

more than two or three mean free paths is generated by the spectrum of unscattered photons. This spectrum peaks near 3 Mev (see Fig. 5) reflecting very strongly the shape of the absorption coefficient curve. About one-third of the scattered radiation is produced in single collisions; the remainder cannot result from many more interactions since the distributions at the

various angles peak at energies only slightly lower than the maxima of the single scattered spectra (see Fig. 3). The lowest energies are suppressed by the high photoelectric cross section in lead and in this particular case by the presence of the boundary.

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Cyclotron Resonance: Method for Determining Collision Cross Sections for Low-Energy Electrons

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The Boltzmann transfer equation is solved for the distribution function of electrons moving under the influence of a constant magnetic field and a high-frequency electric field. The power absorbed from the microwave is calculated for the two customary assumptions: (a) constant mean free time, and (b) constant mean free path.

For the case of constant mean free time, the absorption spectrum exhibits a "Lorentzian" resonance peak. The half-width of this line gives a direct measure of the collision frequency. Experimental data in support of this result are presented. The constant mean free path assumption leads to integrals which can be evaluated closely by the saddle-point method, and the results are presented in graphical form. It is found that the line shape is not very sensitive to the dependence of the collision cross section upon electron velocity.

OBSERVATION of cyclotron resonance with slow electrons in gases presents an opportunity for determining collision cross sections. The purpose of the present paper is to examine the simpler details of this problem, both from the experimental and the theoretical points of view. It is possible to derive the relevant equations for the case of a constant mean free time by following the method of Lorentz, which involves nothing more than a damping term and the force terms representing the electric and magnetic fields in the oscillator equation for the electron. Here, all results are derived from the Boltzmann transfer equation, which is the required starting point for investigations that envisage more general situations. A typical calculation shows, however, that equality of resonance half-width and collision frequency is practically valid even when this quantity depends on electron velocities, as it does in the case of a constant mean free path.

I. BOLTZMANN TRANSFER EQUATION

The distribution function will be approximated by a three-term series. One term describes the undisturbed isotropic distribution; the others account for the anisotropy induced by the fields.

The Boltzmann transfer equation which describes the velocity distribution of electrons subject to a constant magnetic field $\mathbf{H}$ and an oscillating electric field $\mathbf{E} \cos \omega t$ reads

$$\frac{\partial f}{\partial t} + \gamma \cos \omega t \cdot \nabla f + (\omega_b \times \mathbf{v}) \cdot \nabla f = \Delta f / \Delta t, \quad (1)$$

$$\gamma = e\mathbf{E}/m; \quad \omega_b = -e\mathbf{H}/mc; \quad \mathbf{E} = \mathbf{k}E; \quad \mathbf{H} = \mathbf{j}H,$$

where $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ denote unit vectors in the v_x , v_y , and v_z directions respectively. $f(\mathbf{v}, t)$ is the distribution function denoting the density of electrons with velocities about $\mathbf{v}$, and ∇ is the gradient operator in velocity space. $\Delta f / \Delta t$ indicates the rate of change in $f(\mathbf{v}, t)$ due to elastic collisions. We neglect inelastic collisions and electron-ion collisions and assume that the electrons take on that velocity distribution which corresponds to the combined action of elastic collisions and the fields. The absence of a space-gradient term in (1) implies furthermore that the space distribution of the electrons is uniform.

When only an electric field is present, it is customary¹ to expand $f(\mathbf{v}, t)$ as a series of Legendre polynomials in $v_z/v = \cos \theta$. The presence of the magnetic field changes the symmetry about the polar (v_z) axis. It becomes necessary to add a term to describe the

* Supported by the Office of Naval Research.

