# Polarization of Cosmic-Ray  $\mu$  Mesons: Experiment\*

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This paper describes an experimental determination of the polarization of low-energy cosmic-ray  $\mu$  mesons at sea level. A brass plate was placed in a horizontal position inside a magnetic solenoid. Particles which arrived from directions near the vertical and stopped in the plate were detected by a coincidence-anticoincidence counter telescope. Stopped negative  $\mu$  mesons were destroyed by nuclear capture. Stopped positive  $\mu$  mesons decayed into electrons which were detected by delayed coincidence counters placed above and below the plate. The upward and downward fluxes of the decay electrons leaving the absorber were measured alternately with and without a depolarizing magnetic field. The results of the measurements demonstrate that (1) cosmic-ray  $\mu$  mesons are polarized, (2) the ratio between the downward fluxes of electrons from the decay of  $\mu$  mesons stopped in a brass plate with and without a depolarizing magnetic field is 1.052 $\pm$ 0.016, and (3) the indicated polarization of stopped positive  $\mu$  mesons is 0.19 $\pm$ 0.06 if the data are interpreted according to the two-component neutrino theory of  $\mu$ -meson decay. The r sults are consistent with theoretical predictions based on the production spectrum of  $\pi$  mesons as found in other experiments.

## I. INTRODUCTION

XPERIMENTS on artificially produced mesons<sup>1-3</sup> have established the following properties of the  $\pi \rightarrow \mu \rightarrow e$  decay sequence. (1) The decay of  $\pi$  mesons at rest gives rise to longitudinally polarized  $\mu$  mesons. (2) The distribution in the direction of electrons from the decay of polarized  $\mu$  mesons at rest is anisotropic with a maximum in the backward direction.

Most cosmic-ray  $\mu$  mesons which come to rest in a thin absorber at sea level arise from the decay of  $\pi$ mesons. These  $\mu$  mesons arrive with energies in a narrow range, so that they can be produced in the backward decay of relatively high-energy  $\pi$  mesons or in the forward decay of relatively low-energy  $\pi$  mesons. Since the intensity of  $\pi$  mesons is higher at the lower energy, most of the  $\mu$  mesons which stop in the absorber have been produced in the forward decay of their parent  $\pi$ . mesons. It is apparent, therefore, that cosmic-ray  $\mu$ mesons may be partially polarized, and that their polarization may be indicated by an asymmetry in the direction distribution of their decay electrons. The degree of polarization must depend on the relative numbers and properties of the unstable particles which give rise to  $\mu$  mesons and on their energy spectra, so that a measurement of the polarization can provide a check on our understanding of the role of unstable particles in the propagation of cosmic rays in the atmosphere. In the preceding paper4 Hayakawa has developed the theory of the polarization of cosmic-ray  $\mu$  mesons, and has estimated the expected experimental polarization. The purpose of this experiment was to measure the polarization.

#### II. EXPERIMENTAL ARRANGEMENT

As in the work of Garwin  $et \ al.,<sup>1</sup>$  the experiment method consisted of measuring the effect of a magnetic field on the direction distribution of decay electrons from  $\mu$  mesons which come to rest in an absorber. Negative  $\mu$  mesons at rest in matter rapidly form  $\mu$ -mesonic atoms and, as a consequence, are rapidly depolarized. Thus the direction distribution of decay electrons from stopped negative  $\mu$  mesons is nearly isotropic. In order, therefore, to avoid diluting the anisotropy of the direction distribution of decay electrons from positive  $\mu$ mesons, we chose brass for our absorber material so that the nuclear absorption rate of stopped negative  $\mu$ mesons would be high compared to their rate of radioactive decay. We chose the thickness of the brass absorber to be greater than the maximum effective range of the decay electrons so that the Aux from either face of the absorber slab would be approximately equal to the flux from a semi-infinite absorber. The maximum energy of electrons from the decay of  $\mu$  mesons is 53 Mev. Since the average range of 53-Mev electrons in brass is approximately 12.3 g  $\text{cm}^{-2}$ ,<sup>5</sup> we used a brass plate of thickness  $1.91$  cm corresponding to  $16.0$  g cm<sup>-2</sup>. The lateral dimensions of the absorber were 35.5 cm  $\times$ 60.8 cm, and its total weight was 34.7 kg.

