

tained with the help of an advanced or a half-retarded, half-advanced Green's function.⁷

Lubanski's method can be generalized and applied to all Eqs. (7). If a retarded Green's function is used, this leads in the second order to equations of motion which include radiation damping. The equation for the i th particle is

$$m^i \frac{d}{d\tau^i} \left[\left(1 - \frac{1}{2} \gamma^{i\rho\sigma} v_\rho^i v_\sigma^i - \frac{1}{4} \gamma^{i\sigma\sigma} v^{i\mu} + \gamma^{i\mu\rho} v_\rho^i \right) \right]$$

$$= m^i \left[- \frac{1}{2} \frac{\partial \gamma^{i\rho\sigma}}{\partial z_\mu^i} v_\rho^i v_\sigma^i - \frac{1}{4} \frac{\partial \gamma^{i\sigma\sigma}}{\partial z_\mu^i} \right]$$

$$+ \frac{11}{3} G(m^i)^2 \left[\frac{d^2 v^{i\mu}}{d(\tau^i)^2} + v^{i\mu} \frac{dv_\rho^i}{d\tau^i} \frac{dv^{i\rho}}{d\tau^i} \right]. \quad (10)$$

Here $\gamma^{i\rho\sigma}$ is the retarded solution (9), with the i th term in the summation omitted, and evaluated at $x_\mu = z_\mu^i$. If a time-symmetric Green's function is used, the equation for the i th particle obtained is of the form (10) without the damping terms, and the γ 's are the half-retarded, half-advanced solutions of (6); an application of the Wheeler-Feynman method⁸ will again lead to Eq. (10) with retarded interactions and including radiation damping.

Equations (10) are Lorentz-invariant and reduce to the Newtonian equations in the nonrelativistic limit, if radiation damping is neglected. For a body moving around a fixed mass, the advance of the perihelion obtained is 7/6 of the correct value, if radiation effects are not considered.⁹ Quite generally these effects are much too small to be observable. For a system of particles they are much smaller than appears at first glance from Eqs. (10) because of partial cancellation due to the fact that in the nonrelativistic limit the gravitational dipole moment of the system about the center-of-mass vanishes.¹⁰

This feature is not revealed by a fixed-mass approximation to the solution of Eq. (10). It similarly remains hidden if one replaces the consideration of a system of interacting particles by that of a particle moving in a "background field." Our method can be applied with minor modifications to the problem of the derivation of the equations of motion of such a "test particle" beyond the customary approximation.¹¹ The method can also be generalized to include higher poles and nongravitational fields in the energy-momentum tensor. The details of all these calculations will be published elsewhere. We only note one significant result: if the gravitational and nongravitational interactions are of the same order of magnitude, we obtain, e.g., for particles carrying electric charges e^i , on the right hand side of Eq. (10) the additional expression

$$e^i \frac{1}{3} F^{i\mu\nu} v_\nu^i + \frac{2}{3} (e^i)^2 \left[\frac{d^2 v^{i\mu}}{d(\tau^i)^2} + v^{i\mu} \frac{dv_\rho^i}{d\tau^i} \frac{dv^{i\rho}}{d\tau^i} \right], \quad (11)$$

where ${}_1F^{i\mu\nu}$ is the retarded electromagnetic field due to all particles except the i th one. Therefore the gravitational and the electromagnetic radiation damping terms are of the same form, and thus it appears that gravitational radiation effects have as much reality as electromagnetic ones.

¹ J. N. Goldberg, Phys. Rev. **99**, 1873 (1955); A. E. Scheidegger, Phys. Rev. **99**, 1883 (1955), and references given in these papers.
² Einstein, Infeld, and Hoffmann, Ann. Math. **39**, 66 (1938). For further references see, e.g., L. Infeld, Acta Phys. Polon. **13**, 187 (1954).

³ A. Einstein, Berlin Ber. 688 (1916).

⁴ P. A. M. Dirac, Proc. Roy. Soc. (London) **A167**, 148 (1938).

⁵ In the formulas of this paper raising and lowering of indices is understood to be by means of the Minkowski metric tensor $\eta_{\mu\nu}$. The usual summation convention holds for Greek indices, but not for the superscripts i labeling the singularities.

⁶ J. Lubanski, Acta Phys. Polon. **6**, 356 (1937).

⁷ We do not consider any solutions of the homogeneous equations belonging to Eqs. (6) or (7) which are independent of the motion of the singularities.

⁸ J. A. Wheeler and R. P. Feynman, Revs. Modern Phys. **17**, 157 (1945); **21**, 425 (1949); P. Havas, Phys. Rev. **74**, 456 (1948).

⁹ In this limit Eq. (10) reduces to one of the equations considered by F. J. Belinfante, Phys. Rev. **89**, 914 (1953), who also obtained the same advance of the perihelion.

¹⁰ This is the main reason why a nonrelativistic approximation method leads to damping terms in a much higher order than the present calculation (see J. N. Goldberg, reference 1). The reason why no damping has been obtained in relativistic approximation methods suggested by other authors [F. J. Belinfante, reference 9; B. Bertotti, Nuovo cimento **4**, 898 (1956); J. N. Goldberg, Bull. Am. Phys. Soc. Ser. II, **2**, 239 (1957)] to obtain Lorentz-invariant equations of motion will be discussed in a forthcoming detailed account of our method.

¹¹ M. Mathison, Z. Physik **67**, 270 (1931); Acta Phys. Polon. **6**, 163 (1937); L. Infeld and A. Schild, Revs. Modern Phys. **21**, 408 (1949).

