Renormalizable Theory for Fermi Interactions

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A possible model for renormalizable Fermi interactions is discussed. In this theory, Fermi interactions are replaced by indirect two-stage processes which are transmitted by some Bose fields with zero spin having only renormalizable interactions with Fermi particles.

1. INTRODUCTION

We now have three typical interactions of elementary particles (i.e., the electromagnetic, the π-mesonic, and the Fermi interactions). As is well known, the electromagnetic and the π-mesonic interactions have renormalizable forms in the current relativistic field theory. An assumption of renormalizable Yukawa interactions for strange particles also seems compatible, if we assume that they have a spin of $\frac{1}{2}$ or zero. We might think that the only exception to renormalizability is the Fermi interactions at present.

In practice, we would not need any renormalizable theory for Fermi interactions, because they always seem to be very weak and the lowest perturbation might be sufficiently correct. We might, however, wish to set up a renormalizable field theory for Fermi interactions for theoretical completeness of the field theory, if this is possible.

We shall propose a possible model in the present paper. The well-known typical Fermi interactions — *beta decay*, *μ-e decay*, and *μ capture* — will be replaced by the corresponding two-stage processes which are transmitted by virtual boson fields with zero spin. For beta decay and μ decay we must introduce new boson fields other than pion and K fields. The quanta accompanying these fields will turn out to have mass heavier than a nucleon. A classification of a family to which these quanta belong is suggested and the related problems also are discussed.

2. BETA DECAY

In formulating the interaction form of beta decay, we could have various possibilities, for example, the Fermi direct interactions of nucleons and leptons or the Yukawa model in which a Bose field is introduced as the intermediate agent of the interactions between nucleons and leptons. We know that these well-known interactions cannot be brought into a renormalizable theory of beta-decay interactions.

Especially in the latter theory, we must introduce some charged Bose field with spin ≥ 1 in order to get the Gamow-Teller selection rules. The interactions of such a field with other elementary particles will, as is well known, destroy the renormalizability of the field theory.

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On the other hand, Pursey[1] recently tried to set up a renormalizable possibility for the Fermi direct interactions. In his theory a special form of the Fermi interaction was assumed without parity conservation and it was shown that the extent of primitive divergence could be reduced to three types only. However, he found that the theory suffers from the serious difficulty of non-Hermiticity of the beta-decay Hamiltonian.

Some years ago we pointed out that there remains another possibility for a renormalizable field theory for Fermi interactions.[2] The present paper is intended to reconstruct our theory in the new light of nonconservation of parity in the weak interactions, recently predicted by Lee and Yang[3] and confirmed by Wu *et al.*, Garwin *et al.*, and Friedman *et al.*[4] We could modify the Yukawa model of beta decay in the following manner. We assume that a Bose field does not have the usual source of a nucleon or lepton pair, but a source consisting of *a nucleon and a lepton* (see Fig. 1). *In order to get nucleon stability in such a theory, the Bose particle must be charged and have mass heavier than a nucleon. The renormalizability condition requires that it must have zero spin.*

Here we want to make use of the two-component neutrino theory of Lee and Yang,[5] Landau,[6] and

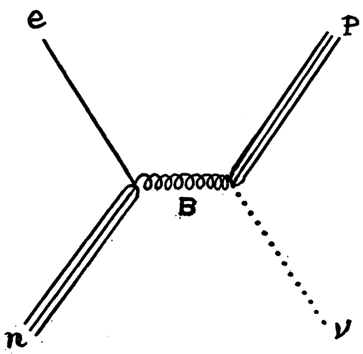

FIG. 1. Diagram for beta decay. A B meson is exchanged between (n,e) and (p,ν) pairs.

