Oscillatory Magneto-Absorption in Semiconductors*

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Infrared magneto-absorption has been investigated in thin samples (~10 microns) of germanium, indium arsenide, and indium antimonide in magnetic fields up to 37 kilogauss. At photon energies greater than the direct transition energy gap in germanium and also greater than the energy gaps in indium arsenide and indium antimonide, the magneto-absorption exhibited oscillatory behavior. This is associated with interband transitions between quantized Landau levels of the valence and conduction bands. By extrapolating a plot of the photon energies of the transmission minima as a function of magnetic field to zero field, we obtained accurate determinations of the energy gaps. For the direct transition in germanium, $E_g=0.803\pm0.001$ ev at ~298°K, 0.890 ± 0.001 ev at ~77°K, and 0.897 ± 0.001 ev at ~4°K. In indium antimonide, $E_g=0.180\pm0.002$ ev at ~298°K, and in indium arsenide, the result was 0.360 ± 0.002 ev at ~298°K. From the accurate determination of the energy gap made possible by these experiments, it is apparent that there is considerable absorption below the energy gap, probably due to transitions involving both photons and optical phonons. Utilizing the theory of Luttinger and Kohn, effective masses were evaluated from the higher quantum transitions in germanium to give values of the conduction band electron mass at $k=0$ of $(0.036\pm0.002)m_0$ at room temperature, and $(0.043\pm0.002)m_0$ and $(0.041\pm0.002)m_0$ at ~77°K and ~4°K, respectively, which is consistent with the corresponding change in energy gap. For indium arsenide, the conduction band electron mass was found to be approximately $0.03m_0$. For indium antimonide, a value for the effective mass in the conduction band of $\sim0.014m_0$, consistent with the cyclotron resonance results, was obtained by assuming that in this case the magnetic levels in the conduction band are appreciably split by spin-orbit interaction. The anisotropy of the magneto-absorption effect was measured for germanium and indium antimonide. For germanium, the anisotropy agreed with predictions from theory and the results of microwave cyclotron resonance. For indium antimonide, the anisotropy was very small for the first two minima.

I. INTRODUCTION

In an early study of the magneto-absorption in germanium, the Lincoln Laboratory group reported the presence of fine structure in the form of oscillations in the transmission of infrared radiation at energies just above the direct-transition absorption edge.[1] The name assigned to this phenomenon by Zwerdling and Lax in their subsequent presentation[2] of the quantitative results for germanium was the *oscillatory magneto-absorption effect*. This phenomenon was independently observed by Burstein and Picus[3] in indium antimonide and was called by them the interband magneto-optic effect. The Lincoln group also reported further experimental results[4] in germanium, indium antimonide, and indium arsenide, including the anisotropy of the effect and detailed theoretical interpretation of the experimental results for germanium. The object of this paper is to present the details of these further results for germanium, indium antimonide, and indium arsenide taken at room temperature as well as more recent data for germanium obtained at 77°K and 4°K. The theoretical interpretation will be given in a subsequent paper.[5]

The earlier investigations of the behavior of the absorption edge in semiconductors in dc magnetic fields[1,6] and in pulsed magnetic fields as high as 200 000 gauss[1] had indicated that the apparent shift of the gap was nonlinear as a function of magnetic field. Such a nonlinear shift appeared to be inconsistent with the simple Landau theory applied to parabolic bands using the effective-mass approximation, which states that the magnetic levels are given by

$$E_n=(n+\tfrac{1}{2})\hbar\omega_c+\hbar^2k_z^2/2m^*, \quad (1)$$

where n is the quantum number of the level, $\omega_c=eB/m^*$ is the cyclotron frequency of the carrier in the magnetic field $\mathbf{B}$, m^* is the effective mass, and k_z is the wave number parallel to the magnetic field. The levels in the band are quantized in the coordinate system transverse to the magnetic field taken as the z direction. These are harmonic oscillator-like levels which, for two simple bands in a semiconductor, give a gap shift:

$$\Delta E_g=\tfrac{1}{2}\hbar(\omega_{c1}+\omega_{c2}). \quad (2)$$

With this model in mind, we have attempted to analyze data obtained for indium antimonide and indium arsenide at higher fields where the slope of the gap shift curves were a maximum. We further assumed that the mass associated with the valence band was rather large and attributed the shift principally to the conduction electrons. The effective mass values deduced in this way were larger than those determined from cyclotron resonance measurements.[7-9] In order to clarify the situation, it was logical to carry out a similar set of experiments for the direct transition in ger-

* The research reported in this document was supported jointly by the Army, Navy, and Air Force under contract with the Massachusetts Institute of Technology.

