Possible Nonlocal Effects in μ Decay

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(Received August 26, 1957)

Possible nonlocal effects in μ decay are investigated phenomenologically. It is shown that the possible difference between the experimental ρ value from $\frac{3}{4}$ can be attributed to such nonlocal phenomena.

RECENT measurements[1–3] on the energy spectrum of electrons from μ decay indicates that the experimental value of the Michel parameter[4] ρ is near 0.68 which is somewhat lower than the theoretical value, $\rho=\frac{3}{4}$, as obtained by using a two-component theory of neutrino[5–7] with no derivative coupling terms in the interaction Hamiltonian. It is possible that all the Fermi-type interactions such as μ decay, β decay, etc., do not actually occur with four spin-$\frac{1}{2}$ fields interacting at *precisely* the same space-time point. Phenomenologically, it might be more appropriate to describe these interactions by a "nonlocal Lagrangian" with these four spinor fields interacting at different space-time points over an extension $\sim 10^{-13}-10^{-14}$ cm. The customary way of using a Hamiltonian without derivative coupling terms is, then, only a first approximation.

The theoretical consistency or inconsistency of a general "nonlocal Lagrangian" is not investigated here. We restrict ourselves to some "nonlocal Lagrangians" of forms suggested by local Lagrangians after the elimination of a virtual field of heavy mesons. *Phenomenological calculations* are then made to determine whether the possible deviation of the experimental ρ value from $\frac{3}{4}$ can be attributed to such nonlocal effects.

Throughout this paper we shall assume that the neutrino field is described by the two-component theory and that the law of conservation of leptons is valid.

I.

To make our analysis definite, we consider first the case that the μ decay,

$$\mu^- \to e^- + \nu + \bar{\nu}, \tag{1}$$

is represented by a "nonlocal Lagrangian" of the form

$$L_I = \sum_i f_i \int [\psi_e^\dagger O_i \psi_\mu(x)] K_i(x-x') \times [\psi_\nu^\dagger O_i \psi_\nu(x')] d^4x d^4x', \tag{2}$$

where the neutrino field satisfies the supplementary condition

$$\gamma_5 \psi_\nu = -\psi_\nu. \tag{3}$$

Because of Eq. (3), in Eq. (2) the S-, P-, and T-type couplings do not exist. The index i in Eq. (2) runs over only the V- and A-type couplings with O_i specifying the corresponding (4×4) matrices.[5] The $K_i(x-x')$ are assumed to be some invariant functions of $(x-x')$.[8] We assume further that the space-time extensions of $K_i(x-x')$ are much smaller than the inverse of the energy momentum transfer involved in the decay process. Thus, it is appropriate to expand $K_i(x-x')$ into a power series in terms of derivatives of $\delta^4(x-x')$,

$$K_i(x-x') = \delta^4(x-x') + m^{-2}\epsilon_i(\partial^2/\partial x_\lambda^2)\delta^4(x-x') + \cdots, \quad (i=V,A), \tag{4}$$

where m is taken to be the mass of the μ meson. $|\epsilon_i/m^2|^{\frac{1}{2}}$ is the length characterizing the nonlocal effects (in units $\hbar=c=1$).

It is convenient to introduce f_1, f_2, ϵ_1, ϵ_2 defined by

$$\begin{aligned} f_1 &= f_A + f_V, \\ f_2 &= f_A - f_V, \\ f_1\epsilon_1 &= f_A\epsilon_A + f_V\epsilon_V, \\ f_2\epsilon_2 &= f_A\epsilon_A - f_V\epsilon_V. \end{aligned} \tag{5}$$

The mean life τ of the μ meson is related to these constants by

$$\tau^{-1} = m^5[|f_1|^2(1+\tfrac{3}{5}\bar{\epsilon}_1) + |f_2|^2(1+\tfrac{3}{5}\bar{\epsilon}_2)](3\times 2^9\pi^3)^{-1}, \tag{6}$$

where

$$\bar{\epsilon}_i = \tfrac{1}{2}(\epsilon_i + \epsilon_i^*), \quad (i=1,2). \tag{7}$$

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[1] Sargent, Rinehart, Lederman, and Rogers, Phys. Rev. **99**, 885 (1955). Their value is $\rho=0.68\pm0.10$.

[2] L. Rosenson, Phys. Rev. (to be published). Rosenson gives $\rho=0.67\pm0.05$.

[3] K. Crowe, Bull. Am. Phys. Soc. Ser. II, **2**, 206 (1957). Crowe gives $\rho=0.68\pm0.02$.

[4] L. Michel, Proc. Phys. Soc. (London) **A63**, 514 (1950).