¹ H. Margenau, Phys. Rev. **69**, 508 (1946).

azimuthal (φ) dependence. We take

$$f(\mathbf{v}, t) = f_0(v, t) + \frac{\mathbf{v} \cdot \mathbf{b}(v, t)}{v}. \quad (2)$$

Intuition suggests and subsequent calculation shows that²

$$\mathbf{b}(v, t) = \mathbf{i}g_1(v, t) + \mathbf{k}f_1(v, t). \quad (3)$$

It is well to note that (2) may be regarded as an expansion of $f(\mathbf{v}, t)$ in spherical harmonics since it is equivalent to the series

$$f(\mathbf{v}, t) = F_0(v, t)Y_0^0(\theta, \varphi) + F_1Y_1^0 + G_1(Y_1^1 + Y_1^{-1}) \quad (4)$$

provided θ is the polar angle between v_z and $\mathbf{v}$, and φ the angle between v_x and the projection of $\mathbf{v}$ upon the v_x-v_y plane. The form of $f(\mathbf{v}, t)$ given by (2) will often be used because it proves more convenient for the ensuing calculations.

Component Terms of the Boltzmann Equation

Substitution from (2) and (3) into (1) yields terms which are written and discussed separately below.

Time derivative.—No manipulation is necessary, and direct substitution of (2) into (1) gives

$$\frac{\partial f}{\partial t} = \frac{\partial f_0}{\partial t} + \frac{\mathbf{v} \cdot \frac{\partial \mathbf{b}}{\partial t}}{v}. \quad (5)$$

Electric field term.—Since

$$\nabla f_0 = -\frac{\mathbf{v} \cdot \frac{\partial f_0}{\partial \mathbf{v}}}{v} \quad (6)$$

and

$$\nabla \left(\frac{\mathbf{v} \cdot \mathbf{b}}{v} \right) = \frac{\mathbf{b}}{v} + \frac{\mathbf{v}}{v} \left[v_x \frac{\partial}{\partial v} \left(\frac{g_1}{v} \right) + v_z \frac{\partial}{\partial v} \left(\frac{f_1}{v} \right) \right], \quad (7)$$

the electric field term becomes

$$\cos \omega t \gamma \cdot \nabla f = \cos \omega t \gamma \left[\frac{f_1}{v} + \frac{v_x^2}{v} \frac{\partial}{\partial v} \left(\frac{f_1}{v} \right) + \frac{v_x v_z}{v} \frac{\partial}{\partial v} \left(\frac{g_1}{v} \right) + \frac{\mathbf{v} \cdot \mathbf{k}}{v} \frac{\partial f_0}{\partial v} \right]. \quad (8)$$

Magnetic field term.—Using (6) and (7), we find the contribution of the magnetic field term to be

$$(\omega_b \times \mathbf{v}) \cdot \nabla f = -\mathbf{v} \cdot (\omega_b \times \mathbf{b})/v. \quad (9)$$

Collision terms.—Only elastic collisions are considered, and we follow Chapman and Cowling³ in

² Were it not for the fact that $\mathbf{E}$ and $\mathbf{H}$ are perpendicular, it would be necessary to include a third term in $\mathbf{b}$ to account for electron motion along $\mathbf{H}$. See R. Jancel and T. Kahan, *Nuovo cimento* **12**, 573 (1954).

³ S. Chapman and T. G. Cowling, *The Mathematical Theory of Nonuniform Gases* (Cambridge University Press, Cambridge, 1939), pp. 348 ff.

taking

$$\frac{\Delta f_0}{\Delta t} = \frac{m}{Mv^2} \frac{\partial}{\partial v} \left(\frac{v^4 f_0}{\lambda} \right) + \frac{kT}{Mv^2} \frac{\partial}{\partial v} \left(\frac{v^3}{\lambda} \frac{\partial f_0}{\partial v} \right), \quad (10)$$

$$\frac{\Delta}{\Delta t} \left(\frac{\mathbf{v} \cdot \mathbf{b}}{v} \right) = -\frac{v}{\lambda} \left(\frac{\mathbf{v} \cdot \mathbf{b}}{v} \right). \quad (11)$$

Here λ is the mean free path for electrons, M the mass of the gas molecules, k the Boltzmann constant, and T the absolute temperature.