[1] Zwerdling, Keyes, Foner, Kolm, and Lax, Phys. Rev. **104**, 1805 (1956).
[2] S. Zwerdling and B. Lax, Phys. Rev. **106**, 51 (1957).
[3] E. Burstein and G. S. Picus, Phys. Rev. **105**, 1123 (1957).
[4] S. Zwerdling and B. Lax, Bull. Am. Phys. Soc. Ser. II, **2**, 141 (1957); Lax, Zwerdling, and Roth, Bull. Am. Phys. Soc. Ser. II, **2**, 141 (1957); Roth, Lax, and Zwerdling, Bull. Am. Phys. Soc. Ser. II, **2**, 141 (1957).
[5] Roth, Lax, and Zwerdling (to be published).
[6] Burstein, Picus, Gebbie, and Blatt, Phys. Rev. **103**, 826 (1956).
[7] Dresselhaus, Kip, Kittel, and Wagoner, Phys. Rev. **98**, 556 (1955).
[8] Burstein, Picus, and Gebbie, Phys. Rev. **103**, 825 (1956).
[9] Keyes, Zwerdling, Foner, Kolm, and Lax, Phys. Rev. **104**, 1804 (1956).

manium. This was inspired by the work of Dash and Newman[10] who measured the direct-transition absorption edge in thin samples of germanium 2–3 μ thick. The existence of a small electron mass $\sim 0.034\ m_0$ at $k=0$ as predicted by Dresselhaus, Kip, and Kittel[11] indicated that this gap shift for germanium should be comparable to that for indium arsenide. The initial experiments were made on a 4 μ thick sample of germanium supplied by Dash, using dc fields up to 36 kilogauss, and showed the expected shift of the absorption edge and also the oscillatory magneto-absorption effect at somewhat higher photon energies. The interpretation of the latter effect demonstrated the validity of the fundamental model as represented by Eq. (1).

II. EXPERIMENTAL TECHNIQUES

A simplified schematic diagram of the experimental apparatus is shown in Fig. 1. The infrared radiation source was a tungsten filament lamp for the germanium experiments and a globar for the indium arsenide and indium antimonide experiments. Both sources were stabilized, the former by means of a servo system employing an infrared-sensitive phototube. The radiation was focused by means of a suitable front-surface reflecting optical system upon the sample located between the tapered pole pieces of a 12-inch Varian electromagnet. The latter was capable of producing dc magnetic fields up to 36 900 gauss in an air gap of $\frac{1}{4}$ inch with pole-face diameter of $\frac{7}{8}$ inch as measured by a rotating coil gaussmeter. The transmitted radiation was refocused by a similar optical system upon the entrance slit of a standard Perkin-Elmer single-pass monochromator employing a dense flint-glass prism for germanium, a LiF prism for indium arsenide, and a CaF_2 prism for indium antimonide, so as to provide maximum refractive dispersion in each case. The signal was detected by a thermocouple, then amplified, rectified, and recorded on a strip chart recorder synchronized with the wavelength drive. Separate traces were obtained by scanning the same wavelength region of interest in each case, using a series of values of constant magnetic field. A very slow wavelength scanning rate was used in order to maximize the accuracy of determining the photon energy of the various transmission minima from the prior calibration of monochromator drum readings *versus* wavelength. For the germanium experiments at liquid nitrogen and liquid helium temperatures, a special all-glass Dewar was built, the most important feature of which was the incorporation of two optical absorption cells of square cross section and different size which composed the inner and outer walls of the Dewar. This portion of the Dewar which contained the sample had a dimension so as to fit between $\frac{5}{8}$-inch air-gap pole pieces of $\frac{7}{8}$-inch pole-face diameter, capable of achieving 30 300 gauss. During the course of such experiments on germanium, the samples were directly in contact with the refrigerant, which together with the glass walls were transparent in the wavelength region of interest for germanium.

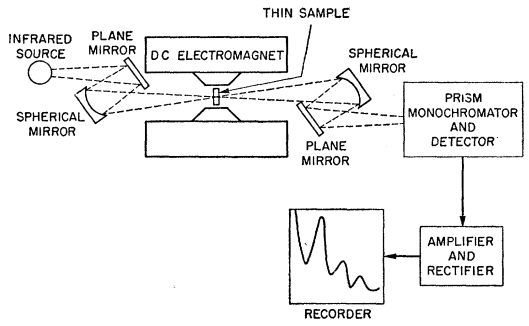


FIG. 1. Schematic diagram of experimental apparatus for the oscillatory magneto-absorption measurements.