If polarized  $\mu$  mesons come to rest in a field of a strength such that their Larmor precession period is small compared to their lifetime of 2.09  $\mu$ sec, any dependence of the decay electron intensity on the angle between the direction of decay and the direction of polarization will be greatly reduced. Clearly, this effect provides the simplest means for detecting and identifying the polarization of  $\mu$  mesons without evaluating the geometrical efficiencies of the decay electron detectors. Specifically, the fluxes of electrons emerging upward

<sup>~</sup> This work was supported in part by the joint program of the 0%ce of Naval Research and the U. S. Atomic Energy Commission.

f On leave of absence from the Chacaltaya Laboratory, La Paz, Bolivia. <sup>1</sup> Garwin, Lederman, and Weinrich, Phys. Rev. 105, 1415

<sup>(1957).</sup> ' J.J. Friedman and V. L. Telegdi, Phys. Rev. 106, <sup>1290</sup> (1957);

A. Abashian *et al.*, Phys. Rev. 105, 1927 (1957).<br><sup>3</sup> I. Pless and R. W. Williams (private communication, 1957).<br><sup>4</sup> S. Hayakawa, Phys. Rev. 109, 1533 (1957) preceding paper.

<sup>&</sup>lt;sup>5</sup> R. R. Wilson, Phys. Rev. 84, 100 (1951).

and downward from the surfaces of the absorber can be measured with and without a depolarizing field, and the differences can be attributed to the effects of polarization. The intensity of the field required for depolarization may be estimated from the gyromagnetic ratio of  $\mu$  mesons which is known to be close to 2.0.<sup>1</sup> The corresponding Larmor period is equal to  $2.09 \mu$ sec, in a field of 35 gauss so that the intensity of the depolarizing field must be large compared to 35 gauss. We made provision for establishing the required field by placing the absorber inside a magnetic solenoid with which a field intensity exceeding 80 gauss could be maintained throughout the absorber. The solenoid was 3600 feet of No. 16 magnet wire wound in a double layer on a form which enclosed a useful rectangular volume with the dimensions 3 in. $\times$ 15 in. $\times$ 30 in. The locations of the solenoid and the absorber in the apparatus are shown in Fig. 1.

We detected particles that arrived from directions near the vertical and stopped in or near the absorber by a coincidence-anticoincidence telescope consisting of three scintillation counters and a tray of Geiger counters (see Fig. I). The scintillators were cylindrical slabs of polystyrene fluor<sup>6</sup> 107 cm in diameter and 7.6 cm in thickness, and the photomultipliers were all DuMont type 6364 with photosurfaces 5 inches in diameter. Two of the scintillation counters  $S_1$  and  $S_2$  were placed several feet above the absorber. Coincident signals from these counters, together with a coincident pulse from the upper tray of Geiger counters  $G_1$  we shall denote such an event by  $(S_1+S_2+G_1)$  indicated the arrival of a particle from the acceptable solid angle. The third scintillation counter  $S_3$  was placed just beneath the solenoid so as to intercept the entire beam defined by the three counters  $S_1$ ,  $S_2$ , and  $G_1$ .  $(S_1+S_2+G_1)$  events with which no coincident  $S_3$  pulses were associated [we shall denote these by  $(S_1+S_2+G_1-S_3)$  were produced primarily by particles that stopped somewhere in the material between the bottom of  $G_1$  and the top of  $S_3$ . Three-quarters of the total mass of material, contained within the beam between the sensitive volumes of  $G_1$ and  $S_3$  was brass absorber and this was, correspondingly, the approximate fraction of  $(S_1+S_2+G_1-S_3)$  events which were produced by particles which stopped inside the absorber.