II. SOLUTION FOR DISTRIBUTION FUNCTION

When the preceding results are collected, the transfer equation, (1), takes the form

$$\frac{\partial f_0}{\partial t} + \cos \omega t \gamma \left[\frac{f_1}{v} + \frac{v_x^2}{v} \frac{\partial}{\partial v} \left(\frac{f_1}{v} \right) + \frac{v_x v_z}{v} \frac{\partial}{\partial v} \left(\frac{g_1}{v} \right) \right] - \frac{\Delta f_0}{\Delta t} + \frac{\mathbf{v}}{v} \cdot \left[\frac{\partial \mathbf{b}}{\partial t} + \nu_c \mathbf{b} - \omega_b \times \mathbf{b} + \cos \omega t \gamma \frac{\partial f_0}{\partial v} \right] = 0, \quad (12)$$

where $\nu_c = v/\lambda$ is the collision frequency for electrons. Equation (12) is of the form

$$F(v, t) + \frac{\mathbf{v} \cdot \mathbf{B}(v, t)}{v} = \Phi_0 Y_0^0 + \Phi_1 Y_1^0 + \Gamma_1 (Y_1^1 + Y_1^{-1}),$$

and we take the necessary average over directions by first multiplying the equation by Y_0^0 and integrating, then by Y_1^0 , etc. In each case the orthogonality of the spherical harmonics eliminates all but one coefficient. The following two equations result from this operation:

$$\frac{\partial f_0}{\partial t} + \frac{\cos \omega t}{3v^2} \frac{\partial}{\partial v} (v^2 \gamma \cdot \mathbf{b}) = \frac{m}{Mv^2} \frac{\partial}{\partial v} \left(\frac{v^4 f_0}{\lambda} \right) + \frac{kT}{Mv^2} \frac{\partial}{\partial v} \left(\frac{v^3}{\lambda} \frac{\partial f_0}{\partial v} \right), \quad (13)$$

$$\left(\frac{\partial}{\partial t} + \nu_c - \omega_b \times \right) \mathbf{b} = -\cos \omega t \gamma \frac{\partial f_0}{\partial v}. \quad (14)$$

The solution of (14) is elementary and is given in the Appendix. We find

$$\mathbf{b} = -(\partial f_0 / \partial v) \mathbf{Q}, \quad (15)$$

where

$$\mathbf{Q} = \text{Re} \left\{ \frac{[\gamma(j\omega + \nu_c) + \omega_b \times \gamma] e^{j\omega t}}{\omega_b^2 + (j\omega + \nu_c)^2} \right\}. \quad (16)$$

To solve (13), we assume $f_0(v, t) = f_0(v)$. Then (13) reads

$$\frac{\partial}{\partial v} \left(\frac{v^2 \cos \omega t \gamma \cdot \mathbf{b}}{3} - \frac{mv^4 f_0}{M\lambda} - \frac{kTv^3}{M\lambda} \frac{\partial f_0}{\partial v} \right) = 0. \quad (17)$$

We now perform the time average of $\cos\omega t \boldsymbol{\gamma} \cdot \mathbf{b}$ and obtain.

$$\langle \cos\omega t \boldsymbol{\gamma} \cdot \mathbf{b} \rangle_t = -\frac{1}{2} \nu_c A(\omega, \nu_c) (\partial f_0 / \partial v), \quad (18)$$

where for brevity we have defined

$$A(\omega, \nu_c) = \frac{\gamma^2}{2} \left(\frac{1}{\nu_c^2 + (\omega - \omega_b)^2} + \frac{1}{\nu_c^2 + (\omega + \omega_b)^2} \right). \quad (19)$$

Equations (17) and (18) lead to the solution

$$f_0(v) = C \exp \left(- \int_0^v \frac{m v dv}{kT + MA/6} \right), \quad (20)$$

which, for $\omega_b = 0$, is identical with that obtained in reference 1. We will evaluate the integral in (20) under the assumption of (a) constant mean free time and (b) constant mean free path.

