ω_b/ω. For comparison we plot this same quantity for the case of constant mean free time. We have taken $\alpha=10^4$, $x_1=(1-\omega_b/\omega)^2\times10^4$ and $\alpha=10^6$, $x_1=(1-\omega_b/\omega)^2\times10^6$, which correspond to $M=3680m$ (molecular hydrogen), $E=18$ volts per cm, $kT'=0.06$ ev, $\omega=5.00\times10^{10}$ cps, and $\lambda=0.03$ cm and 0.3 cm, respectively. The values of ν_c used in Fig. 2 have been computed by equating (32) and (34) (at $\omega=\omega_b$), using the above values of λ, kT', and α.

It is gratifying to note that the curves for constant λ and for constant ν_c agree so closely; for many purposes, then, the simple theory of constant ν_c may be relied upon to give quite meaningful results.‡

APPENDIX

We wish to solve Eq. (14) for $\mathbf{b}$:

$$\left(\frac{\partial}{\partial t}+\nu_c-\boldsymbol{\omega}_b\times\right)\mathbf{b}=-\tfrac{1}{2}(e^{j\omega t}+e^{-j\omega t})\boldsymbol{\gamma}\frac{\partial f_0}{\partial v}.$$

One operates on both sides of (14) with $[(\partial/\partial t)+\nu_c+\boldsymbol{\omega}_b\times]$ to get

$$\left(\frac{\partial}{\partial t}+\nu_c+j\omega_b\right)\left(\frac{\partial}{\partial t}+\nu_c-j\omega_b\right)\mathbf{b} =-\frac{1}{2}\frac{\partial f_0}{\partial v}\{[\boldsymbol{\gamma}(j\omega+\nu_c)+\boldsymbol{\omega}_b\times\boldsymbol{\gamma}]e^{j\omega t} +[\boldsymbol{\gamma}(-j\omega+\nu_c)+\boldsymbol{\omega}_b\times\boldsymbol{\gamma}]e^{-j\omega t}\}, \quad \text{(I.1)}$$

where $\boldsymbol{\omega}_b\times\boldsymbol{\omega}_b\times$ has been replaced by $-\omega_b^2$ since $\boldsymbol{\omega}_b\cdot\mathbf{b}=0$. Define

$$D=\partial/\partial t;\quad r=-\nu_c+j\omega_b,$$

$$-\frac{1}{2}\frac{\partial f_0}{\partial v}[\boldsymbol{\gamma}(j\omega+\nu_c)+\boldsymbol{\omega}_b\times\boldsymbol{\gamma}]e^{j\omega t}=\mathbf{B}(\omega,t).$$

Then (I.1) becomes

$$(D-r)(D-r^*)\mathbf{b}=\mathbf{B}(\omega,t)+\mathbf{B}^*(\omega,t), \quad \text{(I.2)}$$

which has the solution

$$\mathbf{b}=\frac{\mathbf{B}}{(r-j\omega)(r^*-j\omega)}+\frac{\mathbf{B}^*}{(r+j\omega)(r^*+j\omega)}.$$

In terms of our definitions, this is

$$\mathbf{b}=-\frac{\partial f_0}{\partial v}\mathrm{Re}\left\{\frac{[\boldsymbol{\gamma}(j\omega+\nu_c)+\boldsymbol{\omega}_b\times\boldsymbol{\gamma}]}{\omega_b^2+(j\omega+\nu_c)^2}e^{j\omega t}\right\}. \quad \text{(I.3)}$$

‡ *Note added in proof.*—Further computations show that for similar parameters a curve almost indistinguishable from a resonance curve results even when λ is proportional to $1/v$.

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Theory of Moving Striations

L. Pekárek
Physical Institute of Czechoslovak Academy of Sciences, Praha, Czechoslovakia

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It is pointed out that the author's previous experiments on moving striations indicate that undamped sinusoidal wave solutions (treated in recent theoretical studies of striations) cannot correctly represent moving striations.