[1] D. L. Pursey (to be published).
[2] Y. Tanikawa, *Proceedings of the International Conference of Theoretical Physics, Kyoto and Tokyo, 1953* (Science Council of Japan, Tokyo, 1954), p. 369. For references to the literature see this paper.
[3] T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956).
[4] Wu, Amber, Haywood, Hoppes, and Hudson, Phys. Rev. **105**, 1413 (1957); Garwin, Lederman, and Weinrich, Phys. Rev. **105**, 1415 (1957); J. I. Friedman and V. L. Telegdi, Phys. Rev. **105**, 1681 (1957).
[5] T. D. Lee and C. N. Yang, Phys. Rev. **105**, 1671 (1957).
[6] L. Landau, Nuclear Phys. **3**, 127 (1957).

Salam.[7] For a massless neutrino, we assume that the boson B interacts only with the neutrino component for which $\gamma_5\nu(x)=-\nu(x)$. Then we can assume the interaction Hamiltonian responsible for beta decay[8] to be

$$H_{\text{int}}=\int\{g\bar{p}(x)(1-\gamma_5)\nu(x)\mathrm{B}(x)+f\bar{n}(x)(1+\gamma_5)\times e(x)\mathrm{B}(x)+\text{Hermitian conjugate}\}d^3x, \quad (1)$$

where $p(x)$, $n(x)$, $e(x)$, $\nu(x)$ and $\mathrm{B}(x)$ are field operators for the proton, neutron, electron, neutrino, and B meson, respectively; $\bar{p}(x)$ and $\bar{n}(x)$ are adjoint operators defined as $\bar{p}(x)=p^\dagger(x)\gamma_4$ and $\bar{n}(x)=n^\dagger(x)\gamma_4$, and $\dagger$ means Hermitian conjugate.

The Hamiltonian (1) not only does not conserve parity but also is not invariant under charge conjugation. It is, however, *invariant under the combined transformation of space inversion and charge conjugation.*[9,10] Moreover, it becomes invariant under time reversal if we assume the interaction constants g and f to be real.

The S matrix contributing to electron beta decay is given by

$$S_{\text{beta decay}}=gf\int\int\{T(\bar{e}(x_1)(1-\gamma_5)n(x_1)\bar{p}(x_2)\times(1-\gamma_5)\nu(x_2))\langle T(\mathrm{B}^\dagger(x_1)\mathrm{B}(x_2))\rangle_0\}d^4x_1d^4x_2, \quad (2)$$

where T is Wick's chronological operator[11] and $\langle\ \rangle_0$ means the vacuum expectation value. This can be rewritten as follows:

$$\begin{aligned}S_{\text{beta decay}}&=gf\int\int\{T(\bar{e}(x_1)n(x_1)\bar{p}(x_2)(1-\gamma_5)\nu(x_2)\\&\quad+\bar{e}(x_1)\gamma_5n(x_1)\bar{p}(x_2)\gamma_5(1-\gamma_5)\nu(x_2))\\&\quad\times\langle T(\mathrm{B}^\dagger(x_1)\mathrm{B}(x_2))\rangle_0\}d^4x_1d^4x_2 \quad (3)\\&=gf\int\int\{(S'+T'+P')\\&\quad\times\langle T(\mathrm{B}^\dagger(x_1)\mathrm{B}(x_2))\rangle_0\}d^4x_1d^4x_2,\end{aligned}$$

where S', T', and P' are defined by

$$S'=T\left(\bar{p}(x_2)n(x_1)\bar{e}(x_1)\frac{1-\gamma_5}{2}\nu(x_2)\right), \quad (4a)$$

$$T'=T\left(\bar{p}(x_2)\gamma_{[\mu\nu]}n(x_1)\bar{e}(x_1)\gamma_{[\mu\nu]}\frac{1-\gamma_5}{2}\nu(x_2)\right) \quad (4b)$$

$$P'=T\left(\bar{p}(x_2)\gamma_5n(x_1)\bar{e}(x_1)\gamma_5\frac{1-\gamma_5}{2}\nu(x_2)\right), \quad (4c)$$

and

$$\gamma_{[\mu\nu]}=(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)/2i,$$
$$\gamma_5=\gamma_1\gamma_2\gamma_3\gamma_4.$$

In deriving the right-hand side of (3), we used the Pauli[12] and Fierz[13] identity for the γ matrices, and the contraction of $[\mu\nu]$ should be taken only on independent antisymmetric components of $\gamma_{[\mu\nu]}$.

