Spectrum of Turbulent Mixing

R. Bolgiano, Jr.
Cornell University, Ithaca, New York
(Received August 29, 1957)

IN Sec. III of his recent paper[1] Wheelon concludes that the spectrum of mean square fluctuations produced by turbulent mixing of an established gradient is proportional, in the inertial subrange, to k^{-3}. There is a sizable body of radio data[2–5] which, on the basis of scatter propagation theory, has been interpreted[6] as being in agreement with such a law; and Wheelon's analysis, together with the similar result previously published by Villars and Weisskopf,[7] has been taken by many investigators as theoretical confirmation of the experimental findings, thus tending to close the book on this aspect of the subject. This is most unfortunate since Wheelon's work contains a contradiction which, when fully appreciated, places the whole of the k^{-3} theory in grave doubt. Moreover, experimental studies of the spectrum of refractive index fluctuations support a $k^{-5/3}$ law.[8]

As for the inconsistency in Wheelon's development, in considering the inertial subrange he has, by definition thereof, assumed that molecular effects are of negligible importance, i.e., that the spectrum of mean square fluctuations, $S(k)$, is independent of the viscosity ν and the molecular diffusivity D up to and through this range. Proceeding by purely dimensional reasoning, he then arrives at the result

$$S(k)\propto k^{-3}.$$

However, this "dissipation" of fluctuations by molecular diffusion, since it is proportional to the square of the local gradient of the quantity being mixed, is at each value of k given by $2Dk^2S(k)$. Employing Wheelon's k^{-3} form for $S(k)$, this spectral distribution of molecular effects is thus proportional to k^{-1}. Hence, for a wave number k_i well within the inertial subrange one finds the dissipation occurring at that, and smaller values of k, viz.,

$$2D\int_0^{k_i} k^2S(k)dk\propto \log k_i+\text{const},$$

is a significant fraction of the (finite) total dissipation. This in turn implies that, throughout at least a large part of the range for which Wheelon has derived his k^{-3} law, molecular diffusion plays a role of some prominence, in disagreement with his *a priori* assumption.

Note, moreover, that this contradiction is in no way dependent upon his method of analysis. It springs directly from the concept of "inertial subrange," within which interval, and at smaller wave numbers, a negligible fraction of the total dissipation is presumed to occur. It is readily apparent from this definition that the maximum of the dissipation spectrum, $2Dk^2S(k)$, must fall at a value of k which is beyond the upper end of the inertial subrange. Consequently, within this range one is led to the condition

$$S(k)\propto k^{-2+\delta},\quad \delta>0.$$

Wheelon claims for his treatment that it accounts for the fluctuation contribution to each wave number interval resulting from the mixing of the mean gradient by all size eddies simultaneously, whereas he implies that both Silverman's[9] and Batchelor's[10] analyses are less satisfactory in this respect in that they employ a k-independent external source of turbulent input. However, Bolgiano[11] has shown, by extending Batchelor's work to include the effect of the presence of an established gradient, that, even when in-mixing by a whole hierarchy of eddies is considered, one is still led to the $k^{-5/3}$ law in the inertial subrange. Hence, contrary to Wheelon, so-called "mixing-in-gradient" is not at variance with Batchelor's result; and the radio data *cannot* be explained on the grounds of turbulent mixing of mean gradients as Villars and Weisskopf and Wheelon have proposed.

[1] A. D. Wheelon, Phys. Rev. **105**, 1706–1710 (1957).
[2] L. G. Trolese, Proc. Inst. Radio Engrs. **43**, 1300 (1955).
[3] J. H. Chisholm *et al.*, Proc. Inst. Radio Engrs. **43**, 1317 (1955).
[4] K. Bullington *et al.*, Proc. Inst. Radio Engrs. **43**, 1306 (1955).
[5] J. H. Chisholm *et al.*, paper read before the Combined Technical Session, Union Radio-Scientifique Internationale, Washington, D. C., May 23–25, 1957 (unpublished).
[6] K. A. Norton *et al.*, Proc. Inst. Radio Engrs. **43**, 1488 (1955).
[7] F. Villars and V. F. Weisskopf, Proc. Inst. Radio Engrs. **43**, 1232 (1955).
[8] R. Bolgiano, Jr., paper read before Comm. II, Union Radio-Scientifique Internationale, Washington, D. C., May 23–25, 1957 (to be published).
[9] R. A. Silverman, J. Appl. Phys. **27**, 699–705 (1956).
[10] G. K. Batchelor, Cornell University, School of Electrical Engineering Research Report EE-262, September 15, 1955 (unpublished).
[11] R. Bolgiana, Jr., Cornell University School of Electrical Engineering Research Report EE-334, April 30, 1957 (unpublished).