One of the critical parts of these experiments was the preparation of thin samples of suitable cross section. The techniques used were similar to those used and described by Dash and Newman.[10] For germanium, the sample was mounted on optical glass with Allymer thermosetting cement pressed to a thickness of the order of 1 micron. For indium antimonide and indium arsenide, the samples were mounted on either intrinsic germanium or silicon. Two sets of germanium samples were used, one furnished by Dash of thickness $\sim 4\ \mu$ and the other prepared by us of thickness $\sim 7\ \mu$. The samples of the latter were single crystals oriented so that the polished surface was the (110) plane. They consisted of $\frac{1}{4}$-inch diameter disks, which were rotated in the magnetic field so that the field vector was parallel in turn to the [110], [100], and [111] axes. For indium arsenide and indium antimonide, the initial experiments were performed using polycrystalline samples. The indium arsenide samples were $\sim 15\ \mu$ thick and the indium antimonide samples were from 7 to 18 μ thick. Oriented single crystals of indium antimonide were also studied. In order to maximize the sample cross section in the radiation beam so as to obtain optimum resolution, the latter were made in the form of rectangles 0.750×0.150 inch. In this case, it was necessary to have three different samples for the three principal orientations employed which were polished to approximately the same thickness. The thickness of all samples was determined by means of a dial gauge with a precision of 1.25 μ as well as from the optical interference pattern in the infrared at wavelengths beyond the absorption edge. The latter method was relatively simple and merely required a measurement of the spacing between two adjacent maxima and a knowledge of the index of refraction n. The relationship involved is

$$d=\lambda\lambda'/(2n\Delta\lambda), \tag{3}$$

where λ, λ' are the wavelengths of these maxima, $\Delta\lambda$ is the wavelength separation, and d is the sample thickness.

[10] W. C. Dash and R. Newman, Phys. Rev. **99**, 1151 (1955).
[11] Dresselhaus, Kip, and Kittel, Phys. Rev. **98**, 368 (1955).

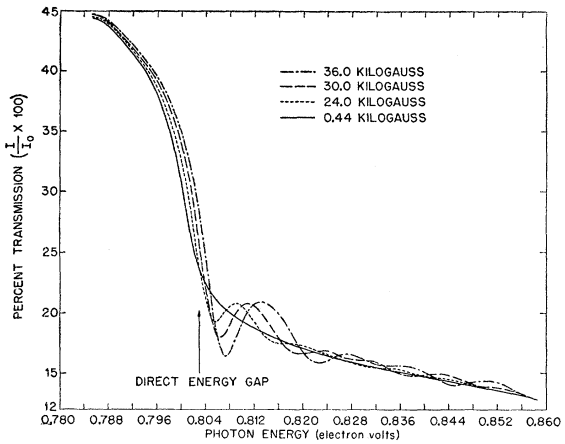


FIG. 2. The magneto-absorption spectrum for the direct transition in germanium.

Experimental Results

In carrying out these experiments, particularly in indium antimonide and indium arsenide, the primary data recorded by the spectrometer system included absorptions due to the atmosphere, to the Allymer cement used for mounting the sample, and to absorptions by the sample not associated with transitions between the quantized levels due to the magnetic field. In order to eliminate the effects of these extraneous absorptions and reduce the data so as to show clearly only the effects on sample transmission due to the presence of the magnetic field, the ratio of the transmitted intensity with a given magnetic field to that at zero magnetic field for the wavelength region of interest was evaluated. This ratio was expressed as $I_t(B)/I_t(0)$, and it can be shown that it is very closely equal to

$$\frac{[1-R(B)]^2}{[1-R(0)]^2}e^{-[\alpha(B)-\alpha(0)]d}, \quad (4)$$

where R is the reflection coefficient and α the absorption coefficient. The minima of this ratio, then, essentially correspond to those wavelengths (or photon energies) for which the differential absorption coefficient $[\alpha(B)-\alpha(0)]$ has maxima. These maxima correspond to the electron transitions between the quantized magnetic or Landau levels from the valence to the conduction bands.

Germanium

In the preceding publication[2] we reported the results of the magneto-absorption in an unoriented single crystal of germanium in the neighborhood of the absorption edge for the direct transition. Figure 2 shows the complete magneto-absorption spectrum in this region of photon energy for a sample $\sim 4\ \mu$ thick. In this particular case, we have plotted the percentage transmission not only to show clearly the effect of the magnetic field on the transmission at energies larger than the absorption edge which is associated with the oscillatory magneto-absorption phenomenon, but also to demonstrate the magneto-absorption effect on the absorption edge itself. We shall discuss the significance of the latter after quantitative analysis of the oscillatory effect.

As mentioned previously, the appropriate presentation of the data at energies above the absorption edge is a plot of $I_t(B)/I_t(0)$. For germanium, these results are shown in Fig. 3, which is a reduction of the data for a sample 7 μ thick with the magnetic field oriented in the [111] direction. The interpretation of these transmission minima is that they correspond to transitions from Landau levels in the valence band to Landau levels in the conduction band at $k=0$. These energy levels for simple parabolic bands are given relative to the extremum of a band by Eq. (1). In a subsequent paper, we shall show that this simple model serves to point out general features such as the linear dependence of the position of the energy levels upon magnetic field for a band which is a quadratic function of momentum, and also if appropriately used, to obtain estimates of the effective masses and highly accurate determinations of the energy gap in these materials. In order to apply the concepts of the simple model to the experimental results, we have plotted the position of the various transmission minima in terms of photon energy as a function of magnetic field and extrapolated the lines obtained to zero magnetic field for room temperature, liquid nitrogen temperature, and liquid helium temperature, as shown in Fig. 4. The two most striking features of these plots are that the functional dependence is indeed linear, and that the lines converge to a photon energy corresponding unequivocally to the energy gap of the transition. For germanium, the energy gap values obtained are 0.803 ± 0.001 ev at ~ 298°K; 0.890 ± 0.001 ev at ~ 77°K; and 0.897 ± 0.001 ev at 4°K. From these values, we calculate the average rate of change of the energy gap with temperature to be 3.9×10^{-4} ev/°K between 77°K and 298°K and 1.0×10^{-4} ev/°K between 4°K and 77°K.