This form is closely related to the usual Fermi combination of scalar, tensor, and pseudoscalar couplings in the two-component theory of the neutrino.[5] This combination is compatible with the present experimental evidence which indicates that the ratio of Fermi and Gamow-Teller interaction constants is not likely to fall outside the range 0.75–1.15.[14] The present theory gives an intrinsic ratio of precisely one. However, if we take into account the pion correction for the vertex of the beta interaction, the renormalized ratio may fall inside the range 0.45–1.3 depending on the model of pion theory.[15]

In the interaction (3), the Fierz interference factor also vanishes. There is, however, another electron energy-dependent factor which comes from the B-meson propagator. We can easily see this factor in the case of neutron beta decay. The energy distribution of the electrons is approximately given by

$$f(\epsilon)=\frac{\epsilon(\epsilon^2-m^2)^{\frac{1}{2}}(\epsilon_0-\epsilon)^2}{(M_\mathrm{B}^2-M^2+2M\epsilon)^2}, \quad (5)$$

where ϵ, ϵ_0, m, M, and M_B are the electron energy, the maximum electron energy, the electron mass, the nucleon mass, and the B-meson mass, resepectively.

The denominator of (5) comes from the B-meson propagator and depends on the electron energy ϵ. The neutron beta-decay experiment, however, seems to exclude other energy dependences than the statistical Fermi factor in the numerator.[16] Therefore we should assume

$$M_\mathrm{B}^2\gg M^2-2M\epsilon,$$

which is sufficiently well satisfied by the B-meson mass:

$$M_\mathrm{B}\gtrsim 2000m. \quad (6)$$

With $M_\mathrm{B}\simeq 2000m$ and the experimental lifetime of the neutron, 12.8 min, we obtain

$$gf/4\pi\simeq 10^{-7}. \quad (7)$$

We can easily see that there is nucleon stability by the following example. If two *nucleons* (not a nucleon-antinucleon pair) were to transform into two leptons in a process involving exchange of virtual B mesons between them, then nucleon stability would not be secured. Our interaction Hamiltonian (1), however,

[7] A. Salam, Nuovo cimento **5**, 229 (1957).

[8] For the second term in the Hamiltonian (1), we implicitly assumed that the β meson interacts only with the electron component of $\gamma_5e(x)=e(x)$. If we assume that the electron has only an electromagnetic mass and is a massless particle without electromagnetic interactions, the assumption might be justified. A similar consideration was given by A. Salam (see reference 7).

[9] G. Lüders, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **28**, No. 5 (1954); W. Pauli in *Niels Bohr and the Development of Physics*, edited by W. Pauli (Pergamon Press, London, 1955).

[10] Lee, Oehme, and Yang, Phys. Rev. **106**, 340 (1957).

[11] G. C. Wick, Phys. Rev. **80**, 268 (1950).

[12] W. Pauli, Zeeman Verhandelungen (Haag, 1935), p. 31.

[13] M. Fierz, Z. Physik **104**, 553 (1937).

[14] E. Feenberg, Phys. Rev. **917**, 1736 (1955).

[15] M. Ross [Phys. Rev. **104**, 1736 (1956)] has independently obtained similar results.

[16] J. M. Robson, Phys. Rev. **83**, 349 (1951).

