Energy Spectrum of Charged Pions from 2.2-Bev Protons on Be*

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Nuclear emulsions have been exposed to the radiations produced by 2.2-Bev protons incident upon an internal beryllium target in the Brookhaven Cosmotron. Particles have been identified and their energies determined by grain counts and multiple scattering measurements. Approximately 100 pion tracks have been studied at each of three laboratory angles, 12°, 18°, and 36° with respect to the direction of the incident beam. Assuming a simple nucleon-nucleon interaction in the target, these data correspond to spectra near 30°, 50°, and 90° in the center-of-mass system of two nucleons. Over this 30° to 90° angular range, the portion of the spectrum near 100 Mev is roughly isotropic, while in the 200- to 300-Mev range it is strongly anisotropic. A comparison of these and other results with recent calculations based on the isobar model reveal certain basic inconsistencies. The observed angular distributions appear qualitatively to be more in agreement with those to be expected from the statistical theory modified to include the final-state particle interactions and the conservation of angular momentum.

I. INTRODUCTION

IN the last few years the spectra of π mesons produced in nucleon-nucleon interactions at energies considerably above the threshold for single pion production (that is, greater than those available from cyclotrons) have been studied by three different techniques, each having its advantages and limitations. First are the Brookhaven cloud chamber experiments in which either neutrons from an internal Cosmotron target[1,2] or the protons of the external beam are incident upon a high-pressure, hydrogen-filled diffusion chamber.[3–6] Magnetic analysis is used to determine the momenta of the particles emerging from the n-p and p-p interactions. Some preliminary results from similar experiments with neutrons at Bevatron energies have also been reported.[7] Two experiments in which studies are made of pion production in p-p interactions induced directly in nuclear emulsions by protons at Bevatron[8] and at Cosmotron[9] energies have also been reported. Since these experiments give the same kind of information as that obtained in more detail with the cloud chamber, they will not be considered as representing a different type of experiment.

A second method by which pion spectra have been studied at these energies is that used in the counter experiments, also conducted at Brookhaven, in which magnetic analysis is employed with a fast scintillation counter telescope as detector to study the spectra of pions from an internal beryllium target.[10] In the third method, the results of which are reported here, nuclear track plates are exposed at close range to radiations from a beryllium target in the Cosmotron. Particles are identified and energies determined by the multiple scattering and grain-count techniques.

Of these three different methods for studying the pion spectra in high-energy nucleon-nucleon interactions, the second probably gives the most accurate measurements of momenta and the best statistical accuracy. However, it has so far been used to study pion spectra at only one laboratory angle. The cloud chamber method is the only one giving direct information concerning the multiplicities of mesons produced in individual events, in so far as the details of each event can be determined. However, the momentum measurements are probably somewhat less accurate than those of the counter experiment, and the identification of particles and the accumulation of good statistics for determining distributions in energy and angle are difficult.

In the emulsion experiment reported here, the momentum measurements by scattering techniques are less accurate than those obtained by magnetic analysis. And, like the counter experiment, this one gives no direct information concerning pion multiplicities. Although this method does not distinguish between positive and negative pions, the distinction between pions and protons is not difficult up to about 1 Bev. Because of the close proximity of the nuclear track plates to the internal target, muon contamination was negligible and excellent discrimination is possible against background tracks. However, the chief advantage of this experiment is that it permits the comparison of pion spectra simultaneously at several different angles over a comparatively large angular range, with considerably better statistical accuracy than can be obtained in

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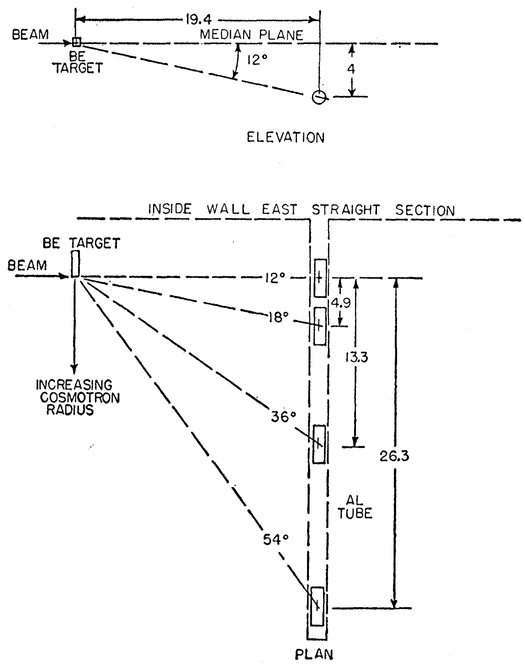