(a) Constant Mean Free Time

For the case of constant mean free time, evaluation of (20) is quite simple since the denominator of the integrand is independent of v . We get for f_0

$$f_0(v) = n \left(\frac{m}{2\pi kT^*} \right)^{\frac{1}{2}} \exp \left(- \frac{mv^2}{2kT^*} \right), \quad (21)$$

where

$$T^* = T(1 + MA/6kT), \quad (22)$$

and the constant C has been fixed by requiring

$$4\pi \int_0^\infty f_0 v^2 dv = n,$$

where n = electron concentration.

(b) Constant Mean Free Path

We rewrite A as

$$A = \frac{\gamma^2 \lambda^2}{2} \left(\frac{1}{v^2 + \lambda^2(\omega - \omega_b)^2} + \frac{1}{v^2 + \lambda^2(\omega + \omega_b)^2} \right), \quad (23)$$

and introduce the parameters

$$\begin{aligned} a &= M\gamma^2\lambda^2/12, & c &= \lambda^2(\omega + \omega_b)^2, \\ z &= v^2, & d &= \lambda^2(\omega - \omega_b)^2. \end{aligned}$$

Equation (20) now reads

$$f_0(v) = C \exp \left(- \int_0^{v^2} \frac{(m/2) dz}{kT + a/(z+c) + a/(z+d)} \right). \quad (24)$$

Since $c \gg v^2 = z$ in order that resonance be observed,⁴ we simplify the calculation greatly by taking $a/(z+c) \doteq a/c$. On letting

$$T' = T(1 + a/c kT), \quad (25)$$

⁴ It is necessary that $\omega > \nu_c$ (and thus $\omega\lambda > \nu_c\lambda = v$) if the electrons are to gain enough energy between collisions to exhibit resonance.

we then find on carrying out the integration:

$$f_0(v) = C \exp(-x^2) [1 + x^2/(x_1 + \alpha)]^\alpha, \quad (26)$$

with

$$\alpha = \frac{M}{24m} \left(\frac{eE\lambda}{kT'} \right)^2, \quad x^2 = mv^2/2kT', \quad x_1 = md/2kT'.$$

A more manageable form is available by expanding

$$\ln f_0 = \ln C - x^2 + \alpha \left[\frac{x^2}{x_1 + \alpha} - \frac{x^4}{2(x_1 + \alpha)^2} + \frac{x^6}{3(x_1 + \alpha)^3} - \dots \right]$$

Terms of order x^6 and higher may be neglected, with the result that

$$f_0(v) = C \exp \left[- \frac{x_1}{x_1 + \alpha} x^2 - \frac{\alpha}{2(x_1 + \alpha)^2} x^4 \right], \quad (27)$$

where

$$\begin{aligned} C &= \frac{n}{4\pi} \left(\frac{m}{2kT'} \right)^{\frac{1}{2}} \left\{ \int_0^\infty x^2 dx \right. \\ &\quad \times \exp \left[- \frac{x_1}{x_1 + \alpha} x^2 - \frac{\alpha x^4}{2(x_1 + \alpha)^2} \right] \Bigg\}^{-1}. \end{aligned} \quad (28)$$

III. CURRENT DENSITY AND POWER ABSORPTION

The current density, $\mathbf{I}$, is given by

$$\mathbf{I} = \frac{4\pi e}{3} \int_0^\infty \mathbf{b} v^3 dv. \quad (29)$$

Because of (15), $\mathbf{I}$ assumes the form

$$\mathbf{I} = - \frac{4\pi e}{3} \int_0^\infty \mathbf{Q} \frac{\partial f_0}{\partial v} v^3 dv. \quad (30)$$