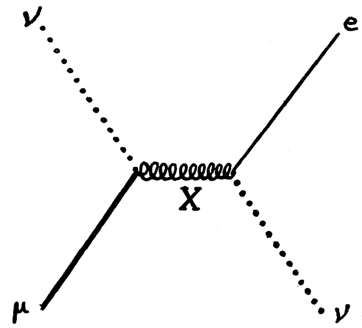

FIG. 2. Diagram for μ-e decay. A X meson is exchanged between (μ,ν) and (ν,e) pairs.

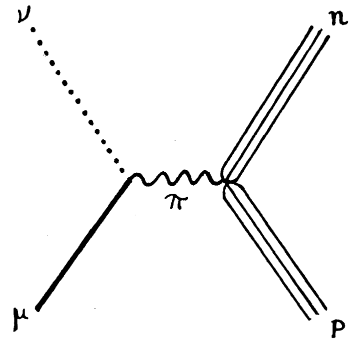

FIG. 3. Diagram for μ capture. A pion is exchanged between (μ,ν) and (p,n) pairs.

forbids the process in which a positively or negatively charged B meson is absorbed by a *nucleon* (not antinucleon); therefore two nucleons never transform into two leptons.

3. μ-e DECAY

In order to understand the μ-e decay by the same principle, we must introduce another Bose field X for the corresponding interaction. The X meson cannot be identified with the β meson, because the nucleon cannot be stable if the β meson has primary interactions with a pair consisting of a muon and an electron. The μ-e decay process is shown in Fig. 2. We assume the following interaction Hamiltonian to be responsible for the μ-e decay:

$$H_{\text{int}}=\int\{g'\bar{\mu}(x)(1-\gamma_5)\nu(x)\mathrm{X}(x)+f'\bar{e}(x)(1-\gamma_5)\times\nu(x)\mathrm{X}(x)+\text{Hermitian conjugate}\}d^3x, \quad (8)$$

where $\mu(x)$ and $\mathrm{X}(x)$ are muon and X-meson field operators, respectively. Here again we assume that the X meson interacts only with the neutrino component for which $\gamma_5\nu(x)=-\nu(x)$.

The S matrix contributing to the μ-e decay is given by

$$S_{\mu-e}=g'f'\int\int\{T(\bar{\nu}(x_1)(1+\gamma_5)\mu(x_1)\bar{e}(x_2)(1-\gamma_5)\times\nu(x_2))\langle T(\mathrm{X}^\dagger(x_1)\mathrm{X}(x_2))\rangle_0\}d^4x_1d^4x_2. \quad (9)$$

This can be rewritten as

$$S_{\mu-e}=g'f'\int\int\{T(\bar{e}(x_2)\mu(x_1)\bar{\nu}(x_2)(1-\gamma_5)\nu(x_2)-\bar{e}(x_2)\gamma_5\mu(x_1)\bar{\nu}(x_2)\gamma_5(1-\gamma_5)\nu(x_2))\times\langle T(\mathrm{X}^\dagger(x_1)\mathrm{X}(x_2))\rangle_0\}d^4x_1d^4x_2.$$

$$=g'f'\int\int\{(V'+A')\langle T(\mathrm{X}^\dagger(x_1)\mathrm{X}(x_2))\rangle_0\}\times d^4x_1d^4x_2, \quad (10)$$

where

$$V'=T\left(\bar{e}(x_2)\gamma_\mu\mu(x_1)\bar{\nu}(x_1)\gamma_\mu\frac{1-\gamma_5}{2}\nu(x_2)\right), \quad (11a)$$

and

$$A'=T\left(\bar{e}(x_2)\gamma_\mu\gamma_5\mu(x_1)\bar{\nu}(x_1)\gamma_\mu\gamma_5\frac{1-\gamma_5}{2}\nu(x_2)\right). \quad (11b)$$

This S matrix actually corresponds to that of the usual combination of vector and axial-vector Fermi interactions in the two-component theory of the neutrino,[5–7] which gives a Michel parameter $\rho=\frac{3}{4}$.[17]

The mass of the X meson can be determined from the electron energy distribution of μ-e decay. The propagator $\langle T(\mathrm{X}^\dagger(x_1)\mathrm{X}(x_2))\rangle_0$ for μ-e decay should be approximately independent of the electron energy. The condition is given by

$$M_{\mathrm{X}}\gg m_\mu, \quad (12)$$

where M_{X} and m_μ are the masses of the X meson and the muon, respectively.