FIG. 1. Experimental arrangement for exposure of plates in Cosmotron.

comparable scanning times in the cloud chamber experiments.

II. EXPERIMENTAL TECHNIQUES

A. Plate Exposures

Ilford G5 1×3 inch nuclear track plates 400 microns thick were exposed to radiations from a plunged internal beryllium target in the east straight section of the Cosmotron. The geometrical arrangement is shown in Fig. 1. Plates were included at other positions along the axis of the plate holder, but only those in the positions shown have been studied in detail. The angles indicated are the true polar angles which particle paths from the target to the center of each plate position make with the incident beam direction. The plates were aligned so as to lie in a plane determined by the center of the target and the axis of the plate holder.

For each exposure, a series of plates in a Bakelite holder with $\frac{1}{8}$-inch walls at the plate positions was inserted into a $\frac{1}{16}$-inch wall aluminum tube, mounted on the side wall and projecting horizontally into the east straight section, 19.4 inches "downstream" from the target and 4 inches below the median plane so as to be out of the primary beam during the initial part of the acceleration period. Using fast rf cutoff at the end of this period, the target is plunged to a position just inside the beam and, as the magnetic field continues to increase, the beam spirals in and strikes only the outer edge of the $\frac{1}{2}\times\frac{1}{2}\times 6$ inch beryllium target. A larger, fixed, copper "clipper" was placed in the south straight section at a radius of one inch less than that to which the Be target is plunged. This prevents protons which have lost a small amount of energy by one traversal of the Be target, and which are oscillating about the equilibrium orbit, from making a second traversal at some distance in from the outer edge. The effectiveness of this arrangement was checked by studying the activity induced in a thin copper foil placed along the back of the target. It was found that nearly all of the activity was confined to the first $\frac{1}{16}$ inch. Thus the target constituted a narrow source $\frac{1}{2}$ inch in height.

All plate processing through part of the fixing was done at 5°C with very slow thermal changes (about one hour) from room temperature to 5°C and back to minimize distortion due to thermal shock. The plates were treated with glycerine to minimize shrinkage after processing.

B. Particle Identification

Most of the data reported here have been obtained from plates at laboratory angles of 12°, 18°, and 36°. Some data from a plate exposed at 54° in an earlier experiment will also be given to show the charged pion and proton energy distributions at that angle, but intensity comparisons with the data at the smaller angles are not possible. Measurements on the particle tracks were made with a Koristka Model MS2 microscope with a micrometer eyepiece and oil immersion objective having an over-all magnification of 770×. A discussion of the accuracy of these measurements is given later in this section.

Because of the low primary beam intensities used in these exposures, background tracks were not a serious problem. Identification of tracks from the target was effected by means of scatter plots of azimuth and elevation angles in the emulsion. Tracks from the target form a distinct group in such plots. Also, because the plates were comparatively close to the target, the muon contamination was very low. On the average, about 6% of pions with 100-Mev energy will decay between the target and the plate at 36°. The average

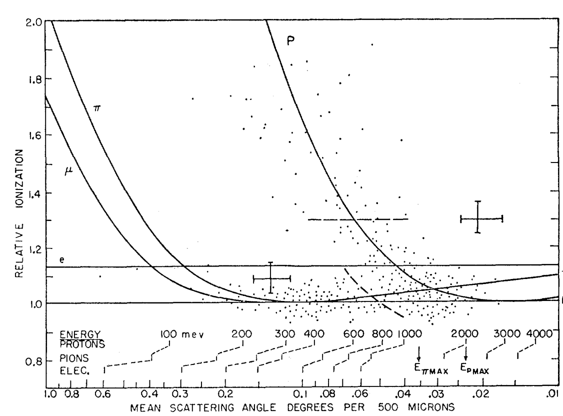

FIG. 2. Relative ionization *vs* mean scattering angle for particles at 12° in the laboratory system.

energy of all pions observed is over 400 Mev so that less than 2% of all mesons observed are likely to be muons.