[5] T. D. Lee and C. N. Yang, Phys. Rev. **105**, 1671 (1957).

[6] A. Salam, Nuovo cimento **5**, 299 (1957).

[7] L. Landau, Nuclear Phys. **3**, 127 (1957).

[8] It is of interest to notice that a more general form of $K_i(x-x')$ is a tensor function $K_{\mu\nu}(x-x')$ which may be written as

$$K_{\mu\nu}(x-x') = \delta_{\mu\nu}K(x-x') + \frac{\partial^2}{\partial x_\mu \partial x_\nu}K'(x-x'),$$

with K and K' both being scalar functions. However, in the present case the contribution of the second term $(\partial^2/\partial x_\mu\partial x_\nu)K'$ is identically zero. Thus one may assume K_i to be simply a scalar function.

The energy spectrum of the electron is no longer describable by a single Michel parameter. For a μ^- at rest with its spin completely polarized, the normalized electron distribution can be written as

$$dN = x^2 dx d\Omega (4\pi)^{-1}\{(6-4x)+ag_1(x)+bg_2(x) + \xi\cos\theta[(2-4x)+cg_1(x)+dg_2(x)]\}, \quad (8)$$

where x= (electron momentum)/(maximum electron momentum),

$$g_1(x)=\tfrac{1}{3}(3-4x), \quad (9)$$

$$g_2(x)=\tfrac{1}{3}(3-8x+5x^2), \quad (10)$$

θ= angle between electron momentum and the spin direction of μ^-, Ω= solid angle of electron momentum, and, a, b, c, d, ξ are numerical parameters. The shape functions $g_1(x)$ and $g_2(x)$ satisfy the following conditions:

$$g_2(x)=0 \quad \text{at} \quad x=1;$$

$$g_i(x)=1 \quad \text{at} \quad x=0;$$

$$\int_0^1 x^2 g_i(x)dx=0, \quad (i=1,2). \quad (11)$$

Because of this choice of shape functions, the energy spectrum of the electron depends rather insensitively on the parameter b.

For the Lagrangian L_{I}, the parameters a, b, c, d, and ξ are given by

$$\xi=\frac{|f_1|^2(1+\frac{1}{5}\bar{\epsilon}_1)-|f_2|^2(1+\frac{1}{5}\bar{\epsilon}_2)}{|f_1|^2(1+\frac{3}{5}\bar{\epsilon}_1)+|f_2|^2(1+\frac{3}{5}\bar{\epsilon}_2)}, \quad (12)$$

$$a=(18/5)(|f_1|^2+|f_2|^2)^{-1}(|f_1|^2\bar{\epsilon}_1+|f_2|^2\bar{\epsilon}_2), \quad (13)$$

$$b=(24/5)(|f_1|^2+|f_2|^2)^{-1}(|f_1|^2\bar{\epsilon}_1+|f_2|^2\bar{\epsilon}_2), \quad (14)$$

$$c=-(6/5)(|f_1|^2-|f_2|^2)^{-1}(|f_1|^2\bar{\epsilon}_1-|f_2|^2\bar{\epsilon}_2), \quad (15)$$

and

$$d=(24/5)(|f_1|^2-|f_2|^2)^{-1}(|f_1|^2\bar{\epsilon}_1-|f_2|^2\bar{\epsilon}_2). \quad (16)$$

The electrons emitted are all longitudinally polarized. In the rest system of μ^-, the helicity (previously called "spirality")[5] of e^- is given by[9]

$$\mathcal{H}=\frac{-|f_1|^2(1+\frac{3}{5}\bar{\epsilon}_1)+|f_2|^2(1+\frac{3}{5}\bar{\epsilon}_2)}{|f_1|^2(1+\frac{3}{5}\bar{\epsilon}_1)+|f_2|^2(1+\frac{3}{5}\bar{\epsilon}_2)}, \quad (17)$$

where $\mathcal{H}$ is defined to be the average value of the electron-spin operator in units of $(\frac{1}{2}\hbar)$ along the electron momentum. Equation (17) is independent of the polarization state of the μ meson.

