Scattering of High-Energy Electrons from C^{12} on the Intermediate-Coupling Model

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The nuclear matrix elements of $1p$-nuclei for intermediate coupling, which have been worked out in a previous paper, are used here in the case of C^{12} to calculate the elastic form factor and the inelastic form factor corresponding to the excitation of the nucleus to its 4.43-Mev level. For the 7.65-Mev level only the monopole transition matrix element has been calculated. The difficulty that has been faced in making a tentative 2^+ assignment to the 9.61-Mev level has been pointed out by discussing the observed magnitude of inelastic 4.43-Mev scattering and position of the 15.09-Mev ($J=1$, $T=1$) level.

1. INTRODUCTION

IN a previous paper[1] a cross-section formula was worked out on the intermediate-coupling model of ($1p$)-shell[2] nuclei for high-energy electron scattering using the Møller potential and Born approximation method. It can be easily seen that the detailed formula for the differential cross section given by Amaldi *et al.*[3] gets much simplified insofar as all the terms containing $\mathbf{J}_{if}$ become very small compared to those containing only ρ_{if}. Omitting all terms containing $\mathbf{J}_{if}$, one gets ultimately the simple formula

$$\sigma_{if}(\theta)=\frac{Z^2e^4}{4E_i^2}\frac{\cos^2(\theta/2)}{\sin^4(\theta/2)}\times\left|\frac{1}{Z}\langle\Psi_f|\textstyle\sum_j\exp(-i\mathbf{K}\cdot\mathbf{R}^{(j)})\tau_-^{(j)}|\Psi_i\rangle\right|. \quad (1)$$

In this form $1/Z$ times the nuclear matrix element can be interpreted as the form factor F.

In this paper we have calculated for C^{12} the elastic form factor and the inelastic form factor corresponding to the excitation of the nucleus to its 4.43-Mev level (2^+) using the formula for nuclear matrix element derived in paper I. For the 7.65-Mev level (0^+) we have calculated only the monopole transition matrix element, and have refrained from calculating the detailed behavior of the differential cross section with angle. The difficulty that we have faced in making a tentative assignment of 2^+ to the 9.61-Mev level has been pointed out by an examination of the possibility of simultaneously reproducing with the intermediate-coupling model this level and the 15.09-Mev level at their correct positions and also the observed order of magnitude of the 4.43-Mev inelastic scattering. The experimental data made use of for comparison with the calculated values have been taken from the paper of Fregeau,[4] and the incident electron energy for these data is 187 Mev.

In making intermediate-coupling calculations for the nuclear wave function Ψ, a departure has been made from paper I in that here we have used a total nuclear potential of the form

$$V=\sum_{j=1}^{A}V(R^{(j)})+\left[\sum_{j=1}^{A}\alpha\mathbf{l}^{(j)}\cdot\mathbf{s}^{(j)}+\sum_{i<j=1}^{A}V(R^{(ij)})\right]. \quad (2)$$

The zero-order potential $V(R^{(j)})$ has been taken, as before, to be the isotropic oscillator potential with well-parameter a_0; but the higher order potential, enclosed in the square bracket, for which matrices corresponding to different (J,T) are to be set up with the LS-coupling states as basis, has been assumed to consist of a single-particle type spin-orbit interaction (in contrast to the two-particle Case-Pais type of paper I) and a central interaction $V(R^{(ij)})$ given by Rosenfeld's prescription,

$$V(R^{(ij)})=V_0f(R^{(ij)})\{-0.13V_W+0.93V_M+0.46V_B-0.26V_H\}. \quad (3)$$

A Gaussian form of radial dependence,

$$f(R^{(ij)})=\exp[-(R^{(ij)})^2/r_0^2], \quad (4)$$

has been assumed. Following Inglis[5] and Lane,[6] we have introduced the intermediate-coupling parameter $\zeta=\alpha/K$, where K is the exchange integral for the central interaction. Our work differs from the manner of analysis of C^{12} data by other authors[4,7] in two respects: (i) we have used intermediate-coupling wave functions which have already reproduced many observed features of ($1p$)-nuclei, and (ii) we have analyzed the data in terms of the oscillator well-parameter a_0 and the intermediate-coupling parameter ζ, both of which can be physically interpreted.

In Sec. II we have referred to the relevant formulas of paper I made use of herein, and have pointed out

[1] M. K. Pal and S. Mukherjee, Phys. Rev. **106**, 811 (1957); to be referred to as paper I.

[2] In paper I we designated the same shell by ($2p$) in conformity with Jahn, Elliott, etc. [See, e.g., Proc. Roy. Soc. (London) **A218**, 345 (1953).] In this paper we are using the shell nomenclature used by B. F. Sherman and D. G. Ravenhall, Phys. Rev. **103**, 949 (1956).

[3] Amaldi, Fidecaro, and Mariani, Nuovo cimento **7**, 758 (1950).

[4] J. R. Fregeau, Phys. Rev. **104**, 225 (1956).

[5] D. R. Inglis, Revs. Modern Phys. **25**, 390 (1953).

[6] A. M. Lane, Proc. Phys. Soc. (London) **A66**, 977 (1953); **A67**, 167 (1954); **A68**, 189, 197 (1955).