 $(S_1+S_2+G_1-S_3)$  events were detected by the electronic arrangements indicated in the block diagram shown in Fig. 2. 1.2  $\mu$ sec after each such event, a gate of length 10  $\mu$ sec was initiated, and the subsequent occurrence of a pulse from  $G_1$  or  $G_2$  during the gate was recorded on one or the other of two registers. We shall demonstrate that most of these events, denoted by  $(S_1+S_2+G_1-S_3+G_1')$  and  $(S_1+S_2+G_1-S_3+G_2')$ , were caused by the decay of  $\mu$  mesons which had traversed  $S_1$ ,  $S_2$ , and  $G_1$  and stopped in the absorber. Since we wanted to be able to detect two particles traversing  $G_1$  within 1.2 usec of one another, and the deadtime of a tray of Geiger tubes connected in parallel is long compared to this time, it was necessary to connect the tubes in  $G_1$  to separate blocking oscillators which gave short output pulses. The length of these pulses determined the length of the delay in the initiation of the gate which was necessary to prevent false  $(S_1+S_2+G_1-S_3+G_1')$  events caused by overlapping prompt  $G_1$ pulses from particles which stopped but did not decay. It was, of course, important that the delay be as short as possible to avoid excessive loss of useful events.

In contrast to  $G_1$ ,  $G_2$  was a tray of Geiger tubes connected in parallel since all desirable  $(S_1+S_2+G_1-S_3)$ events, namely those in which a  $\mu$  meson stopped in the absorber, left  $G_2$  undisturbed and ready to detect any decay electron ejected downward. Furthermore, undesirable  $(S_1+S_2+G_1-S_3)$  events, namely those in which a  $\mu$  meson stopped below the plane containing the axis of the tubes in  $G_2$  but above  $S_3$ , almost always triggered  $G_2$  promptly and left the entire tray insensitive to a traversal by any other particle during a deadtime much longer than the length of the gate.

A system of relays with two sets of positions controlled the currents to the solenoid and the connections to the recording registers. The system was alternated every 15 minutes between the two sets of positions by a timing device. In one position the currents in the two layers of the solenoid winding were in opposite senses



FIG. 1. Schematic diagram (to scale) showing arrangement of counters, absorber, and solenoid, All djmensions indicated are in cm,

<sup>&</sup>lt;sup>3</sup> Clark, Scherb, and Smith, Rev. Sci. Instr. 28, 433 (1957).



Fro. 2. Block diagram of electronic apparatus. The dotted line encloses auxiliary apparatus used in the determination of the lifetime.

so that the field was zero, the numbers of  $(S_1+S_2+G_1-S_3+G_2')$  events were recorded on the registers labeled "up-off" and "down-off," respectively, and the elapsed time with zero field was recorded on a register labeled "time-off" driven by a scale of 256 attached to the 60-cycle line frequency.



FIG. 3. Radioactive decay plot for  $(S_1+S_2+G_1-S_3+G_1')$  and Fig. 5. Kadioactive decay piot for  $(31+32+61-33+61)$  and<br> $(31+52+67-53+62)$  events. The straight line has a slope of<br> $2.18 \mu$ sec,

Similarly, in the other set of positions the currents in the two layers of the solenoid winding were in the same sense so that the field exceeded 80 gauss everywhere in the absorber, the numbers of  $(S_1+S_2+G_1-S_3+G_1')$ and  $(S_1+S_2+G_1-S_3+G_2')$  events were recorded on the registers labeled "up-on" and "down-on" respectively, and the elapsed time was recorded on a register labeled "time-on."