In all of the above formulas we neglect terms that are proportional to either the mass of the electron or ϵ_i^2. For μ^+ decay, all of the corresponding formulas are identical with Eqs. (6)–(17) except that the expressions for ξ and $\mathcal{H}$ should be multiplied by a minus sign. The seven quantities τ, ξ, a, b, c, d, and $\mathcal{H}$ are all in principle measurable. The particular form of the Lagrangian L_{I} imposes some definite relationships between these quantities:

$$a=\tfrac{3}{4}b, \quad c=-\tfrac{1}{4}d, \qquad \mathcal{H}=-\xi(1-\tfrac{1}{3}c). \quad (18)$$

The present experiments are not accurate enough to test the validity of these relationships. Nevertheless, these experiments can already be used to give the magnitudes of the constants f_i and ϵ_i. We list the following conclusions:

1. From the observed angular asymmetry in π-μ-e decay[10,11] together with the assumption of a conservation law of leptons,[12] the parameter ξ is found to be

$$\xi=-0.87\pm0.12. \quad (19)$$

By comparing with Eq. (12), we find

$$|f_2|^2 \gg |f_1|^2. \quad (20)$$

2. If the energy spectrum is found to be approximately described by a single Michel parameter ρ, then by performing a least-squares fit between the present spectrum, Eq. (8), and the spectrum described by a single Michel parameter, the parameter a is determined to be

$$a=[8(\tfrac{3}{4}-\rho)]\left[\frac{1+0.166(b/a)}{1+0.332(b/a)+0.074(b/a)^2}\right]. \quad (21)$$

In obtaining (21) we assume that the parameter b is linearly proportional to a. For the Lagrangian L_{I}, b and a are related by Eq. (18). Thus, we have

$$a=(0.78)[8(\tfrac{3}{4}-\rho)]. \quad (22)$$

If the best ρ value is 0.68, then by using Eqs. (13) and (20) the constant $\bar{\epsilon}_2$ is

$$\bar{\epsilon}_2 \cong 0.12.$$

In this case, it is possible to assume that the nonlocal effects can possibly be due to some intermediate processes involving the virtual emission and absorption of a particle with mass $\cong(\sqrt{8})m$.

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[9] The helicity $\mathcal{H}$ depends slightly on the electron energy. For electrons with a definite x,

$$\mathcal{H}(x)=[|f_1|^2(1+2\bar{\epsilon}_1-2\bar{\epsilon}_1 x)+|f_2|^2(1+2\bar{\epsilon}_2-2\bar{\epsilon}_2 x)]^{-1} \times[-|f_1|^2(1+2\bar{\epsilon}_1-2\bar{\epsilon}_1 x)+|f_2|^2(1+2\bar{\epsilon}_2-2\bar{\epsilon}_2 x)].$$

Equation (17) gives the value of the $\mathcal{H}$ averaged over the energy spectrum of e^-.

[10] Garwin, Lederman, and Weinrich, Phys. Rev. **105**, 1415 (1957); J. I. Friedman and V. L. Telegdi, Phys. Rev. **105**, 1681 (1957).

[11] For other experimental results on angular asymmetry in π-μ-e decay see, e.g., *Proceedings of the Seventh Annual Rochester Conference on High-Energy Nuclear Physics* (Interscience Publishers, Inc., New York, 1957). A critical study of these experimental results has recently been made by D. H. Wilkinson (to be published). Wilkinson's value for ξ is given in Eq. (19).

[12] The minus sign in Eq. (19) is the result of assuming that the lepton number is conserved in weak interactions. See T. D. Lee, *Proceedings of the Seventh Annual Rochester Conference on High-Energy Nuclear Physics*, (Interscience Publishers, Inc., New York, 1957).

II.

In this section, we shall consider a different form of the "nonlocal Lagrangian" for the same process,

$$\mu^{\pm}\to e^{\pm}+\nu+\bar{\nu}.$$

The corresponding Lagrangian L_{II} is assumed to be

$$L_{\mathrm{II}}=\sum_i \int C_i[\psi_e^\dagger O_i\psi_\nu(x)]K_i(x-x')\times[\psi_\nu^\dagger O_i\psi_\mu(x')]d^4xd^4x', \quad (23)$$

where i runs over the usual S-, V-, T-, P-, and A-type couplings with O_i the corresponding matrices. The $K_i(x-x')$ again are assumed to be some invariant functions of $(x-x')$. This particular form L_{II} is most convenient to use in a discussion of the so-called universal Fermi interactions.[13]

It is easy to see that in Eq. (23) the tensor coupling term is identically zero. The spinor part of the scalar term is identical with that of the pseudoscalar term and, similarly, the spinor part of the vector term is identical with that of the axial-vector term. It is convenient to introduce

$$C_1K_1(x-x')=C_SK_S(x-x')-C_PK_P(x-x'),$$
and
$$C_2K_2(x-x')=C_VK_V(x-x')+C_AK_A(x-x'). \quad (24)$$

As in Eq. (4), we expand K_i into a power series

$$K_i(x-x')=\delta^4(x-x')+m^{-2}\zeta_i(\partial^2/\partial x_\lambda^2)\delta^4(x-x')+\cdots,\quad (i=1,2). \quad (25)$$