[7] R. A. Ferrell and W. M. Visscher, Phys. Rev. **104**, 475 (1956).

TABLE I. Energy matrix for $J=0$, $T=0$, with $V=-0.13V_W+0.93V_M+0.46V_B-0.26V_H+\alpha\sum_i \mathbf{l}^{(i)}\cdot\mathbf{s}^{(i)}$.

$[\lambda]LTS$	[44]000	[431]101	[422]000	[422]202	[332]101
A. Matrix elements in terms of direct and exchange integrals.[a]					
[44]000	$9.56L+12.92K$	$-\frac{2}{3}\alpha\sqrt{6}$	$-0.4K\sqrt{10}$	0	0
[431]101		$6.76L+14.02K+\frac{3}{4}\alpha$	$-\frac{1}{3}\alpha\sqrt{15}$	$(7/12)\alpha\sqrt{6}$	$(\frac{1}{4}\alpha-0.1K)\sqrt{15}$
[422]000			$3.98L+20.06K$	0	$-\alpha$
[422]202				$6.74L+10.78K+\frac{3}{4}\alpha$	$\frac{1}{4}\alpha\sqrt{10}$
[332]101					$3.04L+19.38K+(5/4)\alpha$
B. Matrix elements in terms of the intermediate-coupling parameters ζ.[a,b]					
[44]000	70.28	$-\frac{2}{3}\zeta\sqrt{6}$	$-0.4\sqrt{10}$	0	0
[431]101		$54.58+\frac{3}{4}\zeta$	$-\frac{1}{3}\zeta\sqrt{15}$	$(7/12)\zeta\sqrt{6}$	$(\frac{1}{4}\zeta-0.1)\sqrt{15}$
[422]000			43.94	0	$-\zeta$
[422]202				$51.22+\frac{3}{4}\zeta$	$\frac{1}{4}\zeta\sqrt{10}$
[332]101					$37.62+(5/4)\zeta$

[a] The matrix is symmetric.
[b] The matrix elements in B all have to be multiplied by the exchange integral K.

the necessity of considering the 7.65-Mev level as arising from configurations other than $(1s)^4(1p)$.[8] Section III deals with the elastic scattering. In Sec. IV we give our treatment of the monopole transition matrix element of the 7.65-Mev level. Section V describes results of our calculations for the inelastic 4.43-Mev scattering. In Sec. VI we examine the difficulties that we have faced in making a 2^+ assignment to the 9.61-Mev level in view of the observed magnitude of inelastic 4.43-Mev scattering and the position of 15.09-Mev level. Section VII contains concluding remarks.

II. SCATTERING FORMULAS AND 7.65-MEV LEVEL CONFIGURATION

By a diagonalization of the energy matrices for different (J,T) corresponding to the potential terms enclosed in the square bracket in Eq. (2) one obtains the eigenvalues and eigencolumns. The former give the shifts of the (J,T) levels concerned from the zero-order configuration-energy value and the latter enable one to express the intermediate-coupling wave functions as a superposition of the LS-coupling wave functions used as basis in the above matrix representation. One writes, with known coefficients $C_{[\lambda]LS}$,

$$\Psi(JTMM_T)=\sum_{([\lambda]LS)} C_{[\lambda]LS} \times\Psi((1s)^4(1p)^n[\lambda]LTS,\,JMM_T). \quad (5)$$

The general matrix element in the form factor is

$$\langle(1s)^4(1p)^n[\lambda]LTS,\,JMM_T \times|\textstyle\sum_j \exp(-i\mathbf{K}\cdot\mathbf{R}^{(j)})\tau_-^{(j)}| \times(1s)^4(1p)^n[\lambda']L'T'S',J'M'M_T\rangle. \quad (6)$$

Within the s-shell and p-shell this gives respectively formulas (3a) and (3b) of paper I.[8] To obtain $|F|^2$, one has to sum and average over the final and initial M values according to the following formula:

$$|F|^2=(2J'+1)^{-1}\textstyle\sum_M\sum_{M'}|F_{MM'}|^2. \quad (7)$$

Now both the 7.65-Mev level and the ground level of C^{12} are $J=0$, $T=0$. The Clebsch-Gordan coefficients,

$$\begin{bmatrix} T' & 1 & T \\ M_T & 0 & M_T\end{bmatrix} \text{ and } \begin{bmatrix} J' & 2 & J \\ M & 0 & M\end{bmatrix},$$

vanish by the triangular inequality rule when $T=T'=0$, $J=J'=0$. The remaining terms of the matrix elements require $L=L'$, $S=S'$, $J=J'$, $M=M'$ for their nonvanishing.