Polarization of μ^+ Mesons from the Decay of K^+ Mesons*

Charles A. Coombes, Bruce Cork, William Galbraith,† Glen R. Lambertson, and William A. Wenzel
Radiation Laboratory, University of California, Berkeley, California
(Received October 14, 1957)

THE dilemma posed by the apparently irreconcilable τ and θ decay modes of the K meson led Lee and Yang to re-examine the principle of conservation of parity in decay processes.[1] They concluded that parity might not be conserved in weak interactions and suggested some experimental tests of their predictions. The first of these concerned the angular distribution of electrons in the β decay of oriented nuclei, and the

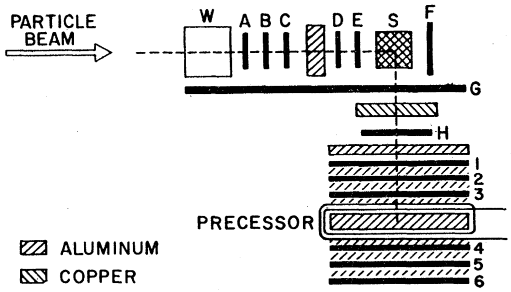

FIG. 1. Side elevation of detector system. A through H and 1 through 6 are plastic scintillation counters. W is a water Čerenkov detector. S is the K^+ stopper, a 3-in. cube of polystyrene. A $K_{\mu 2}^+$ meson stopping in S and its decay μ stopping in the precessor are indicated by the counter response $-W+A+B+C+D+E-F+G+H+1+2+3-4$. Decay positrons are indicated by $+3+2+(1)-4-G$ or $+4+5+(6)-3-G$. G acts as a guard counter against possible background of scattered π^+ mesons entering the counter system 1, 2, $\cdots$6.

experiment on the decay of Co^{60} nuclei revealed an asymmetry in the electron distribution.[2] Experiments on the $\pi^+ \rightarrow \mu^+ \rightarrow e^+$ decay chain also showed an asymmetry in the angular distribution of the positrons,[3-5] in agreement with the two-component theory of the neutrino. The angular distribution of the positrons is of the form

$$N(\theta)d\Omega = N(\pi/2)(1+a\cos\theta)d\Omega, \quad (1)$$

where θ is the angle between the original direction of motion of the μ meson and the direction of the emitted positron. The value of a is negative, indicating that the positron is emitted preferentially in the backward direction relative to the incoming μ meson.

We have measured the angular distribution of positrons arising in the decay chain $K_{\mu 2}^+ \rightarrow \mu^+ \rightarrow e^+$. This mode of decay is similar to that of the π meson in that a single neutrino (or antineutrino) is emitted with the μ meson. The K mesons were produced by bombarding a tantalum target with 6.2-Bev protons in the Bevatron. Those emitted within a small solid angle at 88° to the incident proton beam were focused by means of magnetic quadrupole lenses, and momentum-analyzed by a 30° deflection in a magnetic field. The central momentum, as indicated by range measurements, was 480 Mev/c, and the momentum bandwidth, $\pm 3\%$. The detector system, located outside the concrete shielding around the Bevatron, consisted of plastic scintillation counters, a water Čerenkov detector, and absorbers in the arrangement of Fig. 1. This system was shielded magnetically from the stray field of the Bevatron. It was also enclosed in a radiation shield of lead (4 in.) and boron-loaded paraffin wax (12 in.), which reduced the background, due mainly to low-energy neutrons and γ rays.

K^+ mesons came to rest in a block of polystyrene, S, and the μ mesons emitted downwards from those decaying via the $K_{\mu 2}$ mode entered a stack of scintillation counters 1, 2, $\cdots$6. Protons in the incoming particle beam were stopped in the Čerenkov counter, W; π mesons were rejected in our coincidence circuits because first, they produced a pulse in the Čerenkov counter and, second, they passed through S into the anticoincidence counter F. Another coincidence circuit selected those μ mesons which stopped in a 1.5-in. block of aluminum ("precessor"). This block was wound with aluminum wire so that a uniform horizontal magnetic field could be applied to it. A master coincidence indicating a stopped μ^+ meson was used to trigger an oscilloscope trace, and the direction of emission of the positron in the subsequent decay of the μ was indicated by the responses of the scintillators in the stack. These signals were mixed in coincidence pairs to reduce the general background of pulses on the oscilloscope trace.