Some papers have recently been published in *The Physical Review* dealing with the theoretical interpretation of moving striations in electrical discharges.[1,2] Of special interest is the paper of Robertson, as it contains a very profound and thorough analysis of this problem. The general equations of this theory, it seems, take into consideration all the main microphysical processes that could be of basic importance for the production of moving striations.

For comparison with theoretical results, and also for the type of solution required, experiments are considered which were carried out by various methods on spontaneously existing moving striations. The greater part of this experimental material is due to the very thorough and extensive investigations of Donahue and Dieke.[3] All these experiments were carried out on spontaneously existing moving striations in the state of stationary self-excited oscillations in the discharge. In such a state it is difficult, however, to find the conditional dependence and temporal evolution of processes, which cause the existence of moving striations.

It is the purpose of this note to point to some publications,[4,5] which are of direct consequence to the theories quoted, dealing with experiments on moving striations. In contradistinction to experiments mentioned above, a method of artificially introducing small

[1] H. S. Robertson, Phys. Rev. **105**, 368 (1957).
[2] S. Watanabe and N. L. Oleson, Phys. Rev. **99**, 646 (1955); **99**, 1701 (1955).
[3] T. Donahue and G. H. Dieke, Phys. Rev. **81**, 248 (1951).
[4] L. Pekárek, Vestnik Moskov. Univ. No. 3, 73 (1954).
[5] L. Pekárek, Czechoslov. J. Phys. **4**, 295 (1954).

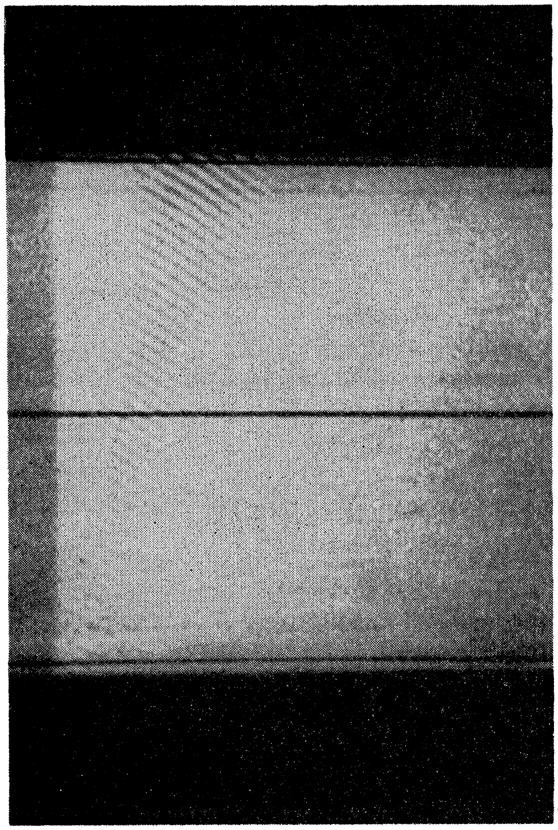


FIG. 1. Photograph of positive column of a glow discharge in neon in a rotating mirror. The axis of the discharge is vertical, the time going from left to right. The dotted line at top and bottom represent time markers (100 microseconds). The three black horizontal lines are markers of distance. The discharge current is 3 ma, and the pressure of the pure neon gas is a few millimeters of Hg.

perturbations was used here for investigating the phenomenon of moving striations, and the transient response of the light intensity of the positive column and of the discharge voltage was recorded by the usual methods.