If the 7.65-Mev level belongs to the same configuration as the ground level, then the intermediate-coupling wave functions of these states are given by expressions like Eq. (5) with coefficients $C^o{}_{[\lambda]LS}$ for the ground state and $C^e{}_{[\lambda]LS}$ for the excited state, the summation in both cases running over the five LS-coupling states of Table I. These coefficients satisfy, for orthogonality of the two states, the relation

$$\sum_{[\lambda]LS} C^o{}_{[\lambda]LS}C^e{}_{[\lambda]LS}=0. \quad (8a)$$

In view of the orthogonality of the fractional parentage coefficients we get

$$\langle\Psi_{7.65}|\textstyle\sum_j \exp(-i\mathbf{K}\cdot\mathbf{R}^{(j)})\tau_-^{(j)}|\Psi_0\rangle =4\mathcal{B}\displaystyle\sum_{[\lambda]LS} C^o{}_{[\lambda]LS}C^e{}_{[\lambda]LS}=0. \quad (8b)$$

Since experimentally inelastic scattering corresponding to this level has been observed, one has to postulate, therefore, that this level does not belong to the configuration $(1s)^4(1p)^8$. Sherman and Ravenhall[9] have demonstrated the same fact by considering the mono-

[8] In paper I these formulas are quoted wrongly; they have to be multiplied by $\frac{1}{2}$ and $(-1)^{L-L'+J'-J}$ respectively.

[9] B. F. Sherman and D. G. Ravenhall, Phys. Rev. 103, 949 (1956).

pole transition matrix element $\langle \Sigma r_p{}^2 \rangle$, or $\langle \Sigma (R^{(i)})^2 \tau^{(i)} \rangle$ according to our notation.

III. ELASTIC FORM FACTOR

With the interaction potential given by the portion enclosed in the square bracket in Eq. (2), we have set up the energy matrix for the ground state $J=0$, $T=0$ of the $(1s)^4(1p)^8$ configuration. This is shown in Table IA, in which L and K are direct and exchange integrals given respectively by[10]

$$L = F^{(0)} + (4/25)F^{(2)} = \tfrac{1}{4}V_0\{3\Theta^3 - \Theta^5 + 3\Theta^7\}, \quad \text{(9a)}$$

$$K = (3/25)F^{(2)} = \tfrac{1}{4}V_0\{\Theta^3 - 2\Theta^5 + \Theta^7\}, \quad \text{(9b)}$$

with

$$\Theta = (2\theta^2+1)^{-\frac{1}{2}},\ \theta = a_0/r_0. \quad \text{(9c)}$$

By a proper choice of θ (i.e., r_0 for given a_0) it is possible to satisfy the requirement $L=6K$ suggested by Inglis.[5] With this relationship between the two integrals and putting $\zeta = \alpha/K$, we get Table IB. We have diagonalized this matrix for $\zeta = 3$, 4.5, 6, 10, and 20. The highest eigenvalue, which gives the ground state, and the corresponding wave functions are given in Table II.

TABLE II. Highest eigenvalue (E_0) and the corresponding eigencolumn of the energy matrix for $J=0$, $T=0$. The eigenvalue given here must be multiplied by $K(=+0.0214V_0$ for $\theta=1)$. The expression for K contains the potential well depth V_0 which is negative and hence the highest eigenvalue is the lowest on the energy-level diagram.

	ζ	0	3	4.5	6	10	20
E_0		70.28	72.03	74.56	78.43	92.09	131.25
Wt. of [44]000	C_1	1	0.939	0.850	0.726	0.503	0.358
Wt. of [431]101	C_2	0	−0.332	−0.481	−0.588	−0.661	−0.651
Wt. of [422]200	C_3	0	0.007	0.072	0.130	0.223	0.272
Wt. of [422]202	C_4	0	−0.088	−0.181	−0.301	−0.440	−0.487
Wt. of [332]101	C_5	0	−0.036	−0.093	−0.160	−0.278	−0.370

To obtain the elastic form factor we note that in the matrix element (3b) of paper I the $\mathfrak{C}$ term drops out through the vanishing of $\begin{bmatrix} J & 2 & J \\ M & 0 & M \end{bmatrix}$. The rest gives for the intermediate-coupling state, again with the use of the orthogonality of the fractional parentage coefficients, the following matrix element

$$\rho_{\rm el} = 2\mathfrak{A} + 4\mathfrak{B} \sum_{[\lambda]LS} C^2{}_{[\lambda]LS} = 2\mathfrak{A} + 4\mathfrak{B}$$

because

$$\sum_{[\lambda]LS} C^2{}_{[\lambda]LS} = 1$$

by the normalization requirement. Hence

$$|F_{\rm el}|^2 = \frac{1}{Z^2}|\rho_{\rm el}|^2 = (1-\tfrac{1}{9}K'^2)^2 \exp(-K'^2/2).$$

[10] For an explanation of the symbols $F^{(0)}$ and $F^{(2)}$, see J. P. Elliott, Proc. Roy. Soc. (London) A218, 345 (1953).

TABLE III. Square modulus of the elastic form factor.

	$K' \diagdown \zeta$	All
$\lvert F_{\rm el}\rvert^2$	0	1
	0.5	0.833
	1.0	0.479
	1.5	0.183
	2.0	0.0417
	2.5	0.0041
First zero of $\lvert F_{\rm el}\rvert^2$ at $K'=$		3.00

It is seen, therefore, that in the case of C^{12} the elastic form factor does not depend on the nature of angular momentum coupling. The values of $|F_{\rm el}|^2$ are tabulated in Table III for different K'. Figure 1 displays the $|F_{\rm el}|^2$ *vs* K' curve and the fit of the experimental data for two different choices of the oscillator well-parameter, *viz.*, $a_0 = 1.64$ and 1.72×10^{-13} cm. The former choice is definitely better and it leads to an rms value of the radius of C^{12} equal to 2.41×10^{-13} cm.