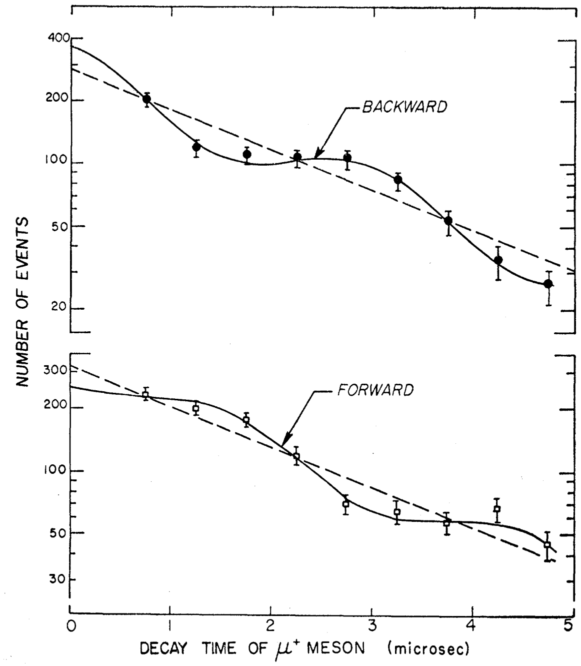

FIG. 2. Detection rate of decay positrons as a function of time. Magnetic field of 25 gauss in precessor.

In carrying out the experiment to determine a in Eq. (1), we applied a magnetic field, first of 25 gauss and later 50 gauss, to the precessor. This field causes the spin direction of the μ meson to precess with a fre-frequency given by $\omega/2\pi = 1.35\times 10^4 B\ \text{sec}^{-1}$, where B is the magnetic flux density in gauss. As a result, a sinusoidal modulation is superimposed upon the normal exponential detection rate of decay positrons; i.e., the counting rate for positrons observed at an angle θ measured with respect to the initial direction of the muons is a function of time having the form

$$N(t) \propto e^{-t/\tau}[1+a_0\cos(\theta+\omega t)]. \quad (2)$$

The amplitude of the modulation then gives a_0, the

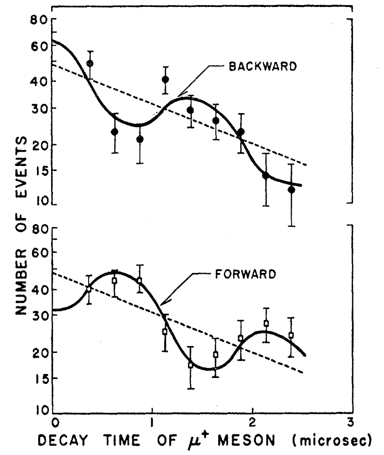

FIG. 3. Detection rate of decay positrons as a function of time. Magnetic field of 50 gauss in precessor.

observed asymmetry. This method has an important advantage over that based on measuring the ratio of the backward and forward decay rates with no magnetic field in that the value of a_0 obtained is independent of many instrumental asymmetries. Furthermore, differences between the backward and forward decay rates can be used to determine any remaining instrumental asymmetries.

The results of the experiment are contained in Figs. 2 and 3. The values of a_0 deduced from these results are given in Table I. In order to compare the results with

TABLE I. Asymmetry of K-μ-e decay. $N(\theta)d\Omega = N(\pi/2)(1+a\cos\theta)d\Omega$.

Precessor field (gauss)	Direction of emission of positrons	a_0	a_0(corr.)
25	Backward	-0.27 ± 0.05	-0.34 ± 0.07
25	Forward	-0.21 ± 0.05	-0.26 ± 0.07
50	Backward	-0.31 ± 0.09	-0.38 ± 0.12
50	Forward	-0.35 ± 0.10	-0.43 ± 0.13
0	Backward/forward	-0.20 ± 0.03	-0.28 ± 0.06
		Average $a_m = -0.31\pm0.04$	

the predictions of the Lee-Yang theory, we have to revise the observed values of a_0 upwards for the following effects. (a) There is a geometrical correction of about 6% which included an edge effect and an apparent depolarization due to the finite angle subtended by the precessor at the position where the K mesons stop and decay. (b) The range curve of the μ meson indicates that about 10% of the μ mesons do not stop in the uniform-field region, and these contribute a background which dilutes the true asymmetry. (c) Averaging the data for calculation purposes into the finite time intervals of Figs. 2 and 3 has the effect of reducing the asymmetry by about 5%. When these corrections are made, we obtain the values of a_0(corr.) given in Table I.