Negative Pion Scattering and the Dispersion Relations*

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RECENTLY the forward scattering dispersion relations have been used to analyze the scattering of negative pions on protons.¹ The result was a very poor fit below resonance with $f^2=0.08$; or a moderately poor fit above resonance with $f^2=0.04$. However $f^2=0.04$ is not consistent with the forward scattering of positive pions. It was inferred that the dispersion relations were violated; or if $f^2=0.04$ was accepted, that charge independence was violated. Subsequent investigations have shown that the effects of electromagnetic or strange particle interactions² would not alter the results appreciably.

We have recalculated the integrals¹ S_- over total cross sections that appear in the dispersion relation. The result has proven to be very sensitive to the chosen energy dependence of the total cross section. This is due to the strong dependence of the integral on the derivative of the cross section with respect to energy.

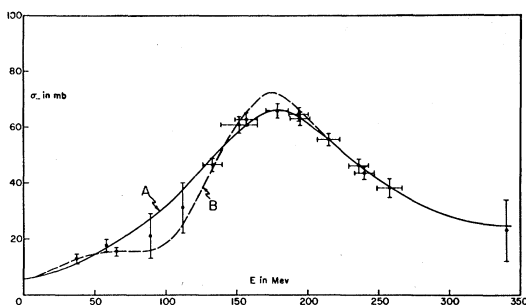


FIG. 1. Total cross section for negative pions. The experimental results are given together with the probable errors. Curve A is similar to that of reference 3. Curve B is more consistent with the dispersion relations.

In Fig. 1 are presented two curves of the cross section *versus* energy which correspond adequately to the data. Curve A is similar to the curve used by Anderson, Davidson, and Kruse.³ Curve B begins to rise more rapidly near 100 Mev, which may be attributed to the influence of some of the p waves. It should be noted that the difference between curves A and B is permitted chiefly by the data near 100 Mev, which has a large experimental error. Curve B also has a somewhat larger maximum than Curve A. The important effect is the substantially greater slope of B over A in the region near 150 Mev, where the previous analysis had achieved a very poor fit. A similar variation is possible in the curve of the positive-pion total cross section, but the dispersion relation for forward scattering of negative pions is not sensitive to this. For higher energies than 350 Mev we use the values indicated by reference 1.

In Fig. 2 the forward scattering resulting from Curve A is plotted for $f^2=0.04$ and $f^2=0.08$. The result is similar to that of reference 1. We have added the experimental value of the forward scattering at 307 Mev, which is now available.⁴

In Fig. 3 the forward scattering resulting from Curve B is plotted for $f^2=0.07$ and $f^2=0.08$. Both of these

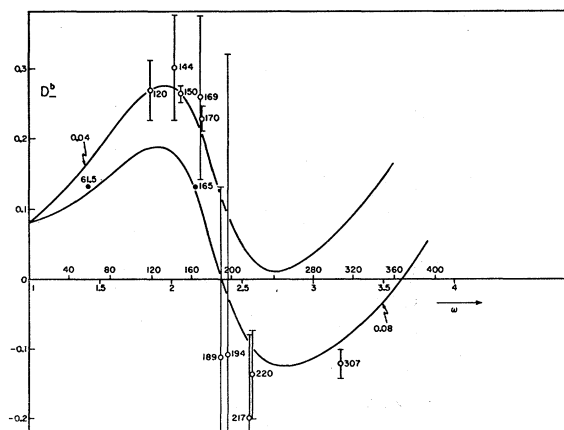


FIG. 2. The forward scattering amplitude for negative pions. The theoretical curves are calculated from Curve A of Fig. 1. The experimental values are as given in reference 1, except that for 307 Mev.⁴ In the latter case only the statistical error is included.

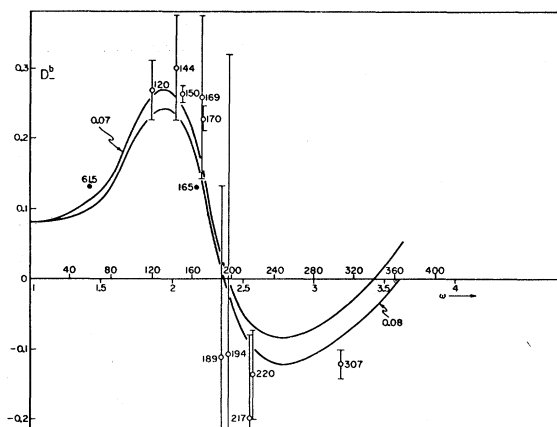


FIG. 3. The forward scattering amplitude for negative pions. The theoretical curves are calculated from Curve B of Fig. 1. The experimental values are the same as in Fig. 2.

values of f^2 are consistent with the positive pion scattering and with photoproduction.⁵ For both values the fit is much better than obtained in reference 1. For $f^2=0.07$, only the 170-Mev forward scattering amplitude is in serious disagreement. The result could be further improved by minor modifications in the total cross-section curve employed.

We conclude that Curve B, or some similar curve through the total cross-section data, does not significantly contradict the dispersion relations or charge independence. It should be possible to distinguish experimentally between Curves A and B by re-examining the 100-Mev region.

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¹ G. Puppi and A. Stanghellini, *Nuovo cimento* **5**, 1256 (1957).

² Agodi, Cini, and Vitale, *Phys. Rev.* **107**, 630 (1957). See this reference for a review and further references.

³ Anderson, Davidson, and Kruse, *Phys. Rev.* **100**, 339 (1955).

⁴ S. M. Korenchenko and V. G. Zinov (private communication).

⁵ Beneventano, Stoppini, Tau, and Bernardini, *Proceedings of the CERN Symposium on High-Energy Accelerators and Pion Physics, Geneva, 1956* (European Organization of Nuclear Research, Geneva, 1956), Vol. 2, p. 259.

Demonstration of Parity Nonconservation in Hyperon Decay*†

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AS is well known, the question of parity conservation in particle decays was raised first by the proper-