IV. MONOPOLE TRANSITION MATRIX ELEMENT FOR THE 7.65-MEV LEVEL

By an extrapolation of the inelastic scattering curve to $K \to 0$ Schiff[11] has evaluated the monopole transition matrix element for the 7.65-Mev level. Fregeau[4] quotes a value $\approx 5 \times 10^{-26}$ cm² for this quantity obtained by this method. Schiff[11] obtained too large a value for this quantity on the collective model and too small a value

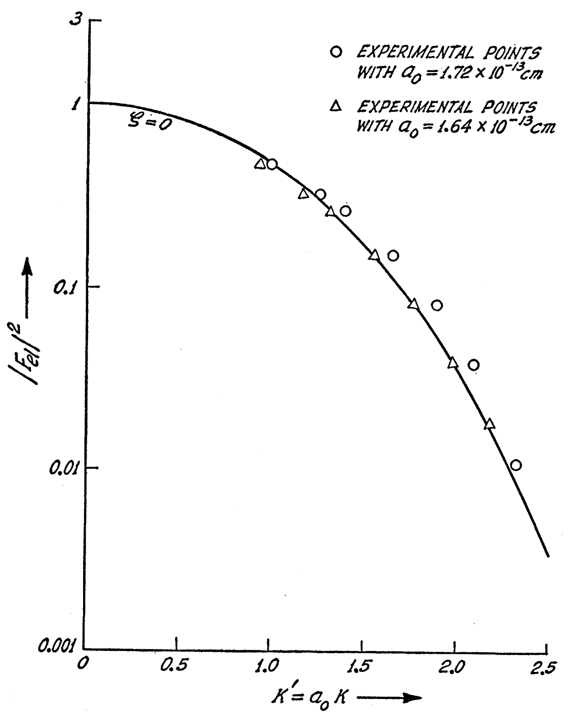

FIG. 1. $|F_{\rm el}|^2$ *versus* K'. Fit of the curve with the experimental values for $a_0 = 1.64 \times 10^{-13}$ cm has been demonstrated.

[11] L. I. Schiff, Phys. Rev. 98, 1281 (1955).

on the jj-coupling shell model. Sherman and Ravenhall[9] have considered the 7.65-Mev level as a mixture of $(1s)^4(1p)^8$ and $(1s)^4(1p)^7(2p)$ configurations, the mixture of the latter in the former being determined by the perturbation method. They also report, in brief, intermediate-coupling calculations with some of the possible states omitted but fail to reproduce the high magnitude of the monopole matrix element.

Following the suggestion of Hill and Wheeler,[12] Ferrell and Visscher[13] have given a treatment of the 0^+ excited level of O^{16}. These last-named authors[7] have recently used the same type of collective ("breathing mode") wave function to describe the 7.65-Mev level of C^{12} and conclude that with 35% LS mode and 65% collective mode of excitation for this level the observed inelastic form factor can be reproduced. The mixture of LS and collective modes is necessary because the former gives too low and the latter too large scattering.

We have used Ferrell and Visscher's way of writing the 7.65-Mev 0^+ state wave function in terms of shell-structure configurations:

$$\Psi(J=0,\, T=0)=(3/13)^{\frac{1}{2}}\Psi((1s)^3(2s)(1p)^8;\, J=0,\, T=0) + (10/13)^{\frac{1}{2}}\Psi((1s)^4(1p)^7(2p);\, J=0,\, T=0). \quad (10)$$

Making certain simplifying assumptions, stated below, and treating the ground state on the intermediate coupling model, instead of pure LS model, we find it possible to reproduce the proper value of the monopole transition matrix element without having to postulate a mixing of different modes of excitation.

The simplifying assumptions mentioned above are regarding the LTS values in $(1s)^3(2s)(1p)^8$ and $(1s)^4(1p)^7(2p)$ that couple to give the final $J=0$, $T=0$. Because $(1s)^3(2s)(1p)^8$ and $(1s)^4(1p)^7(2p)$ will each be about 30 Mev above the ground state configuration $(1s)^4(1p)^8$, it is necessary that the 7.65-Mev level be the lowest states of the above configurations depressed by $\sum V[R^{(ij)})$ and $\sum a\mathbf{l}^{(i)}\cdot\mathbf{s}^{(i)}$ through about 22 Mev. For $(1s)^3(2s)(1p)^8$ we, therefore, assume a coupling

$$\Psi((1s)^3 0\tfrac{1}{2}\tfrac{1}{2},\, (2s)0\tfrac{1}{2}\tfrac{1}{2}: 000;\, (1p)^8[44]000;\, J=0).$$