Since the relative number of K-particles per pion is less than one percent at 2.2 Bev, the problem of particle identification consists of distinguishing between low-energy pions and electrons produced by photons from π^0 decays in the target, and between high-energy pions and protons. These distinctions are made by means of relative ionization *vs* mean scattering angle plots. Such a plot of the data taken from the 12° plate is shown in Fig. 2. The theoretical curves for protons, mesons, and electrons were calculated using the value of scattering constant computed by Voyvodic and Pickup[11] and the energy loss by ionization in emulsions calculated by Sternheimer.[12] The representative errors indicated on two of the points are the standard deviations determined by the number of grains counted and the number of scattering measurements made. The point at the far right probably represents a high-energy deuteron.

Data from the 12° plate were selected for the representative plot of Fig. 2 because this is the case in which the distinction between high-energy pions and protons is least satisfactory by this method. The dashed curve parallel to the theoretical proton curve is drawn arbitrarily about three standard deviations from the proton curve; it intersects the pion curve at about 1 Bev. It was assumed that all points near minimum ionization and to the left of this curve represented pions. This eliminates some high-energy pions which may have energies up to 1.66 Bev at this angle. However, the peak of the pion energy distribution lies between 400 and 600 Mev (see Fig. 3). Few pions should have been missed up to 800 Mev, at which energy the distinction between pions and protons is quite clear, but above that energy the spectrum rapidly becomes unreliable. The percentage of pions missed becomes less at 18° and should be very small at 36°. Thus it is believed that only the high-energy end of the observed pion spectrum at 12° is affected appreciably by this omission.

For several reasons the undetected electron contamination due to pair production by photons from π^0 decays in the target is believed to be small. First, the scanning of plates for tracks was done at a distance of 4 to 6 mm in from the edge to avoid the edge distortion of the emulsion. All lightly ionizing tracks in the correct direction to have come from the target were traced back close to the edge of the emulsion. In this way tracks of electrons from pairs produced in the emulsion could be identified and eliminated. The number of pair origins found was very small. The total thickness of material in the target and plate holder amounted to about one-third (measured in radiation lengths) that scanned in the emulsion, so the number of pairs originating outside the emulsion was even smaller.

[11] L. Voyvodic and E. Pickup, Phys. Rev. **85**, 91 (1952).
[12] R. M. Sternheimer, Phys. Rev. **91**, 256 (1953).