# IIL PERFORMANCE OF THE EQUIPMENT

In Table I we list the various counting rates recorded during a typical run of the equipment with zero field, and the expected rates based on sea level  $\mu$ -meson data.<sup>7</sup> The discriminator levels on  $S_1$ ,  $S_2$ , and  $S_3$  were deliberately made low so that their individual counting rates were large compared to the rates at which they were traversed by  $\mu$  mesons. As a result the rates  $(S_1+S_2), (S_1+S_2+G_1),$  and  $(S_1+S_2+G_1-S_3)$  were essentially independent of small variations in the sensitivities. The calculated value of  $(S_1+S_2)$  was obtained by a crude evaluation of the geometrical factor so that it can only be roughly compared with the observed value. The geometrical factor for  $(S_1+S_2+G_1)$  could be more accurately evaluated. The estimated value for this rate is based on the assumption that the particles traversed  $140 \text{ g cm}^{-2}$  of building material and  $100 \text{ g}$  $cm<sup>-2</sup>$  of lead. The estimated value for the rate of  $(S_1+S_2+G_1-S_3)$  is insensitive to the thickness of absorber overhead because the differential range spectrum is nearly flat at low ranges. The agreement in this case indicates that the apparatus functioned properly and that most of the gates were initiated by  $\mu$  mesons which

<sup>&#</sup>x27; II, Rossi, Revs. Modern Phys. 20, 537 (1948),

Designation	Observed $(\sec^{-1})$	Rate Calculated $(\sec^{-1})$	
S <sub>1</sub>	$1.60 \times 10^{2}$	$1.1 \times 10^2$	(flux of $\mu$ mesons)
$S_2$	$1.60 \times 10^{2}$	$1.1 \times 10^2$	(flux of $\mu$ mesons)
G <sub>1</sub>	$4.12\times10$	$2.5\times10$	(flux of $\mu$ mesons)
G <sub>2</sub>	$4.69\times10$	$2.3\times10$	(flux of $\mu$ mesons)
S <sub>2</sub>	$3.00 \times 10^{2}$	$1.1 \times 10^2$	(flux of $\mu$ mesons)
$S_1+S_2$	$5.40\times10$	$\sim7\times10$	(flux of $\mu$ mesons)
$S_1 + S_2 + G_1$	5.33	5.5	(flux of $\mu$ mesons)
$S_1 + S_2 + G_1 - S_3$	$8.30 \times 10^{-2}$	$6.9 \times 10^{-2}$ in absorber $2.3 \times 10^{-2}$ outside absorber	(stopped $\mu$ mesons)
$S_1 + S_2 + G_1 - S_3 + G_1'$	$1.90\times10^{-3}$	$\cdots$	$(\mu$ -meson decay)
$S_1 + S_2 + G_1 - S_3 + G_2$	$3.22 \times 10^{-3}$	$2.6 \times 10^{-3}$	$(\mu$ -meson decay)

TABLE I. Counting rates of various events recorded during a typical period.

stop in the absorber. Finally, we computed the rate of  $(S_1+S_2+G_1-S_3+G_2')$  by considering in detail the escape of decay electrons from a semi-infinite absorber as will be explained in Sec. IV. The corresponding computation for  $(S_1+S_2+G_1-S_3+G_1')$  was not made because the Geiger tube in  $G_1$  which is triggered by an incoming  $\mu$  meson is insensitive to the decay electron, and an elaborate evaluation of the geometrical efficiency of  $G_1$  would be required for a meaningful estimate.

We made a further check by photographing 13 000 events on an oscilloscope whose sweep was triggered on gate signals caused by  $(S_1+S_2+G_1-S_3)$  events, and on whose deflection plates we displayed the added pulses from  $G_1$  and  $G_2$  with  $G_1$  positive and  $G_2$  negative. The recorded events could be unambiguously classified according to the various cosmic-ray processes which gave rise to the gate signals. The relative numbers of events in the various classifications was entirely consistent with the results shown in Table I when proper account was taken of the accidental coincidences and the contribution of radioactivity to the pulses from  $G_1$ and  $G<sub>2</sub>$ . The total contribution of accidental coincidences to the counting rates for the decay events  $(S_1+S_2+G_1-S_3+G_1')$  and  $(S_1+S_2+G_1-S_3+G_2')$  was less than  $0.1\%$ .