As regards the $(1s)^4(1p)^7(2p)$ configuration there may be as many as thirteen $(1p)^7$ states that can be linked with $(2p)1\frac{1}{2}\frac{1}{2}$ to produce $J=0$, $T=0$. Of these the states of maximum symmetry of $(1p)^7$ are those belonging to [43] and they are two in number, $1\frac{1}{2}\frac{1}{2}$ and $2\frac{1}{2}\frac{1}{2}$. To simplify matters we consider only the former and take

$$\Psi((1s)^4 000;\, (1p)^7[43]1\tfrac{1}{2}\tfrac{1}{2},\, (2p)1\tfrac{1}{2}\tfrac{1}{2}: L=0,\, T=0,\, S=0,\, J=0).$$

By an application of Racah's[14] equation (27) we get the monopole transition matrix element between the 7.65-Mev level and the ground state, given by

$$\begin{aligned}\langle (R^{(i)})^2\tau_-^{(i)}\rangle &= \frac{a_0^2 C_{[44]000}}{(13)^{\frac{1}{2}}}\int_0^\infty \{\sqrt{3}\psi(1s)\psi(2s) + 2(5)^{\frac{1}{2}}a(\Psi,\bar{\Psi}p)\psi(1p)\psi(2p)\}\rho^4 d\rho \\ &= a_0^2 C_{[44]000}(2/13)^{\frac{1}{2}}\{5a(\Psi,\bar{\Psi}p)+\tfrac{3}{2}\} \\ &= a_0^2 C_{[44]000}(13/2)^{\frac{1}{2}}. \end{aligned} \quad (11)$$

$a(\Psi,\bar{\Psi}p)$ is the single-particle fractional parentage coefficient corresponding to the coupling of $\bar{\Psi}([43]1\frac{1}{2}\frac{1}{2})$ with $(1p)1\frac{1}{2}\frac{1}{2}$ to form $\Psi([44]000)$. Its value unity has been used to obtain the last step of Eq. (11). This equation at once suggests the possibility of reducing, with intermediate coupling for the ground state, Ferrell and Visscher's "breathing mode" value because the weight of the state [44]000 in the ground state is less than unity and in fact adjustable with ζ. The values of this monopole matrix element for different ζ and a_0 are tabulated in Table IV.

V. INELASTIC FORM FACTOR FOR 4.43-MEV LEVEL

Morpurgo[15] deduced the value of this quantity on both LS- and jj models and found that the former gives a value lower by a factor of 2.5 and the latter lower by a factor of 6. Ferrell and Visscher[7] have, therefore, treated the 4.43-Mev level in close analogy with their treatment of the 7.65-Mev level. They have postulated a 90% LS mode and 10% collective mode for this state to explain the observed magnitude of the inelastic form factor. We thought that it may be worth while to see if an intermediate-coupling treatment of both the ground and 4.43-Mev levels can increase the value of the form factor for the pure LS model. In fact for intermediate coupling we obtain a number of LS coupling matrix elements in the form factor weighted by suitable numerical factors. If some of these other matrix elements be larger than the single matrix element for pure LS coupling and if there is no chance cancellation among the different matrix elements whose sum has to be evaluated to get the final value, the form factor may increase from the LS value.

In intermediate coupling, eight LS states will be superposed to build up the state $J=2$, $T=0$. In a preliminary calculation we have omitted four of these

TABLE IV. Monopole transition matrix element for the 7.65-Mev level. $\langle\Sigma r_p^2\rangle = a_0^2(13/2)^{\frac{1}{2}}C_1$. (For explanation of C_1 see Table II.)

	$a_0\times10^{13}$ cm $\diagdown\,\zeta$	0	3	4.5	6	10	20
$\langle\Sigma r_p^2\rangle\times10^{26}$ cm^2	1.64	6.86	6.42	5.82	4.98	3.45	2.45
	1.59	6.44	6.04	5.48	4.68	3.24	2.31
	1.72	7.54	7.08	6.41	5.48	3.79	2.70

[12] D. L. Hill and J. A. Wheeler, Phys. Rev. 89, 1102 (1953).
[13] R. A. Ferrell and W. M. Visscher, Phys. Rev. 102, 450 (1956).
[14] G. Racah, Phys. Rev. 63, 367 (1943).
[15] G. Morpurgo, Nuovo cimento 3, 430 (1956).

TABLE V. Energy matrix for $J=2$, $T=0$, with $V=-0.13V_M+0.93V_M+0.46V_B-0.026V_H+\sum_j \alpha(\mathbf{l}^{(j)}\cdot\mathbf{s}^{(j)})$, and omission of four of the states, namely, [422]002, [422]200, [422]202, [332]101.

[λ]*LTS*	[44]200	[321]101	[431]201	[431]301
		A. Matrix elements in terms of direct and exchange integrals.[a]		
[44]200	$9.56L+10.52K$	$-(1/15)\alpha\sqrt{(105)}$	$\frac{1}{6}\alpha\sqrt{(42)}$	$(2/15)\alpha\sqrt{(30)}$
[431]101		$6.76L+14.02K-\frac{3}{8}\alpha$	$-(3/40)\alpha\sqrt{10}$	0
[431]201			$6.76L+13.52K+(1/12)\alpha$	$-(1/15)\alpha\sqrt{(35)}$
[431]301				$6.76L+10.52K+\frac{2}{3}\alpha$
		B. Matrix elements in terms of the intermediate coupling parameter.[a,b]		
[44]200	67.88	$-(1/15)\zeta\sqrt{(105)}$	$\frac{1}{6}\zeta\sqrt{(42)}$	$(2/15)\zeta\sqrt{(30)}$
[431]101		$54.58-\frac{3}{8}\zeta$	$-(3/40)\zeta\sqrt{10}$	0
[431]201			$54.08+(1/12)\zeta$	$-(1/15)\zeta\sqrt{(35)}$
[431]301				$51.08+\frac{2}{3}\zeta$