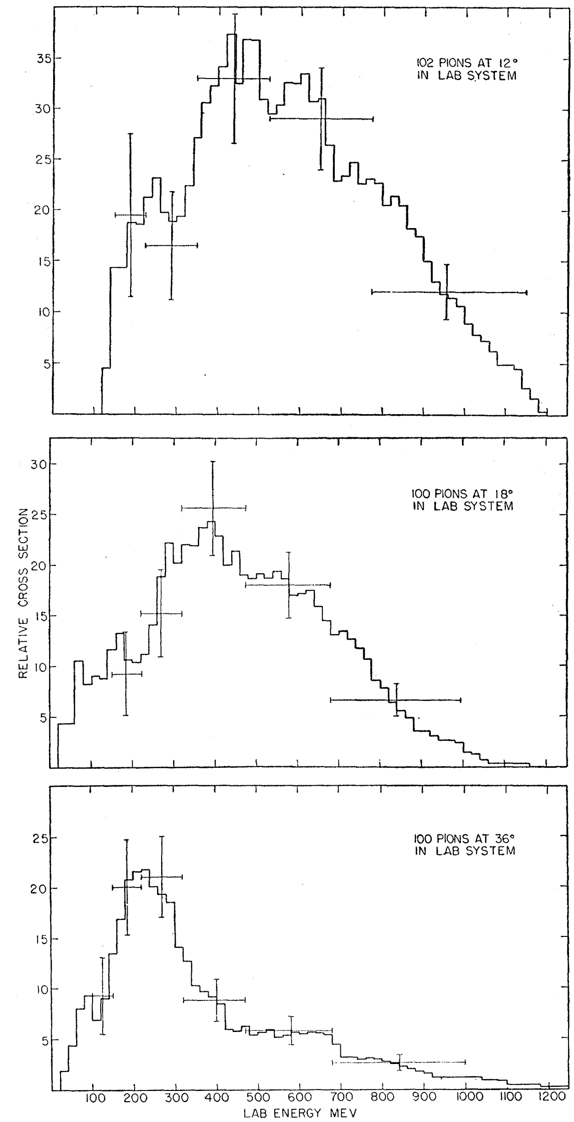

FIG. 3. Energy distributions in the laboratory system for charged pions at 12°, 18°, and 36°.

Second, measurements of the energy of electrons from identified pairs indicate that they seldom had energies over 100 Mev; none was found with an energy greater than 200 Mev. A 100-Mev electron is easily distinguishable from a pion with the same mean scattering angle on the basis of relative ionization. Finally, very few particles of any kind were found in the 12°, 18°, and 36° plates with a mean scattering angle greater than 0.4 degree per 500 microns, corresponding to a 100-Mev pion or a 150-Mev electron. For reasons to be discussed in the next section these particles are not included in the presentation of the data in the center-of-mass system

This experiment was concerned primarily with the pion spectra, but at 12° the protons outnumber pions by

TABLE I. Relative intensities in the laboratory system.

Laboratory angle	Number of pions	Total pion intensity relative to 36°	Number of protons	Total proton intensity relative to 36°	Ratio: number of protons to number of pions
12°	102	2.7±0.7	301	12.0±2.5	3.0±0.5
18°	100	1.7±0.4	166	4.0±0.8	1.7±0.3
36°	100	1.0±0.3	68	1.0±0.3	0.7±0.2
54°	73	...	63[a]	...	0.6±0.3[a]

[a] Combined data from two different plates. See Discussion.

3 to 1. In order to save time, therefore, if, after 10 scattering measurements and the counting of about 250 grains, it was found that a track had a relative ionization greater than 1.3, it was studied no further unless the mean scattering angle indicated that it was probably a low-energy meson. (No such mesons were found at 12°.) This procedure also accounts for the fact that the spread in the proton data in Fig. 2 is considerably greater above a relative ionization of 1.3, indicated by a dashed line, than below. Other similar appropriate criteria were used for the data from the other plates in which particle energies were lower.

C. Accuracy of Measurements

To maximize the accuracy of the scattering measurements on the highest energy tracks, a cell length of 500 microns was used in all measurements. This cell length has the added advantage that it requires exactly one revolution of the driving screw, which minimizes the stage "noise." Because of a number of experimental limitations such as the vertical size of the target, slight misalignments in the plate tilt angles, scattering in the vertical direction, and distortion at the emulsion edges, it was seldom possible to obtain reliable scattering measurements for more than thirty 500-micron cells on any track. If one uses the value of $0.80/n^{\frac{1}{2}}$ times the mean scattering angle for the standard deviation in the mean, where n is the number of second differences measured along the track, the corresponding error in the energy measurement is about $\pm 17\%$ for 30 such measurements. This value of the standard deviation in the mean is recommended by Scott[13] when the scattering is small and is measured in terms of angles between successive chords, omitting second differences which are more than four times the mean.

A detailed analysis of the accuracy of the microscope and the scattering measurements has been made, using standard techniques such as (1) measurements on a Bausch & Lomb ruled line oriented at different angles with respect to the direction of the stage motion, (2) measurements of the scattering and of the scattering differences between pairs of parallel tracks of protons of known energy in a calibration plate, and (3) comparison of emulsion distortion in the calibration and experimental plates. From these measurements it was possible to estimate separately the errors due to the microscope stage and eyepiece, the observer, and emulsion distortion.