One can also make an estimate of the effective mass

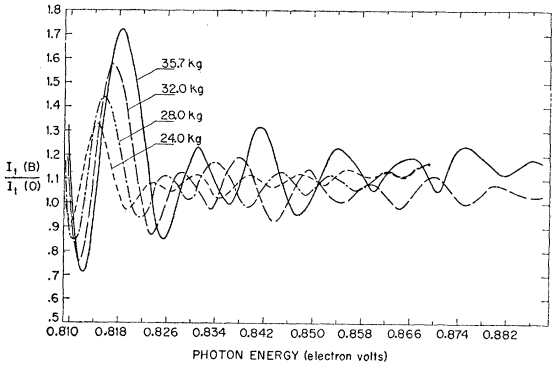


FIG. 3. Oscillatory magneto-absorption in germanium. The ratio of the transmitted signals with and without magnetic field *versus* photon energy for various magnetic field values. **B** along [111] axis.

of the electron at $k=0$ in the Γ_2- band from the slope of these lines. In a previous article[2] we have done this by using the slope of the first line of the room-temperature data. This procedure is inappropriate for the low-temperature data since the slopes of the first lines are vanishingly small. A satisfactory explanation for this is being sought. However, from the detailed theoretical analysis, using the Luttinger-Kohn model,[12] we have shown that the lines 5 and 7 of Fig. 4 correspond to transitions from "heavy"-hole levels of quantum numbers 4 and 6 to electron level 4 and from hole levels 6 and 8 to electron level 6, respectively, according to the selection rules $\Delta n=0$ and $\Delta n=-2$. In accordance with the theory of Luttinger and Kohn, levels of high quantum number are more uniformly spaced in energy, that is, the so-called quantum effects are less significant. Consequently, the energy separation between the fifth and seventh transmission minima is given by

$$\Delta E_{5,7}\approx 2\hbar(\omega_{c1}+\omega_{c2}), \tag{5}$$

where ω_{c1} is the cyclotron frequency for the electron at $k=0$, and ω_{c2} is the cyclotron frequency for the heavy hole in the particular direction taken for the magnetic field, which in this case is the [111] axis. Taking the actual energy separation at 30 kilogauss as shown in Fig. 4, we obtain an energy separation between these minima of 0.021 ev at 300°K. Using the appropriate values for the constants in Eq. (5), we obtain a value of a reduced effective mass $m_r^*=0.033\ m_0$, where

$$\frac{1}{m_r^*}=\frac{1}{m_1^*}+\frac{1}{m_2^*}. \tag{6}$$

Since the heavy-hole mass $m_2^*\sim 0.376\ m_0$[13] along the [111] axis, the electron mass as calculated from Eq. (6) is $m_1^*=(0.036\pm 0.002)\ m_0$. In a similar manner, the calculated electron masses at ~77°K and ~4°K are $(0.043\pm 0.002)m_0$ and $(0.041\pm 0.002)m_0$, respectively. The latter two values are larger than the room temperature mass by ~15%. Such a change is consistent with the variation of the energy gap with temperature.

In Fig. 5 we show results obtained from data at room temperature on a particular sample of germanium oriented so that the magnetic field was successively along the three principal crystallographic directions, [100], [111], and [110], all taken with a magnetic field intensity of 36 000 gauss. It is evident that both the amplitude of the oscillations and the energy values of the respective transmission minima are anisotropic. The complete analysis of the spectrum is rather involved and is treated in the subsequent paper.[5] However, if we use the result of this analysis which shows that the minima 5 and 7 correspond primarily to heavy-hole transitions, we can estimate the expected anisotropy in terms of the change of energy difference between

[12] J. M. Luttinger and W. Kohn, Phys. Rev. **97**, 869 (1955); J. M. Luttinger, Phys. Rev. **102**, 1030 (1956).
[13] Dexter, Zeiger, and Lax, Phys. Rev. **104**, 637 (1956).