[a] The matrix is symmetric.
[b] The matrix elements in B all have to be multiplied by the exchange integral K.

states. This has been done from the argument that these omitted states, namely, [422]002, [422]200, [422]202, [332]101, are not linked by $\alpha\sum \mathbf{l}^{(j)}\cdot\mathbf{s}^{(j)}$ to the state [44]200 (which is the first excited state in pure *LS*-coupling) and hence their mixture with [44]200 in intermediate-coupling will be smaller compared to that of the remaining three states. The resulting 4×4 energy matrix is shown in Table VA and VB. The highest energy value and the eigenfunction for various values of ζ are given in Table VI.

In the present case, inserting known values of J, T, J', T', we get

$$\langle[\lambda]L0S, J=2|\sum_j \exp(-i\mathbf{K}\cdot\mathbf{R}^{(j)})\tau_-^{(j)}| \times[\lambda']L'0S', J'=0\rangle = 4\sqrt{2}\left(\frac{2L+1}{2L'+1}\right)^{\frac{1}{2}} \mathcal{C}\sum_{\bar{\Psi}} a(\Psi,\bar{\Psi}p)a(\Psi',\bar{\Psi}p) \times U(\bar{L}L12;1L'). \quad (12)$$

The nonvanishing matrix elements that will contribute to the form factor are

$$\begin{aligned}
&\langle[44]200, J=2|\sum_j \exp(-i\mathbf{K}\cdot\mathbf{R}^{(j)})\tau_-^{(j)}| \times[44]000, J'=0\rangle = -\tfrac{2}{5}(35)^{\frac{1}{2}}\mathcal{C},\\
&\langle[431]101, J=2|\sum_j \exp(-i\mathbf{K}\cdot\mathbf{R}^{(j)})\tau_-^{(j)}| \times[431]101, J'=0\rangle = (7/10)\sqrt{2}\mathcal{C},\\
&\langle[431]201, J=2|\sum_j \exp(-i\mathbf{K}\cdot\mathbf{R}^{(j)})\tau_-^{(j)}| \times[431]101, J'=0\rangle = -\tfrac{2}{5}(5)^{\frac{1}{2}}\mathcal{C},\\
&\langle[431]301, J=2|\sum_j \exp(-i\mathbf{K}\cdot\mathbf{R}^{(j)})\tau_-^{(j)}| \times[431]101, J'=0\rangle = -\tfrac{2}{3}(7)^{\frac{1}{2}}\mathcal{C}.
\end{aligned} \quad (13)$$

With the help of these results and Tables II and VI for the wave functions we get Table VII for $|F_{\text{in}}|^2$ pertaining to the 4.43 level. The first line of Eq. (14) multiplied with $1/Z$ gives F_{in} for this level with pure *LS* coupling. It may be mentioned that Morpurgo's result for *LS* coupling differs slightly from the value given here because he used an approximate value ($\approx\sqrt{3}$) for his constant Λ. It can be seen from Table VII and Fig. 2 that the intermediate-coupling result is very disheartening and $|F_{\text{in}}|^2$ decreases from the value at the *LS* limit with increase of ζ. Many of the matrix elements do not occur through the $\delta_{S'S}$ selection rule of electron scattering and among the four that occur, one appears with positive and three with negative sign causing enough cancellation. It might be pointed out that the inelastic curves of Fig. 2 cannot distinguish between a_0 values within wide limits because of the large errors in the experimental data.

VI. SPIN ASSIGNMENT OF 9.61-MEV LEVEL; CHOICE OF ζ

If the 9.61-Mev level belongs to the same configuration as the ground level and its J differs from the ground level J' then obviously, through the property of the Clebsch-Gordan coefficient

$$\begin{bmatrix} J' & 2 & J \\ M & 0 & M \end{bmatrix},$$

J can have only one value, and that is 2 (because

TABLE VI. The highest eigenvalue (E_1) and corresponding eigencolumn of the energy matrix for $J=2$, $T=0$. The eigenvalue given here has to be multiplied by K.

	ζ	0	3	4.5	6	10	20
E_1		67.88	69.11	70.43	72.06	77.07	90.98
Wt. of [44]200	C_1	1	0.971	0.942	0.917	0.847	0.781
Wt. of [431]101	C_2	0	−0.113	−0.151	−0.198	−0.259	−0.295
Wt. of [431]201	C_3	0	0.183	0.227	0.305	0.401	0.465
Wt. of [431]301	C_4	0	0.103	0.169	0.201	0.244	0.280

TABLE VII. Square modulus of the inelastic 4.43-Mev form factor.