The interaction constant $g'f'/4\pi$ has the order of magnitude

$$g'f'/4\pi\simeq10^{-7}, \quad (13)$$

determined by the μ-e decay lifetime.

As the X meson should have no primary interaction with nucleons, any value of the mass of the X meson satisfying the condition (12) is not involved in the question of nucleon stability. We might speculate that $M_{\mathrm{X}}>M$ in view of some remarks which will be given on a classification of families of elementary particles in the last section.

4. μ CAPTURE AND OTHER RELATED PROBLEMS

For μ capture, we can return to the original Yukawa model as shown in Fig. 3. As is well known, this modle is compatible with the experimental facts at present.[18] We may assume that the pion interacts only with the neutrino component for which $\gamma_5\nu(x)=-\nu(x)$. The interaction Hamiltonian responsible for this process

[17] L. Michel, Proc. Phys. Soc. (London) **A63**, 514 (1950).
[18] Taketani, Sasaki, and Nakamura, Progr. Theoret. Phys. (Japan) 4, 552 (1949). For references to the literature see this paper.

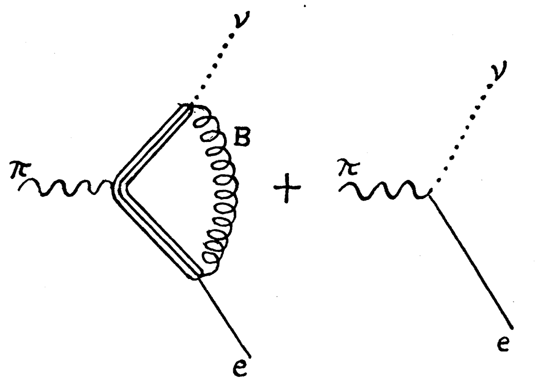

FIG. 4. Diagram for π-e decay. The process in which a B meson is exchanged and the direct process as a renormalizing term.

is given by

$$H_{\text{int}}=\int\{iG\bar{n}(x)\gamma_5 p(x)\varphi(x)+G'\tfrac{1}{2}\bar{\mu}(x)(1-\gamma_5)\times\nu(x)\varphi(x)+\text{Hermitian conjugate}\}d^3x. \quad (14)$$

The first term represents the strong pion-nucleon interaction and the second term gives the π-μ decay interaction. We may note that G' has the value

$$G'^2/4\pi\simeq 10^{-14}. \quad (15)$$

The bosons B, X, and π have different fermion pairs as sources. Then they can have different decay modes:

$$B^{(+)}\rightarrow n+e^{(+)} \quad \text{or} \quad p+\bar{\nu}, \quad (16a)$$

$$X^{(+)}\rightarrow \mu^{(+)}+\nu \quad \text{or} \quad e^{(+)}+\nu, \quad (16b)$$

$$\pi^{(+)}\rightarrow \mu^{(+)}+\nu. \quad (16c)$$

The rates for these decays are given approximately by

$$\frac{1}{\tau_B}=\frac{g^2}{4\pi}\left(\frac{(M_B{}^2-M^2)^2}{M_B{}^2}\right), \quad (17a)$$

$$\frac{1}{\tau_X}=\frac{g'^2}{4\pi}\left(\frac{(M_X{}^2-m_\mu{}^2)^2}{M_X{}^3}\right), \quad (17b)$$

$$\frac{1}{\tau_\pi}=\frac{G'^2}{8\pi}\left(\frac{(m_\pi{}^2-m_\mu{}^2)^2}{m_\pi{}^3}\right). \quad (17c)$$

If we take $g^2/4\pi\simeq 10^{-7}$ and $g'^2/4\pi\simeq 10^{-7}$ (we assume that $g=f$ and $g'=f'$) and $G'^2/4\pi=2.6\times 10^{-14}$, the lifetimes are given by $\tau_B\simeq 10^{-17}$ sec, $\tau_X\simeq 10^{-17}$ sec,[19] and $\tau_\pi=2.54\times 10^{-8}$ sec (experimental value), respectively.