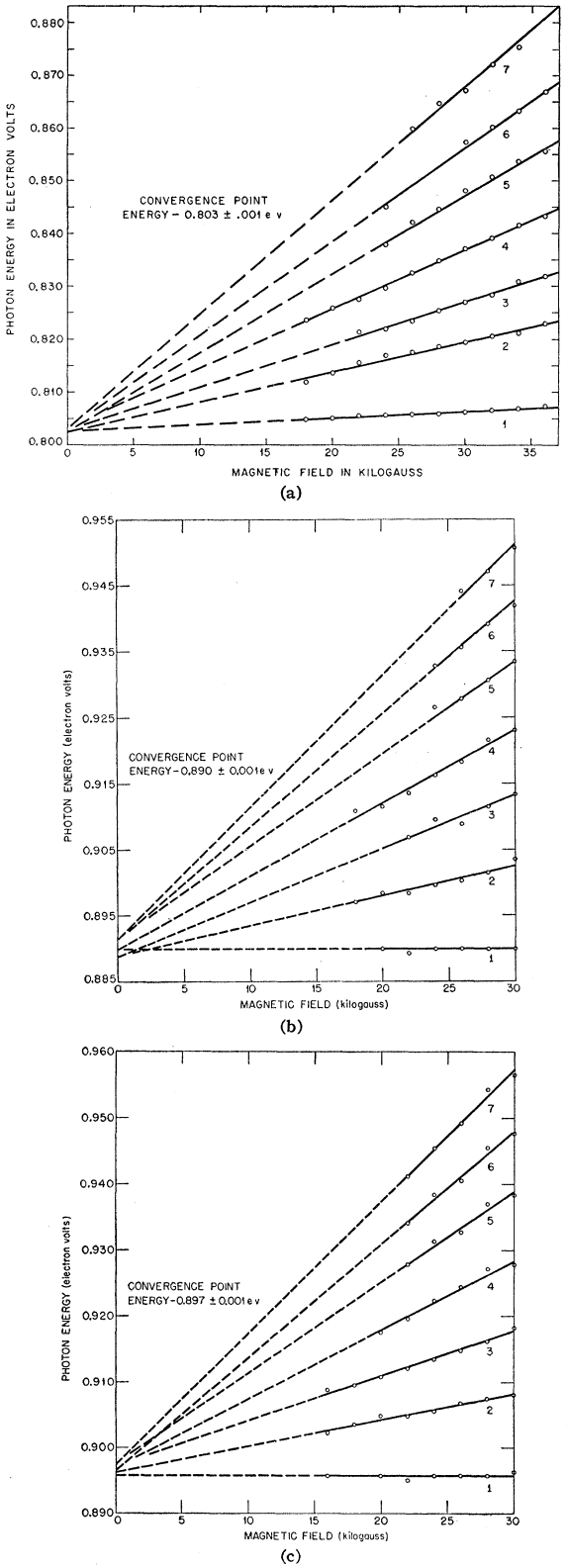


FIG. 4. Photon energy of transmission minima *versus* magnetic field for germanium. (a) Temp. ~298°K; (b) temp. ~77°K; (c) temp. ~4°K.

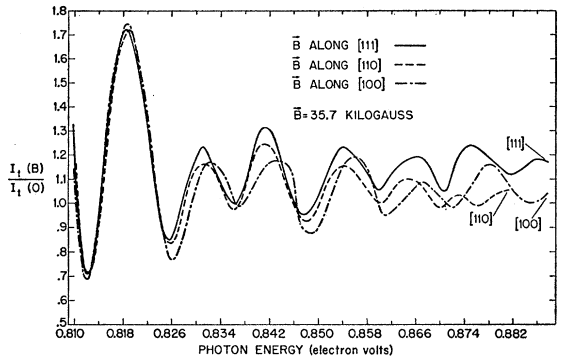


FIG. 5. Anisotropy of the oscillatory magneto-absorption effect in germanium for $B=35.7$ kilogauss.

these minima for the [111] and [100] directions. This is given approximately by

$$\Delta E_{\text{anisotropy}} \approx \bar{n}\hbar[(\omega_{c2})_{111} - (\omega_{c2})_{100}], \qquad (7)$$

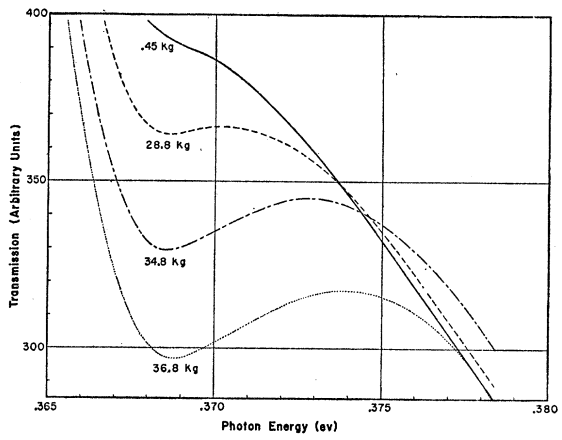
where $\bar{n}$ is an average quantum number of the levels in the valence band for the transitions with $\Delta n=0$ and $\Delta n=-2$. If the values for $m_{111}{}^*$ and $m_{100}{}^*$ are used as obtained from microwave cyclotron resonance,[13] the calculated anisotropy shifts become $\Delta E_5 \approx 0.0018$ ev and $\Delta E_7 \approx 0.0025$ ev in agreement with the experimental values 0.002 ev and 0.003. The calculations for the anisotropy of the other set of transitions as represented by the minima 1 to 4 and 6 of Fig. 5 are more involved and will be tabulated in the subsequent paper.[5]