	$K\diagdown\zeta$	0	3	4.5	6	10	20
$\lvert F_{4.43}\rvert^2$	0	0	0	0	0	0	0
	0.5	0.00022	0.00016	0.00010	0.00005	0.000004	0.0000006
	1.0	0.00263	0.00189	0.00118	0.00056	0.000047	0.0000066
	1.5	0.00672	0.00482	0.00301	0.00143	0.000121	0.0000168
	2.0	0.00930	0.00667	0.00416	0.00198	0.000168	0.0000232
	2.5	0.00739	0.00531	0.00331	0.00158	0.000133	0.0000184

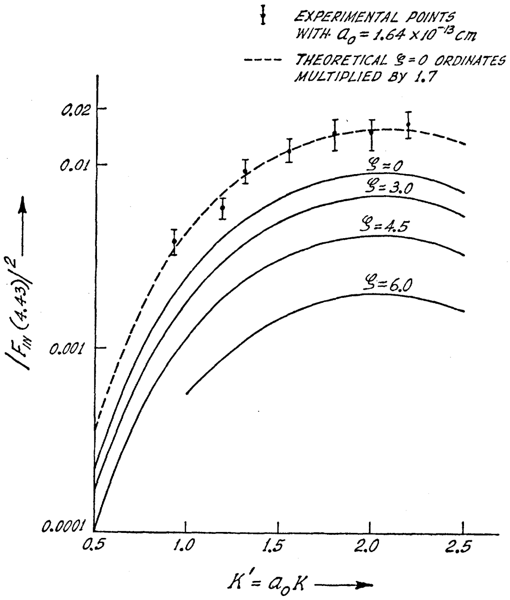

FIG. 2. $|F_{\text{in}}(4.43)|^2$ *versus* K' for different values of ζ. All the theoretical curves lie beneath the experimental points. $\zeta=0$ curve is closest to the experimental points.

$J'=0$). In this eventuality the scattering form factor will contain only the $\mathfrak{C}$ term, just as in the case of 4.43-Mev inelastic scattering. Because the θ dependence of the form factor arises only through $\mathfrak{C}$ one can, therefore, easily explain the parallellism on a logarithmic plot of the two inelastic curves corresponding to the 4.43-Mev level and 9.61-Mev level.

From the α-particle model[16] and also from the fact that the 9.61-Mev level decays by α emission to Be^8, it has been suggested that this level can be either 1^- or 2^+. Fregeau[4] has analyzed the nature of the multipole transition of the 9.61-Mev level by a method suggested by Ravenhall[17] and concludes that this level is 2^+. Ferrell and Visscher,[13] however, have cited a (d,n) experiment by Graue[18] which establishes this level to be 1^-. These authors[13] have, accordingly, analyzed the electron scattering data of this level on the assumption that it is 1^-. The situation seems to be rather puzzling

TABLE VIII. In this table E_3 is the highest eigenvalue of the energy matrix for $J=1$, $T=1$, and E_2 is the second highest eigenvalue of the energy matrix for $J=2$, $T=0$. If we identify the former with the 15.09-Mev level and the latter with the 9.61-Mev level respectively then observed values of the ratios ϵ_1 and ϵ_2 tabulated herein are: $\epsilon_1=3.41$, $\epsilon_2=1.17$ (see reference 20).

$\zeta=$	0	3	4.5	6	10	20
$E_2(9.61)$	54.58	52.70		52.54	44.76	42.04
$E_3(15.09)$	56.24	58.06	60.20	65.25		75.15
$\epsilon_1=(E_0-E_3)/(E_0-E_1)$	5.85	4.79	3.48	2.14		1.39
$\epsilon_2=(E_1-E_2)/(E_0-E_1)$	3.91	5.62		3.06	2.14	1.22

[16] A. E. Glassgold and A. Galonsky, Phys. Rev. **103**, 701 (1956).
[17] D. G. Revenhall, Phys. Rev. **100**, 1797 (1955).
[18] A. Graue, Phil. Mag. **45**, 1205 (1954).

in view of the fact that both the assignments have been claimed to be corroborated by experiments.

Under these circumstances we tried to make a tentative assignment of 2^+ to this level but faced the difficulty discussed below.

We have given in Table VIII the highest eigenvalues of the energy matrices corresponding to $(J=2, T=0)$ and $(J=1, T=1)$ for different values of ζ. The energy levels corresponding to these eigenvalues are obviously to be identified with the ground level, and the observed levels at 4.43 Mev and 15.09 Mev respectively. The calculated ratio as defined in Table VII has been plotted as a function of ζ in Fig. 3. The observed value[19] of this ratio corresponds to a ζ value of 4.5. This may be compared with the value given by Inglis[5] by extrapolation between LS and jj limits.

In the same table we have also included the second highest eigenvalue of the $(J=2, T=0)$ energy matrix.

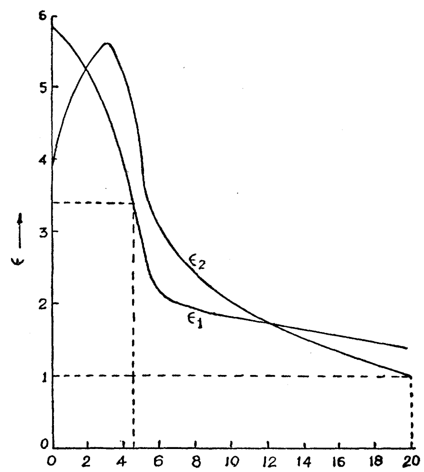

FIG. 3. Choice of $\zeta(=4.5)$ to produce the 4.43-Mev level (2^+, $T=0$) and 15.09-Mev level (1^+, $T=1$) at their observed positions. A ζ value of 20 is shown to be the requirement for producing the 4.43-Mev level (2^+, $T=0$) and the 9.61-Mev level (assumed to be 2^+, $T=0$) at their observed positions.