(a) Constant Mean Free Time

The integration is elementary since $\mathbf{Q}$ is independent of v . A single integration by parts leaves us with

$$\mathbf{I} = e\mathbf{Q}4\pi \int_0^\infty f_0 v^2 dv.$$

Because of the normalization restriction on f_0 , this is simply

$$\mathbf{I} = n e \mathbf{Q}. \quad (31)$$

The average power absorbed by the electrons ($\bar{P}$) is

$$\bar{P} = \langle \mathbf{I} \cdot \mathbf{E} \cos\omega t \rangle_t.$$

We find

$$\bar{P} = \frac{ne^2 E^2}{4m} \left[\frac{\nu_c}{\nu_c^2 + (\omega - \omega_b)^2} + \frac{\nu_c}{\nu_c^2 + (\omega + \omega_b)^2} \right]. \quad (32)$$

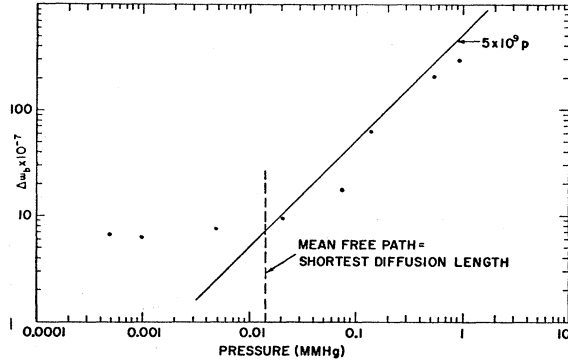


FIG. 1. Half-width of cyclotron-resonance curve as function of pressure p for electrons in hydrogen gas.

The constant mean free time case can be realized experimentally for electrons in hydrogen when the average electron energy is above 3 or 4 volts. Thus, breakdown experiments in hydrogen at cyclotron resonance provide a check of this case.

If we express the power absorbed by the electrons [Eq. (32)] as $\bar{P} = ne^2 E_e^2 / 4m\nu_c$, with

$$E_e^2 = E^2 \left[\frac{\nu_c^2}{\nu_c^2 + (\omega - \omega_b)^2} + \frac{\nu_c^2}{\nu_c^2 + (\omega + \omega_b)^2} \right],$$

it is helpful to view breakdown as a function of magnetic field for a given pressure as a constant E_e process. Thus, using only the second term on the right above when $\omega \approx \omega_b$ and $(\omega + \omega_b)^2 \gg \nu_c^2$, we see that $\bar{P}$ as a function of ω_b should have a half-width equal to ν_c .

Experimental verification of this, and anticipated deviations therefrom below the pressure where diffusion no longer governs loss, is shown in Fig. 1. These data were taken at 3000 Mc/sec frequency at cyclotron resonance in a TM_{010} -mode cylindrical cavity with a uniform magnetic field at right angles to the electric field in the cavity. Each point corresponds to the "half-power" width ($\Delta\omega_b$) of a breakdown curve obtained at the indicated pressure. For comparison the value $\nu_c = 5 \times 10^9 p$ is also plotted where p is pressure in mm Hg.

(b) Constant Mean Free Path

Since we are interested in $(\mathbf{I} \cdot \mathbf{E} \cos \omega t)_t$, we need calculate only the part of $\mathbf{I}$ which is parallel to $\mathbf{E}$ and which varies as $\cos \omega t$. We denote this portion as I_p .

$$I_p = \frac{2\pi e \lambda \gamma \cos \omega t}{3} \int_0^\infty \left(\frac{v^4}{v^2 + c} + \frac{v^4}{v^2 + d} \right) df_0.$$

Substituting for γ and for df_0 from (27) and (28) we get

$$I_p = \frac{ne^2 E \lambda \cos \omega t}{3(2mkT')^{\frac{1}{2}}} \left[\frac{J_5(x_1) + J_5(x_2)}{J^*(x_1)} + 2\beta \frac{J_7(x_1) + J_7(x_2)}{J^*(x_1)} \right], \quad (33)$$

where

$$J_m(x_i) = \int_0^\infty \frac{x^m dx}{x^2 + x_i} \exp(-\eta x^2 - \beta x^4),$$

$$J^*(x_1) = \int_0^\infty x^2 dx \exp(-\eta x^2 - \beta x^4),$$

$$\eta = \frac{x_1}{x_1 + \alpha}, \quad \beta = \frac{\alpha}{2(x_1 + \alpha)^2}, \quad x_2 = \frac{mc}{2kT'}.$$