The distribution function dN can be represented by the same form as Eq. (8). The parameters a, b, c, d, ξ, and the lifetime τ are given by

$$\tau^{-1}=m^5[\tfrac{1}{4}|C_1|^2(1+\tfrac{4}{5}\bar{\zeta}_1)+|C_2|^2(1+\tfrac{3}{5}\bar{\zeta}_2)]\times(3\times2^9\pi^3)^{-1}, \quad (26)$$

$$\xi=\frac{\tfrac{1}{4}|C_1|^2(1+\tfrac{3}{5}\bar{\zeta}_1)-|C_2|^2[1+(6/5)\bar{\zeta}_2]}{\tfrac{1}{4}|C_1|^2(1+\tfrac{4}{5}\bar{\zeta}_1)+|C_2|^2(1+\tfrac{3}{5}\bar{\zeta}_2)}, \quad (27)$$

$$a=-(6/5)(\tfrac{1}{4}|C_1|^2+|C_2|^2)^{-1}\times(\tfrac{1}{4}|C_1|^2\bar{\zeta}_1+2|C_2|^2\bar{\zeta}_2), \quad (28)$$

$$b=-(6/5)(\tfrac{1}{4}|C_1|^2+|C_2|^2)^{-1}(\tfrac{3}{4}|C_1|^2\bar{\zeta}_1+|C_2|^2\bar{\zeta}_2), \quad (29)$$

$$c=(6/5)(\tfrac{1}{4}|C_1|^2-|C_2|^2)^{-1}(\tfrac{1}{2}|C_1|^2\bar{\zeta}_1+|C_2|^2\bar{\zeta}_2), \quad (30)$$

and

$$d=(6/5)(\tfrac{1}{4}|C_1|^2-|C_2|^2)^{-1}(-\tfrac{3}{4}|C_1|^2\bar{\zeta}_1+|C_2|^2\bar{\zeta}_2), \quad (31)$$

where

$$\bar{\zeta}_i=\tfrac{1}{2}(\zeta_i+\zeta_i^*),\quad \text{with}\quad i=1,2.$$

[13] It is important to notice that the corresponding nonlocal effects in β decay are expected to be smaller by a factor $\sim10^{-4}$ as compared to that in μ decay. Thus, it is difficult to detect such possible nonlocal effects in β decay.

The helicity $\mathcal{H}$ for e^-, measured in the rest system of the μ^-, is

$$\mathcal{H}=\frac{-\tfrac{1}{4}|C_1|^2(1+\tfrac{4}{5}\bar{\zeta}_1)+|C_2|^2(1+\tfrac{3}{5}\bar{\zeta}_2)}{\tfrac{1}{4}|C_1|^2(1+\tfrac{4}{5}\bar{\zeta}_1)+|C_2|^2(1+\tfrac{3}{5}\bar{\zeta}_2)}. \quad (32)$$

Again, we list the various conclusions concerning the constants C_1, C_2, ζ_1, and ζ_2.

1. From the observed angular asymmetry in π-μ-e decay and the assumption of a conservation law of leptons, we obtain

$$|C_2|^2\gg\tfrac{1}{4}|C_1|^2. \quad (33)$$

Consequently, if we neglect $\frac{1}{4}|C_1|^2$, as compared to $|C_2|^2$, we have the following approximate relations.

$$\xi d\cong\xi c\cong-b\cong-\tfrac{1}{2}a, \quad (34)$$

and

$$\mathcal{H}\cong-\xi(1+\tfrac{1}{2}c). \quad (35)$$

2. From Eq. (34) and Eq. (21), the parameter a is determined to be

$$a\cong(0.9)[8(\tfrac{3}{4}-\rho)]. \quad (36)$$

Thus, if the best ρ value is 0.68, the constant $\bar{\zeta}_2$ becomes

$$\bar{\zeta}_2\cong-0.21. \quad (37)$$

Because of the minus sign in Eq. (37), it is not possible to attribute the nonlocal effect of L_{II} to some virtual emission and absorption processes of a heavy particle (through local interactions without derivative couplings).[14]

III.