Indium Arsenide

The results for indium arsenide are shown in Fig. 6, where 6(a) corresponds to the raw data as reproduced from the recording chart and 6(b) to the reduced data $I_t(B)/I_t(0)$. This clearly demonstrates the usefulness of the data-reducing scheme for properly resolving the transmission minima. The sample was polycrystalline with n-type impurity concentration $\sim 5\times 10^{16}$ and was relatively thick ~ 18 microns. We were able to resolve only the first two minima as shown in Fig. 6(b), the second only at the higher fields. Nevertheless, for the first minimum we were able to obtain four points as a function of magnetic field which when extrapolated to zero field gives a value of the energy gap $E_g=0.360 \pm 0.002$ ev. From the slope of this line we have also made a rough estimate of the effective mass of the electron, assuming that the mass of the hole was much larger than that of the electron, and hence $\Delta E_g \approx \frac{1}{2}\hbar\omega_{c1}$. The result gives $m_1{}^* \approx 0.03\ m_0$ consistent with the value measured by infrared cyclotron resonance[9] and free carrier absorption.[14]

Indium Antimonide

The reduced data for indium antimonide are shown in Fig. 7(a) for a 7-micron sample and show four minima at 37 kilogauss, of which two are prominent and two are less intense. In taking data on a sample ~ 15 microns thick, we were able to observe the first minimum more distinctly as shown in Fig. 7(b). Data were also taken on a number of other samples and the results were quite reproducible. In order to attempt quantitative interpretations we have again plotted the position of the four minima as a function of magnetic field as shown in Fig. 8. When the lines are extrapolated to zero field, the value of the energy gap at room temperature is $E_g=0.180\pm 0.002$ ev. We have also plotted the data of Burstein and Picus[3] who carried out measurements up to 60 kilogauss. Although their lines do not all converge, these apparently correspond to the first three minima obtained by us. In attempting to analyze the slope of the first line in terms of a simple model as in indium arsenide by letting $\Delta E_g \approx \frac{1}{2}\hbar\omega_{c1}$, where we neglect the mass of the hole, we find that the apparent mass $m_1{}^* \approx 0.024\ m_0$, which is much larger than that obtained from cyclotron resonance.[7-9] An explanation for this, proposed by Roth, is that the Landau levels



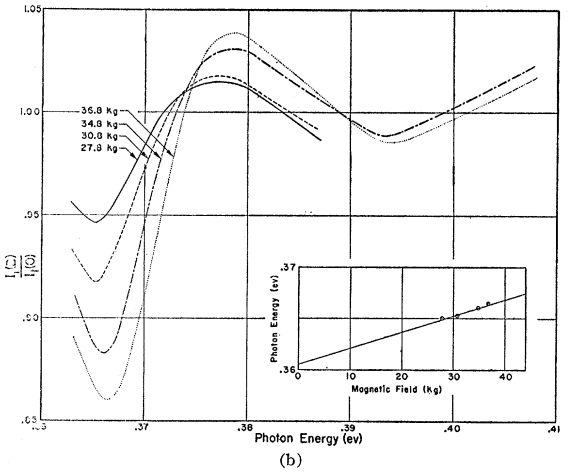


FIG. 6. (a) Transmitted signal *versus* photon energy as a function of magnetic field in InAs. Sample thickness ~ 15 microns. (b) Ratio $I_t(B)/I_t(0)$ *versus* photon energy for same sample of InAs as in 6(a). Insert shows photon energy *versus* magnetic field for the first transmission minimum. Temperature ~ 298°K.

[14] W. G. Spitzer and H. Y. Fan, Phys. Rev. **106**, 882 (1957).

in the conduction band are split into two sets by the effect of the spin-orbit coupling. Although such an effect presumably exists in germanium and indium arsenide, the split there would not be sufficient to be resolved by the prism spectrometer. For indium antimonide, ignoring the fine structure due to the valence band, one can hypothesize that the quantum number of the Landau levels for the electrons associated with the first two lines of Fig. 8 is $n=0$ and for the next two lines is $n=1$. If we ignore the effect of the hole we can show that the mean shift of these two sets of lines is given by $\Delta E\approx(n+\frac{1}{2})\hbar\omega_{c1}$. Using this, we obtain $m_1^*\approx 0.014m_0$ and $m_1^*\approx 0.013m_0$, respectively in good agreement with the values of effective mass of the electron near the bottom of the band as obtained by cyclotron resonance experiments at microwaves[7] and infrared frequencies.[8,9]

We have also attempted to observe anisotropy of the magneto-absorption in indium antimonde by measuring the transmission in two samples which were oriented with the magnetic field parallel to the [111] and [100] axes, respectively. There was a very small observable shift of the first peak of the order of 0.0005 ev and negligible shift of the second peak as shown in Fig. 9.

