the electron distribution of the unperturbed helium atom to calculate Z_1), but this effect does not seem to be by any means as large as was suspected in reference 3. Although the problem of understanding τ_1 cannot be considered completely settled until correlation is taken into account, it seems that there is no essential difficulty involved here.

The situation in regard to τ_2 is quite different. Before the recent measurements[1,2] it was puzzling that there was no lifetime longer than that for plane-wave positrons. Equation (59) was derived in reference 3 to represent the exchange repulsion between a positronium and a helium atom and was expected to decrease considerably the pickoff rate for orthopositronium. The discovery of the long mean life[6] $\tau_2 = (9.1\pm0.5)\times10^{-8}$ sec in liquid helium at 4.2°K and one atmosphere removes this discrepancy, but, going from one extreme to another, confronts us with a pickoff rate so low as to be equally difficult to understand. Subtracting the three-γ rate from τ_2^{-1}, we find $(3.7\pm0.6)\times10^{6}$ sec^{-1}, or

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[1] D. A. L. Paul and R. L. Graham, Phys. Rev. **106**, 16 (1957).

[2] J. Wackerle and R. Stump, Phys. Rev. **106**, 18 (1957).

[3] R. A. Ferrell, Revs. Modern Phys. **28**, 308 (1956). In particular, see pp. 332–3.

[4] Concerning notation, it seems desirable to retain these designations, first introduced by R. E. Bell and R. L. Graham [Phys. Rev. **90**, 644 (1953)], and to designate the very short parapositronium lifetime of 1.24×10^{-10} sec, in those cases where it can be resolved, by the symbol τ_0.

[5] A. Ore, Univ. i Bergen Årbok, Naturvitenskap. Rekke, No. 9 (1949).

[6] We are accepting the value given in reference 1, rather than the value $(12\pm2)\times10^{-8}$ sec of reference 2, because of the smaller experimental error quoted.

a pickoff rate about 80±15 times slower than plane-wave positrons. Since reference 3 was written, the inhibiting effect of the repulsive potential given by Eq. (59) has been calculated[7] and found to be approximately simulated by decreasing the number of electrons per helium atom from two to

$$Z_2 = 0.0784(1-1.872/r_S)^{-3}, \tag{1}$$

where r_S is the radius of the sphere (in Bohr radii) whose volume is equal to the volume per helium atom. (It is identical to the parameter r_s' of reference 3. We have dropped the prime and capitalized the subscript to avoid confusion with the similar parameter in the electron theory of metals.) The radius $r_S = 4.35$ for liquid helium at 4.2°K and one atmosphere. Equation (1) yields $Z_2 = 0.426$, implying a pickoff rate only $(Z_2/2)^{-1} = 4.69$ times slower than that for plane-wave positrons. Thus, there is a discrepancy between the theoretically expected pickoff rate and the experimentally observed one by a factor of about seventeen. It is this discrepancy which we want to consider here.

Evidently there must be some very effective mechanism in liquid helium which prevents the orthopositronium atoms from coming into contact with the helium atoms. One does not actually have to look far to find such a mechanism, for one very direct consequence of Eq. (59) of reference 3 is that it is energetically favorable for a positronium atom to be in a region of lower-than-average helium atom density. Once in such a region, the positronium atom will push the remaining helium atoms away and create a cavity or bubble in the liquid. The outward zero-point kinetic pressure of the positronium atom, arising from its anomalously small mass, will be balanced by the surface tension of the bubble. The determination of the positronium zero-point energy is a simple spherical square-well problem. Although the potential walls at the surface of the bubble are finite, it is easy to establish that no appreciable error is committed by taking them to be infinite.[8] Thus, the zero-point energy is $\pi^2\hbar^2/4ma^2$, where m is the electron mass and a the bubble radius. On the other hand, the increase in surface energy due to the creation of the bubble is $4\pi a^2\sigma$, where σ is the coefficient of surface tension. Minimizing the total energy determines the radius as

$$a = (\pi\hbar^2/16m\sigma)^{\frac{1}{4}}. \tag{2}$$

Substituting[9] $\sigma = 0.10$ erg cm^{-2} gives $a = 22.1$ A, or a diameter of 44.2 A. This can be compared with the interatomic spacing of about 4.60 A. The ratio of these dimensions seems to be sufficient for the present simple picture to apply, at least qualitatively.

We attribute the very low pickoff rate in liquid helium to the bubbles which form immediately around each orthopositronium atom, thereby "protecting" it from contact with the liquid. The small residual rate which is actually found must be attributed to the saturated helium vapor which fills the inside of the bubble. At 4.2°K and one atmosphere, the vapor density corresponds to $r_S = 8.57$. Substitution into Eq. (1) yields $Z_2 = 0.164$, which leads to a pickoff rate of 3.19×10^6 sec^{-1}, in satisfactory agreement with the experimental value. As the temperature is reduced to the λ point, the saturated vapor density becomes so dilute that the pickoff rate becomes completely negligible. At 2.2°K one should expect $\tau_2 = 1.34 \times 10^{-7}$ sec, or only 2% shorter than the free orthopositronium mean life. Although Paul and Graham[1] report no change in passing from 4.2°K to 2.2°K, it is possible that this relatively small increase in the lifetime could go unnoticed. A more crucial test of the present theory would be the variation of τ_2 from 4.2°K and one atmosphere to the critical point at 5.2°K and 2.26 atmospheres. Here the saturated vapor rises to a value which corresponds to $r_S = 5.29$ and $Z_2 = 0.291$. The calculated pickoff rate is 23.7×10^6 sec^{-1}. Including the three-γ rate of 7.3×10^6 sec^{-1} gives a net mean life of 3.2×10^{-8} sec. Thus, upon passing from the boiling point to the critical point τ_2 should decrease by almost a factor of three. This prediction should provide an especially sharp test for the present theory, especially since the trend is just the opposite to what might be expected, based on the decrease in the bulk density of the liquid.

A further check on the above ideas would be provided by positron lifetime measurements in compressed helium gas. The saturated vapor at 4.2°K has a density about one hundred times that of helium at normal temperature and pressure (i.e., 0°C and one atmosphere), while the saturated vapor at 5.2°K is about four times more dense yet. These densities could be achieved at liquid nitrogen temperature by using pressures in the range 0–100 atmospheres. Such data would be very desirable, since they would enable one to avoid possible uncertainties in the derivation of Eq. (1) and to check the bubble hypothesis directly.

In conclusion, we wish to thank Dr. Frank Stern for many helpful discussions.

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[7] The potential of Eq. (59) was weakened by a factor Z' to allow for screening. It is planned to publish a more detailed description of this calculation in the future.

[8] This is at least true in the present case of large bubbles, but is probably not true when the helium is subjected to high pressure. The partial collapse of the bubbles in such cases is still under investigation.

[9] W. H. Keesom, *Helium* (Elsevier Publishing Company, Inc., Amsterdam, 1942), p. 263.