Lastly, we consider a third form of "nonlocal Lagrangian" L_{III} for μ decay,

$$L_{\mathrm{III}}=\sum_i \int C_i'[\psi_e^\dagger O_i\psi_\nu'(x)]K_i'(x-x')\times[\psi_\nu'^\dagger O_i\psi_\mu(x')]d^4xd^4x', \quad (38)$$

where ψ_ν' is the antineutrino field. It satisfies

$$\gamma_5\psi_\nu'=+\psi_\nu'. \quad (39)$$

As in Eq. (24), we introduce

$$C_1'K_1'(x-x')=C_S'K_S'(x-x')-C_P'K_P'(x-x'),$$
$$C_2'K_2'(x-x')=C_V'K_V'(x-x')+C_A'K_A'(x-x'). \quad (40)$$

[14] It may be useful to make a spectral representation for the Fourier transform of K_i,

$$\int K_i(x)\exp(iq_\mu x_\mu)d^4x=\int P_i(M)(M^2+q^2)^{-1}dM.$$

The parameters ζ_i are then related to the spectral function $P_i(M)$ by

$$m^{-2}\zeta_i=\left[\int M^{-4}P_i(M)dM\right]\left[\int M^{-2}P_i(M)dM\right]^{-1}.$$

If the nonlocal effect is due to virtual emission and absorption processes of a single heavy particle of mass M_0 (via a local interaction without derivative couplings), then $P(M)=M^2\delta(M-M_0)$. Consequently ζ_i is positive. If, however, the nonlocal effects are due to some more complicated intermediate processes, then ζ_i may not be positive. We wish to thank G. Chew for an interesting discussion on the possible usefulness of such a spectral representation.

The functions K_i' are then expanded into a power series:

$$K_i'(x-x')=\delta^4(x-x')+m^{-2}\zeta_i'(\partial^2/\partial x_\lambda^2)\delta^4(x-x')+\cdots, \quad (i=1,2). \quad (41)$$

The results of this case can be directly obtained from that of the previous case, L_{II}. If we perform the transformation

$$C_i\to C_i', \quad \zeta_i\to\zeta_i',$$

then, correspondingly, the various observables

$$\tau_{\text{II}}, a_{\text{II}}, b_{\text{II}}, c_{\text{II}}, d_{\text{II}}\to\tau_{\text{III}}, a_{\text{III}}, b_{\text{III}}, c_{\text{III}}, d_{\text{III}},$$

respectively, while

$$\xi_{\text{II}}\to-\xi_{\text{III}}, \quad \text{and} \quad \mathfrak{IC}_{\text{II}}\to-\mathfrak{IC}_{\text{III}}. \quad (42)$$

In Eq. (42) all quantities with subscript II, such as τ_{II}, a_{II}, etc., refer to those deduced from L_{II} and all quantities with subscript III refer to the present case. The expressions for τ_{II}, a_{II}, $\cdots$, $\mathfrak{IC}_{\text{II}}$ are given by Eqs. (26)–(32).

As in Eqs. (33) and (37), we have

$$\tfrac{1}{4}|C_1'|^2\gg|C_2'|^2, \quad \text{and} \quad \bar{\zeta}_1'\cong-0.42. \quad (43)$$

In this case also, it is not possible to attribute the nonlocal effects to some virtual emission and absorption processes of a heavy particle.[14]

There exist some interesting identities between the results of the above three cases, L_{I}, L_{II}, and L_{III}. Let the constants f_i, ϵ_i, C_i, ζ_i, C_i', ζ_i' be related by

$$\begin{aligned}&\zeta_1=\zeta_2'=-2\epsilon_1,\\ &\zeta_2=\zeta_1'=-2\epsilon_2,\\ &\tfrac{1}{4}|C_1|^2=|C_2'|^2=|f_1|^2(1-2\bar{\epsilon}_1),\\ &|C_2|^2=\tfrac{1}{4}|C_1'|^2=|f_2|^2(1-2\bar{\epsilon}_2).\end{aligned} \quad (44)$$

The various observables O for the three cases, denoted by O_{I}, O_{II}, and O_{III}, respectively, satisfy the identity

$$O_{\text{I}}=\tfrac{1}{2}(O_{\text{II}}+O_{\text{III}}), \quad (45)$$

where O can be τ^{-1}, (a/τ), (b/τ), (ξ/τ), $(c\xi/\tau)$, $(d\xi/\tau)$ and $(\mathfrak{IC}/\tau)$, respectively.

IV.

In the above, we have restricted ourselves to some "nonlocal Lagrangians" that are of forms similar to those due to virtual emission and absorption of some heavy quanta through a local Lagrangian with no derivative couplings. Each of these Lagrangians, L_{I}, L_{II}, and L_{III}, introduces some definite but different relationships between various observables. The validity of these interaction forms is subject to direct experimental tests. The interesting result is that if the energy spectrum is approximately described by a single Michel parameter ρ $(=0.68)$, then only the first case L_{I} can possibly result from simple virtual emission and absorption of a heavy quantum via some intermediate processes through a local Lagrangian with no derivative couplings.[14]