We assumed that the pion has a primary interaction with muons, but not with electrons. The π-e decay can occur not through the primary interaction, but through strong pion-nucleon and weak B-meson interactions (see Fig. 4). This process has a logarithmically divergent vertex for which we must expect some renormalization procedure. In the simplest way we may renormalize this vertex by adding the counter-term of the direct π-e-ν interactions according to Dyson's renormalization method.[20]

TABLE I. Families of elementary particles.

Mass \ Type of interaction	Strong	Weak
Heavy	Ξ, Σ, Λ, N; fermion	(B), (X); boson
Light	K, π, (γ); boson	μ, e, ν; fermion
Strangeness	Yes	No

5. CONCLUSION AND SOME REMARKS

We have seen that all typical Fermi interactions would be renormalized in Dyson's method. The essential idea consisted in our assumption that the Fermi interactions might be disentangled into the renormalizable Yukawa type of interactions. Schwinger[21] emphasized analogies between the electromagnetic field and the pion field that appear to be deeper than the similarities upon which Yukawa founded the meson theory. If it should turn out that the renormalizable theory for Fermi interactions described above is correct, one would extend Schwinger's emphasis of analogies between all interactions of elementary particles to include weak interactions such as Fermi interactions.

Finally we shall make some remarks on the classification of elementary particles. One knows that the known elementary particles can be classified into the following families: (1) a strongly interacting baryon family[22] (Ξ,Σ,Λ,N), (2) a strongly interacting "light" boson family [K,π,(γ)], and (3) a weakly interacting fermion family (μ,e,ν). If we put these families into Table I, we easily see one blank square to which the unknown *family of weakly interacting "heavy" bosons* might be expected to belong.

We may guess that the B meson and the X meson should be members of this family. The predicted properties of the B meson and the X meson seem to have just the necessary properties of the family: they are bosons weakly interacting with all other families, they are heavier than a nucleon, and they may have

[19] We have assumed that the X meson also has a mass of order of 2000m.

[20] S. Weinberg, Phys. Rev. **106**, 1301 (1957). He proposed that all infinities appearing in decay processes involving fermion pairs could be removed by adding some counter-terms of direct interactions.

[21] J. Schwinger, Phys. Rev. **104**, 1164 (1956).

[22] A. Pais, *Proceedings of the International Conference of Theoretical Physics, Kyoto and Tokyo, 1953* (Science Council of Japan, Tokyo, 1954). The name "baryon" was first correctly given by Pais, but Belinfante also used that name in a slightly different way [F. J. Belinfante, Phys. Rev. **92**, 145 (1953)].

nucleon number rather than strangeness quantum number.[23]

There seem to be good reasons why it is very difficult to discover experimentally this family of weakly interacting heavy bosons, at present and also in the near future, because members of this family have only very weak interactions with all other families and have very short lifetimes.

[23] The author owes his thanks to Dr. C. N. Yang for suggesting that the B meson and the X meson might have nucleon number one and zero, respectively.

ACKNOWLEDGMENT

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Solutions of the Static Theory Integral Equations for Pion-Nucleon Scattering in the One-Meson Approximation*

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Numerical solutions of the one-meson approximation of the Low equations for elastic pion-nucleon scattering in the fixed-nucleon, extended-source theory are obtained with a Gaussian cutoff function. The validity of the method requires that the scattering amplitudes have no zeros in the complex plane other than at $z=\pm1$. The functions obtained for the (3,3) and (1,1) states satisfy the Low equations within the accuracy of the method, but the (1,3) and (3,1) states are only approximately given. This difficulty with the (1,3) and (3,1) states is correlated with the development of a zero in the corresponding scattering amplitude well before physically interesting values of the parameters (coupling constant and cutoff) are reached.