As an additional check on our observed effects, we measured the backward and forward rates of the positrons with no magnetic field on the precessor (see Fig. 4). Here, in addition to making a geometrical correction, we must increase the observed backward/forward ratio by about 12% to allow for an instrumental asymmetry that favored forward decays over backward. The magnitude of this correction was estimated from the data of Figs. 2 and 3.

The weighted mean value from all the data, treating each result as independent, is

$$a_m = -0.31\pm0.04. \qquad (3)$$

To be detected, the positron from a μ meson decaying at the center of the precessor must have a minimum

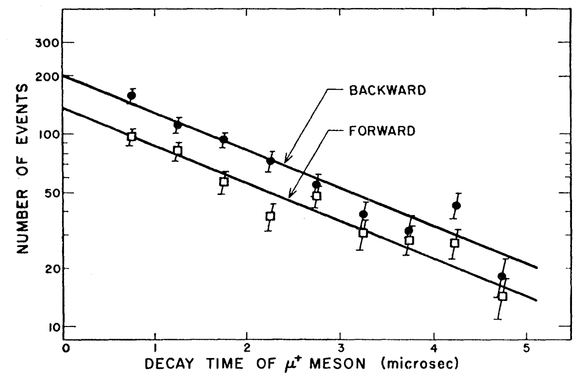

FIG. 4. Detection rate of decay positrons as a function of time. No magnetic field in precessor.

range of 9 g/cm^2 of aluminum, corresponding to a minimum average positron energy of about 25 Mev. This is almost the same energy discrimination as that in the Columbia group's experiment on the $\pi\rightarrow\mu\rightarrow e$ decay chain.[6] They found $a=-0.305\pm0.033$ for positrons of range ≥ 9.3 g/cm^2 carbon.

The theoretical value of a may be found by integrating the electron spectrum given by Lee and Yang above the minimum energy, and is $a_{\text{theor}}\approx -0.43\xi P$. Here ξ is the coupling-mixture parameter of Lee and Yang, and P is the fractional polarization of the μ^+ meson in $K_{\mu 2}$ decay. If we assume that the μ meson is completely polarized, the asymmetry a_m corresponds to a value of $\xi\approx 0.72$. In arriving at this result, we have not made any corrections to our observed data for the effects of radiation by, and Coulomb scattering of, the positrons. Consequently, our value is a lower limit. It is in reasonable agreement with the value of ξ estimated for μ mesons from π-meson decay.[7,8]

We conclude that the μ mesons from $K_{\mu 2}$-meson decay are highly, if not completely, polarized, and the direction of polarization is the same as that occurring in $\pi\rightarrow\mu\rightarrow e$ decay. This result strongly suggests that the spin of the K meson, like that of the π, is zero. With this conclusion we note that the observed direction of

μ polarization is opposite to that predicted by the "attribute" scheme for the fundamental particles.[9,10]

* This work was performed under the auspices of the U. S. Atomic Energy Commission.
† Commonwealth Fellow. On leave of absence from Atomic Energy Research Establishment, Harwell, England.

1 T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956); **105**, 1671 (1957).
2 Wu, Ambler, Hayward, Hoppes, and Hudson, Phys. Rev. **105**, 1413 (1957).
3 Garwin, Lederman, and Weinrich, Phys. Rev. **105**, 1415 (1957).
4 J. Friedman and V. L. Telegdi, Phys. Rev. **105**, 1681 (1957).
5 Cassels, O'Keefe, Rigby, Wetherell, and Wormald, Proc. Phys. Soc. (London) **A70**, 543 (1957).
6 Coffin, Garwin, Lederman, Weinrich, and Berley, Bull. Am. Phys. Soc. Ser. II, **2**, 204 (1957).
7 M. F. Kaplon, *Proceedings of the Seventh Annual Rochester Conference on High-Energy Physics, 1957* (Interscience Publishers, New York, 1957).
8 D. H. Wilkinson (to be published).
9 W. G. Holladay, Phys. Rev. **107**, 637 (1957).
10 R. G. Sachs, Phys. Rev. **99**, 1573 (1955).