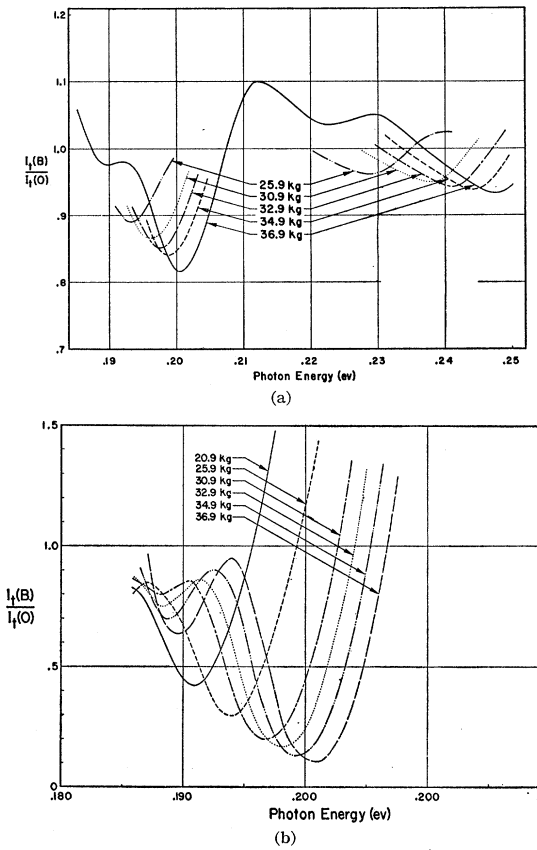


FIG. 7. (a) Ratio $I_t(B)/I_t(0)$ *versus* photon energy for a single crystal of InSb sample ~7 microns thick for various magnetic field values. (b) The same for an oriented InSb sample ~15 microns thick with **B** along [111] axis.

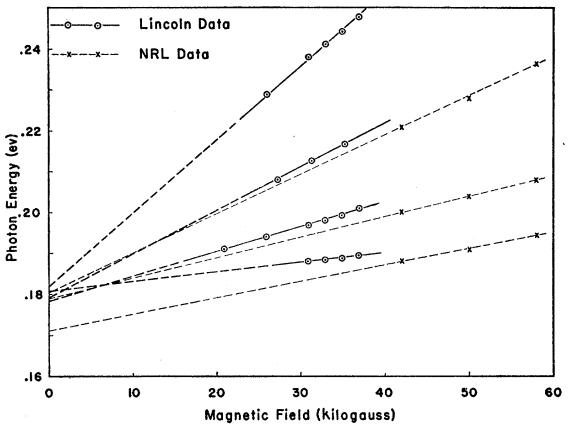


FIG. 8. Photon energy of transmission minima *versus* magnetic field for a single crystal of InSb. Temperature ~298°K.

Apparently the anisotropy effects due to the valence band are not as significant as in germanium. This is consistent with the absence of observable anisotropy in the galvanomagnetic measurements of Frederikse and Hosler,[15] but not with the indications of anisotropy of holes from cyclotron resonance experiments at microwave frequencies.[7]

Discussion

One of the significant results of the oscillatory magneto-absorption experiments is concerned with the location of the energy gap relative to the absorption edge. From Fig. 2 it can be seen that photons of energy as much as ~0.02 ev less than that corresponding to the gap in germanium at $k=0$ are also absorbed. A similar situation exists in indium arsenide and indium antimonide. A conclusion that can be drawn is that a phonon and a photon combine to provide sufficient energy to cause a transition from the valence band to the conduction band. Dumke[16] arrived at this conclusion independently on theoretical grounds and pointed out that an optical phonon would be involved. Consequently, in principle, one can estimate the lattice vibration frequency associated with the optical mode if one analyzes the direct absorption edge in detail. Apparently, from our present measurements in germanium, it corresponds to an energy exceeding 0.02 ev. Since this is not a simple direct transition, the calculation of the transition probability involves a second order perturbation calculation. Consequently, it is not surprising that the absorption coefficient in this region does not follow the $(h\nu-E_g)^{\frac{1}{2}}$ dependence. Hence it is no longer necessary to assume that an indirect transition from a maximum of the valence band displaced from $k=0$ is the explanation of absorption below the gap. This assumption was used by Blount

[15] H. P. R. Frederikse and H. R. Hosler, National Bureau of Standards Report No. 4956, 1956 (unpublished).
[16] W. P. Dumke, Bull. Am. Phys. Soc. Ser. II, 2, 185 (1957).