A best fit to the (3,3)-state data up to 170-Mev pion laboratory energy requires a coupling constant, f^2, less than 0.08. The solution is consistent with the (3,1)-state data, but gives a (1,1)-state phase shift of larger magnitude than experiment appears to permit.

It is found that the cutoff function does not prevent strong interactions at very high energies. Their occurrence appears to be a property of the static model. The contributions of such interactions to various static-theory calculations is briefly discussed. It is shown that in the p-wave part of the relativistic dispersion relations, where there is no cutoff function, if use is made of the low-energy data, then the very high-energy contributions of the static theory are replaced by recoil terms of order μ/M.

1. INTRODUCTION

THE formulation of the p-wave part of the pion-nucleon interaction proposed by Chew and Low[1] has been applied to a number of problems, including elastic pion-nucleon scattering,[1] photoproduction of mesons,[2] pion production in meson-nucleon collisions,[3] and calculation of the electromagnetic properties of nucleons.[4] Because of the degree of success that approximate methods have given in this theory, particularly in predicting the pion-nucleon resonance in the (3,3) state [isotopic spin $\frac{3}{2}$, spin $\frac{3}{2}$], it is of interest to obtain a solution of the Low equations for the scattering amplitudes. These equations are coupled to an infinite set of equations for other processes, and in principle the entire set should be solved simultaneously. However, in the one-meson approximation the scattering equations reduce to a set of three coupled equations which involve only the three scattering functions. We describe here results obtained in an attempt to find that solution of these three equations which is analytic in the coupling constant.[5]

The method of solution consists of an iterative procedure applied to the integral equations for the functions inverse to the scattering amplitudes.[1] These equations are valid if the scattering amplitudes have no zeros in the complex plane, other than thost at $z=\pm1$, which is the case for sufficiently small values of the coupling constant. It has been conjectured[6] that this condition is maintained for values of the coupling

* Supported in part by the U. S. Atomic Energy Commission.

[1] G. F. Chew and F. E. Low, Phys. Rev. **101**, 1570 (1956).

[2] G. F. Chew and F. E. Low, Phys. Rev. **101**, 1579 (1956).

[3] Saul Barshay, Phys. Rev. **103**, 1102 (1956); N. Fukuda and J. S. Kovacs, Phys. Rev. **104**, 1784 (1956); J. Franklin, Phys. Rev. **105**, 1101 (1957).

[4] H. Miyazawa, Phys. Rev. **101**, 1564 (1956); F. Zachariasen, Phys. Rev. **102**, 295 (1956); S. Fubini, Nuovo cimento **3**, 1425 (1956); S. Treiman and R. G. Sachs, Phys. Rev. **103**, 435 (1956); G. Salzman, Phys. Rev. **105**, 1076 (1957).

[5] These equations possess a large number of solutions, only one of which is analytic in the coupling constant. This was shown for both the charged and neutral scalar-meson theories by Castillejo, Dalitz, and Dyson, Phys. Rev. **101**, 453 (1956), who first called attention to the nonuniqueness of the solution. Klein has shown, Phys. Rev. **104**, 1136 (1956), that by taking appropriate linear combinations of the three scattering functions a set of equations is obtained which is equivalent to the original three equations, and whose solutions are generalized Wigner R functions. By this means the proof of nonuniqueness can be extended, as stated by Klein, to the symmetric pseudoscalar p-wave meson theory.

[6] G. F. Chew, *Encyclopedia of Physics* (Springer-Verlag, Berlin, to be published), second edition, Vol. 43.