Figure 1 is a photograph of the positive column taken by means of a rotating mirror, the time going from left to right. The conditions of the discharge were chosen so that no oscillations and no moving striations were present in the absence of disturbances. Into this state of a homogeneous positive column, perturbations were introduced by means of square-wave modulation of the discharge current. The transient response after such a step of current may be seen on the photograph. It is evident that after the disturbance striations arise in the positive column of the discharge. Striations originate earlier at the cathode end (bottom) of the column. Each striation has a finite lifetime which is longer for striations at greater distances from the cathode, i.e., those generated later. In the self-oscillatory state this process, which we call the wave of stratification,[5] repeats because of feedback which can be realized in two channels.[4,5] The individual striations of these waves, however, follow each other and the separate waves of stratification cannot be observed in the self-oscillatory, i.e., the stationary state. The perturbations caused by periodically repeating these waves are observed as pulses propagating in the direction from the cathode to the anode ("negative striations" in the terminology of Donahue and Dieke).

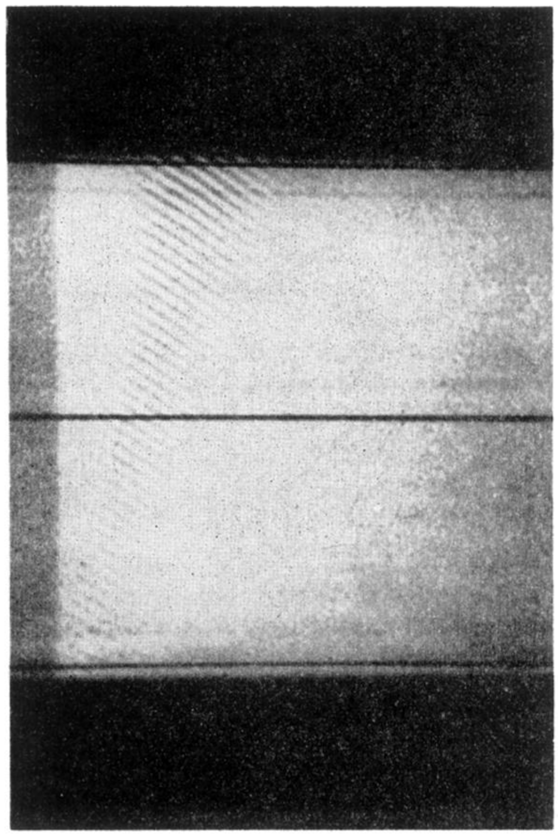
The transient wave of stratification shown in Fig. 1 is typical for the generation of moving striations in inert gases. It is the simplest phenomenon with moving striations; the values of its parameters are independent of processes in the neighborhood of the electrodes, being determined exclusively by the plasma of the positive column. Further, the wave of stratification—when a small perturbation is used in the experiment—is a linear effect indicating that the theory of striations can depart from the theory of small perturbations of the homogeneous state of the positive column.

From the properties of the wave of stratification (Fig. 1) it is at once evident that this "wave" is phenomenologically quite different from a sinusoidal wave. Moreover, it seems that this phenomenon has no direct analogy in the propagation of waves in continuous media. This specificity of the wave of stratification, which cannot be observed in experiments with stationary moving striations, has obviously to be taken into account in formulating any theory of moving striations.

However, in the papers quoted[1,2] the authors look for a solution in the form of undamped sinusoidal waves. This assumption is evidently in contradiction to our experiments published in 1954, and the sinusoidal wave solution cannot therefore correctly represent moving striations. Of course, it does not follow from this conclusion that the original equations, e.g., in such general form as given by Robertson,[1] could not have a solution corresponding to the wave of stratification. This question, however, would need further attention.

We have elaborated a phenomenological theory of the wave of stratification[6,7] based on the conception of a step-by-step generation of the striations due to local perturbation of the space-charge equilibrium at the cathode end of the column. This theory, which explains the delay in the generation of the striations and the increase in their lifetime at increasing distances from the cathode, will be published shortly in the *Czechoslovak Journal of Physics.*

[6] L. Pekárek, Report at the Conference of Physics of Plasmas, October 9, 1956, Leipzig (unpublished).
[7] L. Pekárek, Czechoslov. J. Phys. (to be published).



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