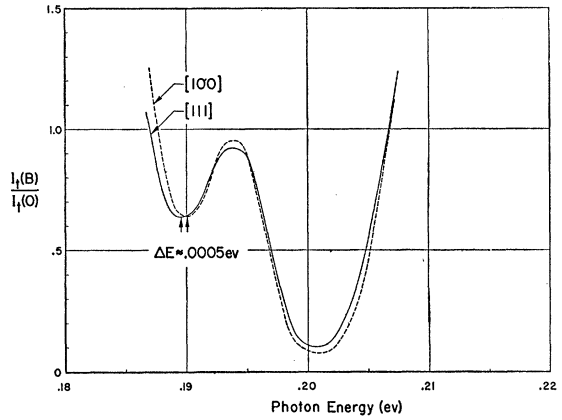


FIG. 9. Anisotropy of the oscillatory magneto-absorption in a single crystal of InSb for $B=36.9$ kilogauss (sample thickness ~18 microns).

et al.[17] in interpreting the absorption edge in indium antimonide based on the results of Roberts and Quarrington.[18]

From the simple analysis of the transitions between Landau levels of higher quantum numbers in germanium, we have shown that one can obtain a fairly good estimate of the effective mass of the conduction-band electron at $k=0$. We have also shown that since the spectrum of some of these transitions can be represented by the parameters associated with the "heavy" holes and the electrons, where for higher quantum numbers the quantum effects described by Luttinger and Kohn are not troublesome, one can obtain a good correlation between the anisotropy observed experimentally and that predicted theoretically from the parameters determined from cyclotron resonance. In principle, it is possible to do the inverse of this and observe the anisotropy of the oscillatory magneto-absorption and from that obtain the energy-momentum parameters of the valence band. Such an experiment might be very profitable for the germanium-silicon alloys where the cyclotron resonance data are not sufficiently well resolved for quantitative interpretation.

It is apparent that for a study of the magnitudes of energy separations corresponding to "heavy-hole" Landau levels, which determines the fine structure predicted by the theory,[5] the spectral resolution of a prism spectrometer is inadequate. The line separation for the heavy hole of $m_2^*=0.376\ m_0$ at 36 kilogauss is ~0.001 ev. In order to resolve such closely spaced lines, we have adapted the spectrometer system to use a grating having 6000 lines/cm, which should be capable of a spectral resolution corresponding to 0.00016 ev at 0.80 ev with 100-micron slits. With such spectral resolution, it will be possible to make a careful study of the absorption edge itself in order to obtain a more accurate estimate of the optical lattice frequency. It should also be possible to look for fine structure in the absorption edge associated with excitons[19] and also to study any related magnetic effects. It will probably be necessary to use liquid helium temperatures to avoid line broadening which would obscure the fine structure of the oscillatory magneto-absorption. The criterion for such resolution to be possible is $\omega_c>1/\tau$, which is very well satisfied at 4°K. Furthermore, it has been shown theoretically[5] that the use of circularly and linearly polarized light will simplify the magneto-absorption spectrum by reducing the number of fine structure lines associated with each minimum, thereby facilitating quantitative interpretation.†

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† *Note added in proof.*—The fine structure of the oscillatory magneto-absorption of the direct transition in germanium predicted theoretically above has now been clearly observed experimentally. The experiments were carried out using a magnetic field of thirty-nine kilogauss, liquid helium temperature, and an infrared grating spectrometer with useful resolving power greater than 20 000. With unpolarized radiation in the photon energy range 0.90 ev to 1.07 ev and the magnetic field oriented parallel to [100], approximately thirty absorption maxima were observed. Linearly polarized infrared radiation with the electric vector first parallel then perpendicular to the magnetic field resolved considerable additional structure. The first five maxima corresponding to those of Fig. 5 (but shifted in energy as a result of the lower temperature) have been thus far resolved into the following prominent components for the two directions of polarization indicated by the symbols (‖) and (⊥):—Maximum 1, (‖) 0.9001 and 0.9045 ev, (⊥) 0.9000 and 0.9046 ev; Maximum 2, (‖) 0.9155, 0.9170, and 0.9202 ev, (⊥) 0.9161 ev; Maximum 3, (‖) 0.9258, 0.9296, and 0.9316 ev, (⊥) 0.9256 and 0.9304 ev; Maximum 4, (‖) 0.9403 and 0.9430 ev, (⊥) 0.9367, 0.9396, 0.9439, and 0.9469 ev; Maximum 5, (‖) 0.9524, 0.9558, and 0.9605 ev (⊥) 0.9539 and 0.9573 ev. Furthermore, many additional fine structure absorption maxima were clearly resolved up to 1.07 ev. Analysis of these data will be reported at a later date.

The new measurements were obtained with the sample in the after-optics, i.e., so that energy from the exit slit was focused on the sample, rather than that from the source, as shown in Fig. 1. These measurements suggest a small increase of the energy gap relative to that previously found at liquid helium temperature. This could be due to a higher sample temperature in the earlier measurements which were made with the total spectral output of the source focused on the sample in the fore-optics. This matter is being further investigated.