DISCUSSION

The quantitative application of the theory presented in this paper is limited by the following considerations:

(i) The assumptions of spherical constant energy surfaces and validity of the simple model of band structure are not correct.

(ii) Equation (2) is not correct for most semiconductors, since the temperature dependence of the zero-field lattice mobility predicted by it is not in agreement with experiment.

(iii) The validity of a Maxwellian distribution, even at low fields, has not been proved and hence the expressions for β may not be quantitatively correct.

However, the theory can be put to considerable use in checking the theories of impurity scattering[1,3] at low temperatures. The common method is to investigate the zero-field mobility μ_0 of the same sample over a wide range of temperature. The zero-field lattice mobility can in general be known by theory or experiment. With the help of Fig. 1 and Eq. (13), we can determine one of the mobilities μ_{I0} or μ_{N0} if the other is known by calculation or otherwise. The value of β obtained by using Eq. (17A) and Fig. 1 may also be compared with that experimentally observed.

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Mobility of Carriers in Nondegenerate Semiconductors at Low Electric Fields

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By using the velocity distribution of carriers in the presence of an electric field E, due to Yamashita and Watanabe, it is shown that the mobility μ is given by $\mu=\mu_0(1+\beta E^2)$ for low fields. The values of β obtained are larger than those arrived at by the assumption that the distribution is Maxwellian in the presence of an electric field. The discrepancy between the theoretical and observed values of β is large enough to justify further careful examination of the theory of the perturbing effect of weak fields.

WITH the usual simplifying assumptions, Conwell[1] has shown that the mobility μ is related to the time of relaxation τ by

$$\mu=\frac{q}{3m}\left\langle\frac{1}{v^2}\frac{d}{dv}(v^3\tau)\right\rangle, \tag{1}$$

where the symbols have their usual meanings.

In discussing the effect of field on mobility, the concept of hot carriers, introduced by Shockley,[2] has often been employed. Further, it has been assumed that the field in effect raises the carriers to a temperature T, greater then the temperature T_0 of the crystal, the velocity distribution still being Maxwellian and given by

$$N(v)dv=Av^2\exp(-\lambda v^2)dv, \tag{2}$$

where $\lambda=m/2kT$, k being the Boltzmann constant.

Further, Shockley[2] argued that in the steady state, the average power gain from the field must balance the average power loss in collisions, or, symbolically,

$$q\mu E^2+(d\mathcal{E}/dt)_c=0. \tag{3}$$

It can easily be shown from Eqs. (1), (2), and (3) that for low fields the relationship between the mobility μ and field E can be expressed as

$$\mu=\mu_0(1+\beta E^2). \tag{4}$$

When scattering due to lattice vibrations is predominant,

$$\tau_L=l/v, \tag{5}$$

where l is the mean free path, constant at a given temperature, and

$$\beta_L=-3\pi\mu_{L0}^2/64c^2=-\alpha, \tag{6}$$

where c is the velocity of sound in the crystal and μ_{L0} denotes the zero-field lattice mobility.

When scattering due to ionized impurities is pre-

[1] E. Conwell, Phys. Rev. **88**, 1379 (1952).
[2] W. Shockley, Bell System Tech. J. **30**, 990 (1951).

dominant,

$$\tau_I \propto v^3, \tag{7}$$

and

$$\beta_I = (3/64c^2)\times 3\pi\mu_{L0}\mu_{I0}, \tag{8}$$

where μ_{I0} is the zero-field impurity mobility.

Despite the elegance of this treatment, it suffers from the defect that Eq. (2) is an approximation of uncertain validity. This defect has been removed by Yamashita and Watanabe,[3] who obtained the velocity distribution of the carriers in the presence of the field by solving the Bloch integral equation in nonpolar semiconductors to the second order of approximation. The velocity distribution they arrived at is given by

$$N(v)dv = Av^2(\lambda_0 v^2 + aE^2)^{aE^2}\exp(-\lambda_0 v^2)dv, \tag{9}$$

where

$$a = \frac{q^2l^2}{6mc^2kT_0} = \frac{3\pi\mu_{L0}^2}{16c^2} = 4\alpha,$$

and $\lambda_0 = m/2kT_0$.

It has been found by Yamashita and Watanabe[3] that Eqs. (1) and (9) lead to

$$\mu \propto E^{-\frac{1}{2}}$$

at high fields, a result similar to that obtained by the former treatment. In this communication we proceed to investigate the dependence of mobility on the field, at low fields, by using Eq. (9).

For low fields, Eq. (9) can be put in the form

$$N(v)dv = Av^2\exp(-\lambda_0 v^2)\{1 + aE^2\ln(\lambda_0 v^2)\}dv. \tag{9A}$$

The total number of carriers is given by

$$N_0 = \int_0^\infty N(v)dv = \frac{A}{2\lambda_0^{\frac{3}{2}}}\left\{\int_0^\infty x^{\frac{1}{2}}e^{-x}dx + aE^2\int_0^\infty x^{\frac{1}{2}}e^{-x}\ln x\, dx\right\}$$
$$= \frac{A}{2\lambda_0^{\frac{3}{2}}}\Pi(\tfrac{1}{2})\{1 + aE^2\Psi(\tfrac{1}{2})\}, \tag{10}$$

where $x = \lambda_0 v^2$ and the functions Π and Ψ are defined by Jahnke and Emde.[4]

If

$$\tau = \sigma v^n, \tag{11}$$

Eqs. (1), (9A), and (10) lead to

$$\mu = \frac{\sigma q A(n+3)}{3mN_0}\int_0^\infty v^{n+2}\exp(-\lambda_0 v^2)\{1 + aE^2\ln(\lambda_0 v^2)\}dv$$
$$= \frac{q(n+3)\sigma}{3m\lambda_0^{n/2}}\frac{\Pi\{(n+1)/2\}}{\Pi(\frac{1}{2})}\left[\frac{1 + aE^2\Psi\{(n+1)/2\}}{1 + aE^2\Psi(\frac{1}{2})}\right]$$
$$= \mu_0(1 + \beta E^2), \tag{4}$$

where μ_0 is the value of μ when $E = 0$ and

$$\beta = a\{\Psi(\tfrac{1}{2}n + \tfrac{1}{2}) - \Psi(\tfrac{1}{2})\}. \tag{12}$$

When the scattering due to lattice vibrations predominates, τ is given by Eq. (5), $n = -1$, and

$$\beta_L = -0.6137a = -2.4548\alpha. \tag{6A}$$

This must be compared with the value $-\alpha$ given by the former treatment.

Measurements of the small quadratic changes in mobility predicted by Eq. (4) have been limited to n-type germanium and are summarized by Gunn.[5] It is seen that the values of β experimentally observed are 10–100 times lower than those predicted by the earlier treatment and hence in even greater disagreement with the results obtained in this paper. However, even if the accuracy of the measurements is doubtful, the discrepancy between theory and experiment is large enough to justify further careful examination of the theory of the perturbing effect of weak fields.

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[3] J. Yamashita and N. Watanabe, Progr. Theoret. Phys. Japan **12**, 443 (1954).

[4] E. Jahnke and F. Emde, *Tables of Functions* (Dover Publications, New York, 1945), fourth edition, p. 9.

[5] J. B. Gunn, *Progress in Semiconductors* (John Wiley and Sons, Inc., New York, 1957). Vol. 3.