The final check on the identification of the decay events was the determination of their decay curve and mean life from the oscillographic records of decay events. Figure 3 shows a plot of the differential distribution in time of the decay events. The data points are obviously consistent with an exponential decay, and if we assume that the decay is, in fact, exponential, the most probable mean life is<sup>8</sup>

### $(2.18 \pm 0.10) \mu$ sec,

which is in agreement with the accepted value for  $\mu$  mesons.

Although there is little reason to expect a magnetic field of the order of 100 gauss to effect significantly the

response of a Geiger tube to an ionizing particle, we carried out the following simple experiment. Ke wrapped a solenoid around a Geiger tube and measured the counting rate of the tube when exposed to  $\gamma$  rays with and without a magnetic field of 800 gauss along the axis of the tube. The difference between the rates was less than the statistical uncertainty of  $0.5\%$ . Although no test was made of the effect of the field on the delays of the Geiger tube pulses, it appears to be safe to assume that the decay rates are unaffected by the direct action of the magnetic field on the Geiger tubes. Furthermore, the solenoid field had no measurable effect on the other rates quoted in Table I so that we conclude that the decay rates were unaffected by direct action of the field on any of the detectors.

The radius of curvature of a 20-Mev electron in a field of 100 gauss is 670 cm which is large compared to the maximum path length of a detected decay electron. This, together with the fact that  $G_1$  and  $G_2$  were symmetrically placed with respect to the magnetic field and the absorber insure that, if the up and down rates are affected at all by the direct action of the magnetic forces on the decay electrons, they must be affected in the same way. Consequently the ratios (down-on)/(downoff) and (up-on)/(up-off) can be reversed relative to unity only through the action of the field on the parityviolating decay of polarized  $\mu$  mesons.

The rate of gates generated by  $(S_1+S_2+G_2-S_3)$ events was high enough so that the daily average rate could be determined with a statistical uncertainty of 1.2%. We found that the daily averages varied within a maximum range of  $5\%$  from the over-all mean. The chi-squared test indicated the presence of variations of the same order in the much lower rates of delayed events  $(S_1+S_2+G_1-S_3+G_1')$  and  $(S_1+S_2+G_1-S_3+G_2')$ . Since these rates were all determined for long periods during which the field was alternately on and off, it is clear that the observed variations were not caused by the variations in the magnetic field. Furthermore, they were small. The most important fact, however, is that

<sup>s</sup> R. Peierls, Proc. Roy. Soc. (London) A149, 467 (1935).



FiG. 4. Histogram plots of expected (solid line) and observed (dotted line) relative frequencies of days on which the ratio (down-on)/(down-off) fell in various intervals. The expected relative frequencies were computed for the case where the average value of the ratio is 1.052.

they cannot effect in a direct way the ratios which express the final result of the measurements.

#### III. EXPERIMENTAL RESULTS ON THE p-MESON POLARIZATION

In Table II we list the total counts in the four decay event registers recorded during a total running time of 1472.7 hours over a period of about two months. Figure 4 shows the statistical distributions for the daily values of the ratio of rates upon which the Gnal result depends.