The combined experimental error from all sources other than the statistical errors, expressed in terms of a mean scattering angle, is 0.015 degree per 500 microns. This introduces an error of about 6% in the measured energy of a 1-Bev pion. The error decreases to 2.5% at 800 Mev and to less than 2% at 600 Mev. Since about 80% of all pions observed had energies less than 800 Mev and the maxima of all distributions lie below 600 Mev, the average error due to these various sources is very few percent. Since the standard deviations in the energies measured in this experiment were quite large compared with the combined experimental errors, no correction has been made in the data for errors of the latter type.

III. RESULTS

Table I summarizes the numbers of particles studied at the various angles and gives the total particle intensity at each angle relative to that at 36°. For reasons discussed in the preceding section, many of the proton data are considerably less accurate than those for the pions, but the results are included for general interest. Also given in the table are the proton-pion ratios at each angle. The data at 54° were obtained in an earlier experiment so that total intensity comparisons with those at other angles are not possible. The proton data and the proton-pion ratio at this angle were obtained from the combined results of measurements on two plates exposed with different primary beam intensities. For experimental reasons the energy measurements in one plate are known to be more accurate than in the other. Only pion data from the more accurate series of measurements at 54° are used in the corresponding histogram discussed later in this section.

The energy distributions in the laboratory system of the pions observed at 12°, 18°, and 36° are shown in Fig. 3. At 54° both the energy transformation and the Jacobian for transforming the differential cross section are slowly varying functions of energy. Consequently, the energy distribution of the pions in the laboratory system at this angle is not significantly different from that in the center-of-mass (c.m.) system. A combined histogram indicating the distributions in both systems is given later.

The plots in Fig. 3 show the relative cross section per unit energy, per unit solid angle, as a function of

[13] W. T. Scott, Phys. Rev. 85, 245 (1952).

energy in the laboratory system. They are obtained as follows: a block of width equal to twice the standard deviation in the energy measurement and of height equal to the reciprocal of that width times the solid angle is plotted for each particle. The blocks are then added in 20-Mev intervals to form the histograms shown. A series of adjoining energy intervals, each of width equal to two standard deviations in the mean energy, is chosen and the number of particles in each interval is used to evaluate the statistical error indicated in Fig. 3.

Since the data were all obtained in the same experiment, the relative cross sections at the different angles may be compared. The 18° and 36° data were obtained the same exposure. The data at 12° were obtained in another exposure with the relative intensity between the two exposures measured by the Cosmotron circulating-beam monitor. This is believed to be accurate to within a very few percent for such relative intensity measurements in a single experiment, when nothing is varied but the beam intensity. Thus, the largest error involved in comparing the cross section at the different angles is the uncertainty in the initial thickness of the emulsions before processing. Ilford emulsions of this type are generally found to be within ±10% of the specified thickness so that this probably represents an upper limit on the error introduced by assuming that the emulsions were all originally exactly 400 microns in thickness.

Two experimental limitations affect these distributions. At the low-energy end of the spectra, a few particles may have been omitted because they had undergone sufficient scattering so as to appear not to come from the target. The number of these is believed to be very small since the angular plots showed that very few tracks occurred at angles greater than ±3° (the chosen angular acceptance aperture) from the mean angle of the group. Further, of those occurring within this angular range, very few had energies less than 100 Mev. For reasons to be given later, these low-energy data are not included in the center-of-mass distributions.

A more serious limitation affecting the high-energy end of the spectrum is the inability with this experimental technique to distinguish between pions with energies around 1 Bev and protons with the same mean scattering angle. Consequently, a significant number of high-energy pions may have been eliminated by the arbitrary cutoff procedure described in Sec. II. Since the errors are large in this energy range, these omissions affect the histograms at energies well below 1 Bev. Thus, it is likely that the actual spectra, particularly at the smaller laboratory angles, do not decrease so rapidly with energy as indicated by the results given here. The effect is seen to be more pronounced in the center-of-mass spectra shown in Fig. 5. However, it is evident that this limitation does not affect appreciably the central portions of these spectra, which include most of the pions.