For $\bar{P}$ we have

$$\bar{P} = \frac{ne^2 E^2 \lambda}{6(2mkT')^{\frac{1}{2}}} \left[\frac{J_5(x_1) + J_5(x_2)}{J^*(x_1)} + 2\beta \frac{J_7(x_1) + J_7(x_2)}{J^*(x_1)} \right]. \quad (34)$$

When the resonant condition ($\omega = \omega_b$, $x_1 = 0$) is satisfied, $\bar{P}$ takes the simpler form

$$\bar{P}_{\max} = \frac{ne^2 E^2 \lambda}{6(2mkT')^{\frac{1}{2}}} \left[\frac{\Gamma(\frac{1}{2})}{\Gamma(\frac{3}{4})(2\alpha)^{\frac{1}{2}}} \right], \quad (35)$$

provided we ignore the nonresonant term $J_7(x_2)$ which is quite insignificant in comparison with $J_7(x_1 = 0)$. Evaluation of $J_m(x_1)$ and $J^*(x_1)$ by the saddle point method[†] introduces an error[‡] of not more than two percent in $\bar{P}$. In Fig. 2 we plot $\bar{P}/\bar{P}_{\max}$ as a function of

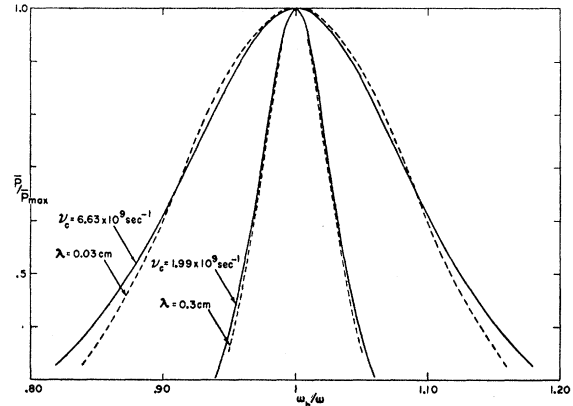


FIG. 2. Power absorbed in cyclotron resonance as function of frequency. The full curves were calculated for constant mean collision frequencies, the dashed curves for constant mean free paths, the parameters for the two cases being adjusted to give the same maximum absorption $P_{\max}$.

[†] For further discussion see H. Margenau [Phys. Rev. (to be published)].

[‡] The error depends rather crucially on the form of the integrand (variable of integration) chosen for these calculations. The most suitable form is ascertained by making checks where exact evaluation is possible or by accurate numerical integration.

[§] For values of ω_b/ω near unity, we need compute only

$$\bar{P} = \frac{ne^2 E^2 \lambda}{6(2mkT')^{\frac{1}{2}}} \left[\frac{J_5(x_1) + 2\beta J_7(x_1)}{J^*(x_1)} \right].$$

ω_b/ω . For comparison we plot this same quantity for the case of constant mean free time. We have taken $\alpha=10^4$, $x_1=(1-\omega_b/\omega)^2 \times 10^4$ and $\alpha=10^6$, $x_1=(1-\omega_b/\omega)^2 \times 10^6$, which correspond to $M=3680m$ (molecular hydrogen), $E=18$ volts per cm, $kT'=0.06$ ev, $\omega=5.00 \times 10^{10}$ cps, and $\lambda=0.03$ cm and 0.3 cm, respectively. The values of ν_c used in Fig. 2 have been computed by equating (32) and (34) (at $\omega=\omega_b$), using the above values of λ , kT' , and α .