The first objective was to check on the existence of a polarization effect. As we have indicated, care was taken to avoid instrumental variations in detection efficiency which might be correlated with the magnetic field strength. Furthermore, the tests described in Sec. II strongly indicate the absence of any such variations. However, the danger of drawing a false conclusion about the existence of a polarization effect can be further reduced by computing from the observed data the ratio

$$
R_0 = \frac{\text{(up-off)} + \text{(down-on)}}{\text{(up-on)} + \text{(down-off)}},
$$

which should be particularly insensitive to any residual instrumental effects correlated directly with the magnetic field. Specifically, if the detection efficiencies of  $G_1$  and  $G_2$  were both less with field on, then the ratios  $(\text{up-off})/(\text{up-on})$  and  $(\text{down-off})/(\text{down-on})$  would tend to be greater than they should be and would not be a

true indication of polarization. However, in the ratio  $R_0$ , any such direct effect would be surpressed, provided it affected both "up" and "down" rates the same way. The value found for this ratio was

### $1.041 \pm 0.012$ ,

where the indicated error is the standard deviation for the distribution of the ratio of two random variables each obeying the Poisson distribution whose mean value is equal to one-half the total number of decay events recorded. The ratio is diferent from unity by more than three standard deviations, and this constitutes the clearest proof that cosmic-ray  $\mu$  mesons are polarized.

Another clear indication of the existence of polarization is the fact that the ratios quoted in Table I are reversed relative to unity as would be expected for the predicted asymmetric decay with a maximum in the upward direction.

As was mentioned earlier, the geometrical evaluation of the detection efficiency of  $G_1$  for decay events is greatly complicated by the fact that the decay electron, if it goes upward, is likely to pass through the Geiger tube which was discharged and rendered insensitive by the parent  $\mu$  meson. Consequently, the most significant ratio for the evaluation of the polarization is (down-on)/(down-oB), and its value, as indicated in Table II, is  $1.052 \pm 0.016$ . We checked the daily variation of this ratio for proper statistical behavior. The observed distribution should be consistent with the hypothesis that the ratio is the quotient of two random variables each distributed according to the Poisson distribution with mean values equal to the average daily numbers of events. The chi-squared test gave a test value which should be exceeded with a probability of  $55\%$  if the hypothesis is true, and this indicates the statistical reliability of the results for this, and by implication, for the other ratios. It is also apparent from Fig. 4 that the expected relative frequencies of the various rates fit the observed frequencies satisfactorily.

## IV. EVALUATION OF THE RESULTS

The experimental results we have described in Sec.III establish the existence of a field-sensitive anisotropy in the decay of stopped cosmic-ray  $\mu$  mesons. We shall now compare the observed anisotropy with that which would be expected for polarized  $\mu$  mesons. Specifically, we shall calculate the upward and downward fluxes of

TABLE II. Summary of total numbers of decay events recorded in each of the four registers corresponding to the four combinations of the direction and field conditions. The last column gives the ratios of the field-on and field-off rates.

	Field Field off on	Rate with field on	
			Rate with field off
Total elapsed time (hr) "down" "up"	736.1 8156	736.6 8582	$\cdots$ $1.052 + 0.016$
	5230	5110	$0.977 + 0.019$

decay electrons from semi-infinite slabs of absorber in which cosmic-ray  $\mu$  mesons come to rest. The results depend on the polarization of the  $\mu$  mesons, and on the way in which the decay probability of  $\mu$  mesons depends on the energy and direction of the ejected electron. Ke shall show that on the basis of the  $\mu$ -meson decay theory of Lee and Yang' the upward (downward) flux is approximately

$$
J_{U(D)} = 0.18nR[1 + (-)0.29\xi P],
$$
 (1)

where *n* is the number of  $\mu$  mesons which stop in one gram of absorber in one second,  $R$  is the average range of a beam of 53-Mev electrons into a semi-infinite slab of absorber,  $\xi$  is a theoreti al parameter, and P is the polarization. We shall also show that the ratio of the observed downward fluxes with and without a depolarizing field gives