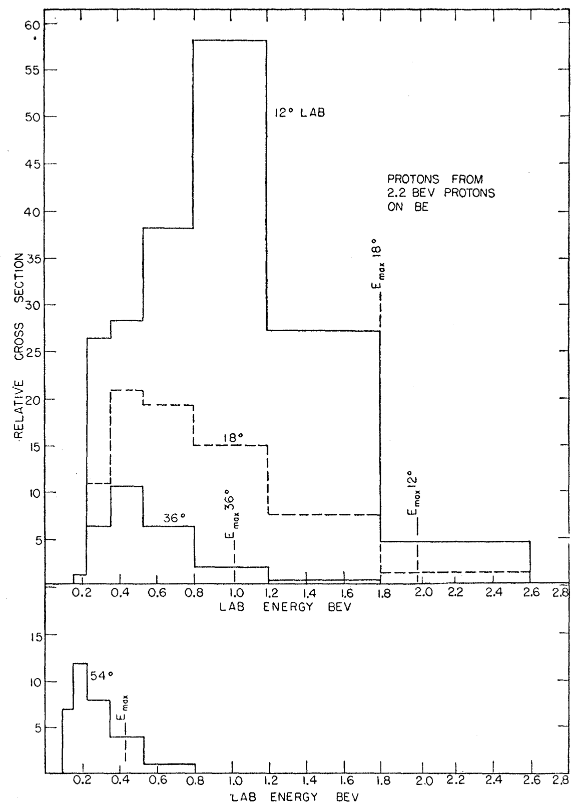

FIG. 4. Energy distributions in the laboratory system for protons at 12°, 18°, 36°, and 54°.

As discussed in the preceding section, the lower energy proton tracks were measured less carefully than the pions. However, the results have proved useful in designing new experiments, and they have been summarized along with the pion data in Table I. In Fig. 4, histograms show the proton energy distributions at 12°, 18°, 36°, and 54° in the laboratory system. The energy intervals are approximately twice the standard deviation in the mean energy. The histogram for the particles observed at 54° is shown separately, since these data were obtained in an earlier experiment and comparisons of intensities with those at other angles cannot be made.

Of considerably more interest are the pion data in the center-of-mass system. It is assumed here that, for 2.2-Bev protons in a nucleus as light as beryllium, one is concerned essentially with simple nucleon-nucleon interactions in the target. Consequently the c.m. system is assumed to be that of two free nucleons. The Brookhaven counter experiments,[5] by the comparison of results obtained with Be and H, have shown that this assumption is warranted, at least for pions at 32° in the lab system which correspond to particles near 90° in this c.m. system. The results obtained at 36° in this experiment are essentially in agreement with those at 32° in the counter experiments. Elementary calculations have been made of the effects of the Fermi motion