The ratio ϵ_2 defined in the table is also plotted against ζ in Fig. 3. The observed value of this ratio corresponds to a ζ value of 20. Because this ζ value is widely different from that quoted in the preceding paragraph by the matching of levels of known spin and parity, we could not make the assignment 2^+ to the 9.61-Mev level.

We, however, point out the approximate nature of our eigenvalues for the state $J=2, T=0$ because we dropped four of the states in constructing the energy matrix. The omission of these states will cause more deviation from the eigenvalues, with none of the states omitted, in the case of the second highest eigenvalues than in the case of the highest. Therefore, our choice of $\zeta=4.5$ has a smaller error than the value of $\zeta=20$, concluded by

[19] F. Ajzenberg and T. Lauritsen, Revs. Modern Phys. **27**, 77 (1955).

us to be requisite for producing a 2^+ level at 9.61 Mev. However, since this second value deviates widely from the first, the conclusion that it would be impossible with a single value of ζ to produce a 2^+ level at 9.61 Mev together with a 2^+ level at 4.43 Mev and a $1^+(T=1)$ level at 15.09 Mev may be taken as credible.

VII. SUMMARY AND CONCLUSIONS

It has been shown that the elastic form factor in the case of C^{12} is not dependent on the nature of angular momentum coupling. This is a consequence of the general result that the matrix element $\rho_{el}{}^{MM}$ for elastic scattering reduces to the simple form

$$2\mathcal{A}+Z_p\mathcal{B}+\Bigg[\sum_{[\lambda]LTS}\sum_{[\lambda']L'TS}C_{[\lambda]LTS}C_{[\lambda']L'TS} \times\Bigg\{\sum_{\bar{\psi}}a(\psi,\bar{\psi}p)a(\psi',\bar{\psi}p)\Bigg(1-\begin{bmatrix}T & 1 & T\\ M_T & 0 & M_T\end{bmatrix} \times U(\bar{T}T\tfrac{1}{2}1;\tfrac{1}{2}T)\Bigg)U(\bar{L}L12;1L')\Bigg\}U(SJL'2;LJ)\Bigg] \times\sqrt{10}\begin{bmatrix}J & 2 & J\\ M & 0 & M\end{bmatrix}\frac{n}{2}\mathcal{C},$$

where Z_p is the $1p$-shell charge number. Since only the $\mathcal{C}$ term above is coupling-dependent, the elastic form factor does not depend on ζ if this term drops out through selection rules on J. By fitting the elastic curve, we have obtained $a_0=1.64\times10^{-13}$ cm, which corresponds to a proton rms radius of 2.41×10^{-13} cm for C^{12}.

We have also shown with certain simplifying assumptions that the proper value of the monopole transition matrix element for the 7.65-Mev level can be produced by taking the "breathing mode" wave function for this level and the intermediate coupling wave function for the ground level. The "breathing mode" wave function can really be looked upon as a wave fraction with mixing of the configurations $(1s)^3(2s)(1p)^8$ and $(1s)^4(1p)^7(2p)$. It may, therefore, be suggested that for arriving at definite conclusions a more general procedure would be to allow adjustable mixture of the above configurations, set up the full energy matrix with all the states having $J=0$, $T=0$ belonging to these two configurations, and check if the highest eigenvalue agrees with the position of this level and if the corresponding wave function produces the observed $|F_{\text{in}}|^2$ when taken along with the ground-state wave function given in this paper. Here we have considered only the monopole transition matrix element because of the simplifying assumptions that we have introduced. Regarding this part of the work, no claim at arriving at conclusions can obviously be made. Only a suggestion is made that the language of intermediate coupling in itself may be adequate to describe the 7.65-Mev level without the import of collective-mode language and of simultaneous excitation of particle and collective modes accidentally coincident in energy (without actually verifying if they are really coincident).

For the inelastic scattering to the 4.43-Mev level we have extended Morpurgo's observation in the LS and jj limits to the intermediate-coupling region. It has been shown that $|F_{\text{in}}|^2$ decreases from the value at the LS limit with increase of ζ. In the LS limit one gets the closest approach to the observed value, the theoretical value still remaining about one-third of the latter. In this connection we would like to mention a contradictory conclusion by Banerjee[20] from the analysis of inelastic p-p scattering data for this level of C^{12} that the jj-coupling wave function is more nearly correct for it. Though the accuracy of p-p scattering data may be greater than that of electron scattering data, the mode of analysis of the p-p data is more liable to uncertainties due to the nuclear interactions involved; this point, therefore, calls for a closer examination.

ACKNOWLEDGMENT

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[20] M. K. Banerjee, Palmer Physical Laboratory, Princeton (private communication).