$$
P\xi = 0.19 \pm 0.06. \tag{2}
$$

Figure 5 is a schematic diagram of the decay of a  $\mu$ meson which has stopped at a depth  $x$  beneath the surface of a semi-infinite slab of absorber. We shall measure  $x$  in units of  $R$ . The angle between the vertical direction and the direction of emission of the electron will be denoted by  $\theta$ . It is clear that the effects of polarization will manifest themselves in a dependence of the decay probability on the angle  $\theta$ . We call  $dN(u, \theta, P) = g(u, \theta, P)du(d\Omega/4\pi)$  the fraction of  $\mu$  mesons with polarization  $P$  which decay into electrons ejected with energy between  $u$  and  $u+du$  and into a solid angle element  $d\Omega$  at an angle  $\theta$  with respect to the direction of polarization. It is convenient to measure the energy in units of the maximum energy of  $E_m = 53$  Mev so that u ranges from zero to one. Finally, we call  $Q(x, \theta, u)$  the probability that an electron of energy  $u$  ejected in a direction  $\theta$  at a depth x in the absorber emerge from the surface. With these symbols we can write an expression for  $J$  which we call the number of decay electrons that emerge from unit area of a semi-infinite absorber slab in unit time.  $J$  is given by the expression

$$
J = nR \int_{\alpha=0}^{\infty} dx \int_{\theta=0}^{\pi/2} d\theta \int_{u=0}^{1} g(u,\theta,P) Q(x,\theta,u) \frac{\sin\theta}{2} du. \quad (3)
$$

If the decay particles were heavy compared to electrons, then straggling and scattering would be relatively unimportant, and to each initial energy  $u$  there would correspond a well-defined range  $r(u)$  subject only to small fluctuations. In this case the function  $Q(x, \theta, u)$ could be well approximated by the expression

$$
Q(x,\theta,u) = \begin{cases} 0, & \frac{x}{\cos\theta} > r(u) \\ 1, & \frac{x}{\cos\theta} < r(u) \end{cases}
$$
 (4)

<sup>2</sup> T. D. Lee and C. N. Yang, Phys. Rev. 105, 1671 (1957).



FIG. 5. Schematic diagram illustrating the decay of a  $\mu$  meson at a depth  $x$  in a semi-infinite absorber.

Actually, electrons above the critical energy  $(\sim 20 \text{ MeV})$ for brass) frequently undergo radiation processes which cause large fluctuations in the rate of energy loss, while electrons below the critical energy are subject to severe Coulomb scattering which tends to randomize their direction. Thus, in reality,  $Q(x, \theta, u)$  is a complicated function which could probably be evaluated only by Monte Carlo methods. Ke have therefore chosen the step function given in Eq. (4) to represent  $Q(x, \theta, u)$ approximately for the escape of the decay electrons.

Wilson' has investigated the ranges of electrons with energies comparable to the critical energy by Monte Carlo methods. He has demonstrated that the average range of electrons can be represented by the formula

$$
r = \frac{X_0}{R} \ln 2 \ln \left( \frac{uE_m}{E_c \ln 2} + 1 \right) - r_s, \tag{5}
$$

where  $E_0$  is the initial energy of the electrons,  $E<sub>c</sub>$  is the critical energy in the absorber,  $X_0$  is the radiation length, and  $r_s$  is a small correction which is made for the effect of Coulomb scattering near the end of the range where the direction of the electron is randomized.<br>Equation (5) provides a nonlinear relationship between  $\boldsymbol{u}$  and  $\boldsymbol{r}$  which makes the required computations difficult. In view of the approximate representation of the function <sup>Q</sup> which we have already adopted, and considering the present accuracy of the experimental value for the decay asymmetry, we have approximated the relation between  $r$  and  $u$  by the linear equation

$$
r = u.\t\t(6)
$$

As yet there is no complete experimental determination of the function  $g(u, \theta, P)$ . We have therefore adopted the relation derived by I.ee and Yang' on the basis of the two component theory of the neutrino, namely

(4) 
$$
g(u,\theta,P) = 2u^2\{(3-2u)+\xi P\cos\theta(1-2u)\}.
$$
 (7)

The evaluation of Eq. (3) on the basis of these assumptions can be carried out analytically and leads to the result given by Eq. (1).