Radiation Damping in General Relativity

PETER HAVAS
Department of Physics, Lehigh University, Bethlehem, Pennsylvania
(Received September 23, 1957)

THE question of the physical reality of gravitational radiation has recently been the subject of some controversy.[1] The discussion has mainly been centered around the "new approximation method" of Einstein, Infeld, and Hoffmann[2] for obtaining the equations of motion in the general theory of relativity. This method is based on the assumption that the time derivative of any field quantity is much smaller than the spatial derivatives. However, an approximation procedure in which time and space coordinates are on the same footing is more in the spirit of the theory of relativity and is better suited for the study of the motion of bodies of high relative velocity and of possible radiation damping effects.

It is possible to obtain Lorentz-invariant equations of motion to any desired degree of accuracy by a consistent development of Einstein's original linear approximation method.[3] In the field equations of general relativity,

$$R_{\mu\nu}-\tfrac{1}{2}g_{\mu\nu}R=-8\pi GT_{\mu\nu}, \tag{1}$$

the metric tensor $g_{\mu\nu}$ is expanded in a power series in some parameter λ, with the Minkowski metric $\eta_{\mu\nu}$ (with signature -2) as the zero-order approximation:

$$g_{\mu\nu}=\eta_{\mu\nu}+\sum_{n=1}^{\infty}\lambda^n\,{}_ng_{\mu\nu}. \tag{2}$$

We shall be interested in the motion of singularities. In the simplest case of N simple poles of the gravitational field (with coordinates $z_\mu{}^i$), no other fields being considered, we take the energy-momentum tensor as

$$T_{\mu\nu}=\sum_{n=1}^{\infty}{}_nT_{\mu\nu}$$
$$=\sum_{i=1}^{N}\int_{-\infty}^{\infty}t^i{}_{\mu\nu}\delta(x_0-z_0{}^i(\tau^i))\delta(x_1-z_1{}^i(\tau^i))\times\delta(x_2-z_2{}^i(\tau^i))\delta(x_3-z_3{}^i(\tau^i))d\tau^i, \tag{3}$$

with

$$t^i{}_{\mu\nu}=\sum_{n=1}^{\infty}\lambda^n\,{}_nt^i{}_{\mu\nu}(\tau^i),$$

where the τ^i are arbitrary parameters; in the following we shall choose them to be the proper times of the particles (in the sense of the special theory of relativity). The representation of singular quantities by integrals of the type (3) is well known from special relativity.[4] We shall make no assumptions about the form of $t^i{}_{\mu\nu}$; this form and the equations of motion are determined as integrability conditions of Eqs. (5) and (7) below.

We introduce the quantity[5]

$$\gamma_{\mu\nu}=g_{\mu\nu}-\tfrac{1}{2}\eta_{\mu\nu}g^\sigma{}_\sigma \tag{4}$$

and impose the coordinate condition

$$\partial\gamma^{\mu\nu}/\partial x^\nu=0. \tag{5}$$

Substituting the series (2) and (3) into (1) and equating the coefficients of each power of λ to zero, we obtain in the lowest order Einstein's equation[3]

$$\Box\,{}_1\gamma_{\mu\nu}=-16\pi G\,{}_1T_{\mu\nu}. \tag{6}$$

The subsequent equations are all of the form

$$\Box\,{}_n\gamma_{\mu\nu}=-16\pi G\,{}_nT_{\mu\nu}+{}_nU_{\mu\nu}. \tag{7}$$

Here the ${}_n\gamma_{\mu\nu}$ are defined by relations analogous to (4), and ${}_nU_{\mu\nu}$ is a known function of the $g_{\mu\nu}$ of orders $n-1$ or lower. Thus our problem reduces to finding the integrability conditions for a series of inhomogeneous wave equations, subject to the condition (5).

In the approximation (6) this problem has been solved by Lubanski.[6] He showed that ${}_1t^i{}_{\mu\nu}$ must necessarily be of the form

$${}_1t^i{}_{\mu\nu}=m^iv^i{}_\mu v^i{}_\nu. \tag{8}$$

Here the m^i are the rest masses and the $v^i{}_\mu\equiv dz^i{}_\mu/d\tau^i$ are the velocities of the particles; in this order they are all constant.

If a retarded Green's function is used, a solution of Eq. (6) is given by

$${}_1\gamma_{\mu\nu}(x)=-4G\sum_{j=1}^{N}\frac{m^jv^j{}_\mu v^j{}_\nu}{\kappa^j}, \tag{9}$$
$$(x_\rho-z^j{}_\rho,\ x^\rho-z^{j\rho})=0,\quad x_0-z^j{}_0>0,$$

where $\kappa^j\equiv(z_\rho{}^j-x_\rho)v^{j\rho}$. Similar solutions can be ob-