It is gratifying to note that the curves for constant λ and for constant ν_c agree so closely; for many purposes, then, the simple theory of constant ν_c may be relied upon to give quite meaningful results.†

APPENDIX

We wish to solve Eq. (14) for $\mathbf{b}$:

$$\left(\frac{\partial}{\partial t} + \nu_c - \omega_b \times\right) \mathbf{b} = -\frac{1}{2}(e^{j\omega t} + e^{-j\omega t}) \gamma \frac{\partial f_0}{\partial v}.$$

One operates on both sides of (14) with $[(\partial/\partial t) + \nu_c + \omega_b \times]$ to get

† Note added in proof.—Further computations show that for similar parameters a curve almost indistinguishable from a resonance curve results even when λ is proportional to $1/\nu$.

$$\begin{aligned} & \left(\frac{\partial}{\partial t} + \nu_c + j\omega_b\right) \left(\frac{\partial}{\partial t} + \nu_c - j\omega_b\right) \mathbf{b} \\ &= -\frac{1}{2} \frac{\partial f_0}{\partial v} \{ [\gamma(j\omega + \nu_c) + \omega_b \times \gamma] e^{j\omega t} \\ & \quad + [\gamma(-j\omega + \nu_c) + \omega_b \times \gamma] e^{-j\omega t} \}, \quad (\text{I.1}) \end{aligned}$$

where $\omega_b \times \omega_b \times$ has been replaced by $-\omega_b^2$ since $\omega_b \cdot \mathbf{b} = 0$. Define

$$D = \partial/\partial t; \quad r = -\nu_c + j\omega_b, \\ -\frac{1}{2} \frac{\partial f_0}{\partial v} [\gamma(j\omega + \nu_c) + \omega_b \times \gamma] e^{j\omega t} = \mathbf{B}(\omega, t).$$

Then (I.1) becomes

$$(D-r)(D-r^*)\mathbf{b} = \mathbf{B}(\omega, t) + \mathbf{B}^*(\omega, t), \quad (\text{I.2})$$

which has the solution

$$\mathbf{b} = \frac{\mathbf{B}}{(r-j\omega)(r^*-j\omega)} + \frac{\mathbf{B}^*}{(r+j\omega)(r^*+j\omega)}.$$

In terms of our definitions, this is

$$\mathbf{b} = -\frac{\partial f_0}{\partial v} \text{Re} \left\{ \frac{[\gamma(j\omega + \nu_c) + \omega_b \times \gamma] e^{j\omega t}}{\omega_b^2 + (j\omega + \nu_c)^2} \right\}. \quad (\text{I.3})$$

Theory of Moving Striations

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It is pointed out that the author's previous experiments on moving striations indicate that undamped sinusoidal wave solutions (treated in recent theoretical studies of striations) cannot correctly represent moving striations.

SOME papers have recently been published in *The Physical Review* dealing with the theoretical interpretation of moving striations in electrical discharges.^{1,2} Of special interest is the paper of Robertson, as it contains a very profound and thorough analysis of this problem. The general equations of this theory, it seems, take into consideration all the main microphysical processes that could be of basic importance for the production of moving striations.

For comparison with theoretical results, and also for the type of solution required, experiments are considered which were carried out by various methods on spontaneously existing moving striations. The greater

part of this experimental material is due to the very thorough and extensive investigations of Donahue and Dieke.³ All these experiments were carried out on spontaneously existing moving striations in the state of stationary self-excited oscillations in the discharge. In such a state it is difficult, however, to find the conditional dependence and temporal evolution of processes, which cause the existence of moving striations.

It is the purpose of this note to point to some publications,^{4,5} which are of direct consequence to the theories quoted, dealing with experiments on moving striations. In contradistinction to experiments mentioned above, a method of artificially introducing small

¹ H. S. Robertson, Phys. Rev. **105**, 368 (1957).

² S. Watanabe and N. L. Oleson, Phys. Rev. **99**, 646 (1955); **99**, 1701 (1955).

³ T. Donahue and G. H. Dieke, Phys. Rev. **81**, 248 (1951).

⁴ L. Pekárek, Vestnik Moskov. Univ. No. 3, 73 (1954).

⁵ L. Pekárek, Czechoslov. J. Phys. **4**, 295 (1954).