Before the experimental results can be used with Eq. (1) to determine a value for  $\zeta P$ , we must estimate the importance of the following three effects.

(1) A small fraction of the negative  $\mu$  mesons which have been brought to rest and have formed mesic atoms decay before they are captured. If we assume that they are depolarized through spin orbit magnetic interactions within the mesic atoms, then they must contribute an isotropic background of decay electrons which tends to dilute the observed effect. However, the lifetime of negative  $\mu$  mesons in brass is about 0.1 microsecond so that the fraction which are left when the delayed coincidence gate is initiated about 1.2 microsecond later is negligibly small.

(2) With the field on, the average decay probability in a given direction is a time average of the instantaneous probability weighted according to the survival probability for the  $\mu$  mesons. At a time t after a  $\mu$  meson has come to rest its polarization will have precessed by an angle of  $2\pi (t/T)$  where T is the Larmor period. The probability that it survives to the time  $t$  and then decays during the time interval dt is  $e^{-(t/\tau)}dt$  where  $\tau$  is the lifetime. In the Lee-Yang theory the decay probability is proportional to

## $a+\xi P \cos\theta$ ,

where  $\theta$  is the angle between the decay direction and the polarization. After precession through an angle  $\omega t$ , the original direction specified by the polar angles  $\theta$  and  $\varphi$ now makes an angle  $\theta'$  with the polarization which is given by

$$
\cos\theta' = \sin\theta \sin\varphi \sin 2\pi (t/T) + \cos\theta \cos 2\pi (t/T).
$$

Thus the instantaneous decay probability at time  $t$  in directions making an angle of  $\theta$  with the vertical is proportional to

$$
a + \xi P \cos \theta'.
$$

Finally, the value of this probability averaged over all azimuth angles  $\varphi$  and time is proportional to

$$
\left[\frac{e^{(t_0/\tau)}}{\tau}\right] \int_{t_1}^{\infty} (a + \xi P \cos\theta') e^{-(t/\tau)} dt = (a + \xi P' \cos\theta),\tag{8a}
$$

where

$$
P' = P \frac{\sin(2\pi(t_1/T) + \arctan(T/2\pi\tau))}{(1 + (2\pi\tau/T)^2)^{\frac{1}{2}}}.
$$
 (8b)

The observed flux of decay electrons in the presence of the field can now be calculated as before, but with  $P$ replaced by P'. In our case  $\tau = 2.09$  usec,  $T \approx 0.8$  usec so that, regardless of the phase of the sinusoidal function, we have

 $P' < 0.063P$ .

For our particular value of the gate delay  $t_1$  which was 1.2  $\mu$ sec, the sinusoidal function is nearly zero. Thus we shall assume that the magnetic field rendered the decay average probability perfectly isotropic.

(3) P is the polarization of the cosmic-ray  $\mu$  mesons in the direction of their flight. Since the telescope accepts mesons which arrive within a solid cone whose half-angle is about 25°, the observed polarization will be reduced by an amount which depends on the zenith angle distribution of mesons. We shall assume a cosine squared law for this distribution so that the observed polarization is 0.96P.

The ratio of the downward fluxes with and without field can now be written according to Eq. (1),

$$
\frac{J_U + J_D}{2J_D} = \frac{1}{1 - 0.29 \times 0.96 \xi P}
$$

If we set this equal to the observed ratio given in Table II we find for  $\zeta P$  the value given in Eq. (2). According to the theory of Hayakawa<sup>4</sup> the corresponding value of  $\alpha$ , which is the exponent in the  $\pi$ -meson production spectrum, is  $2.0 \pm 0.6$  if we assume that  $\xi$ -1.0. This result is consistent with other values of  $\alpha^{10}$  derived from the range spectrum of  $\mu$  mesons.

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<sup>10</sup> S. Olbert, Phys. Rev. 96, 1400 (1954).