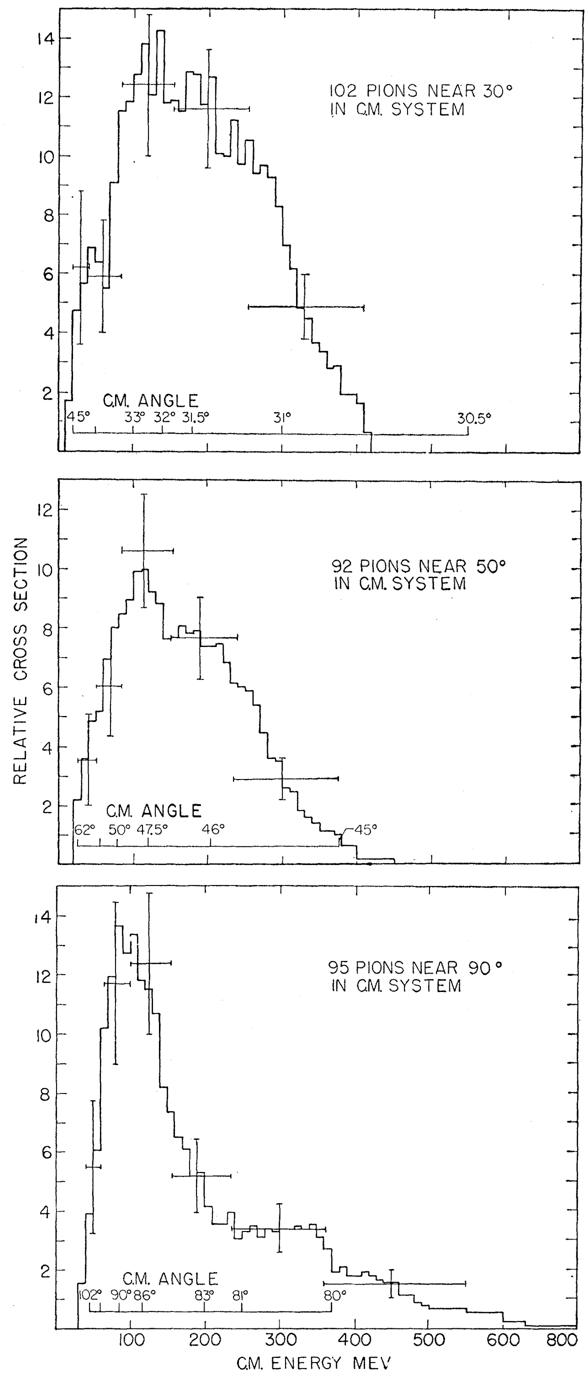

FIG. 5. Energy distributions in the center-of-mass system of charged pions near 30°, 50°, and 90°.

of the target nucleons on the energy of pions observed at the angles used in this experiment. These indicate that the uncertainty in the measured pion energy caused by this motion is for some directions a little greater, and in others a little less, at 12° than it is at 36°; so it is perhaps not unreasonable to assume that on the average the effects of the Fermi motion are not greatly different at these two angles and that they may also be neglected at the smaller angles in this experiment.

Figure 5 shows the data transformed to the center-of-mass system of two free nucleons. The data are transformed for each particle separately and then added in 10-Mev intervals. Since the transformation introduces appreciable spread in the angles at which the particles occur in the c.m. system, particularly at the lowest energies, pions having lab energies less than 150 Mev were omitted in the c.m. plots of the data obtained at 18° and those with energies less than 100 Mev were omitted from the 36° data. No pions with energies less than 150 Mev were found in the 12° plate. Such low-energy particles occur at c.m. angles greatly different from those of the majority of particles in the spectrum. Only eight were omitted from the 18° data and five from the data at 36°. Thus, the 12° data represent pions occurring between about 31° and 40° in the c.m. system; the 18° data, those between 45° and 60°; and the 36° data, those between 80° and 100°.

Figure 6 shows the c.m. spectrum of pions obtained in the earlier experiment at a lab angle of 54°, corresponding to particles between 106° and 125° in the c.m. system. Since good information is not available as to the relative intensities of the incident beam in the two experiments, the spectrum in Fig. 6 is to be compared with those in Fig. 5 only in terms of the energy distributions. Figure 6 also represents, to a sufficiently good approximation for the data involved, the energy spectrum at 54° in the lab system. The corresponding lab energies given at the top of the figure are not greatly different from the c.m. energies over most of the spectrum. Also, since the Jacobian for the transformation varies by only about 20% and almost linearly with velocity over the entire range of particle energies involved, the energy distributions in the two systems are not too different.

The center-of-mass spectra given in Figs. 5 and 6 all have a maximum near 100 Mev and exhibit a marked increase in the incidence of higher energy particles at the smaller angles. The total integrated intensity at 30° is about 50% greater than that at 90°; whereas that at 50° is slightly less than that at 90°. The latter result is probably an experimental error due to the uncertainty in the initial thickness of the nuclear emulsions. On the basis of what is presently known about the angular distributions of pions in similar experiments and about the models proposed to account for such distributions, it is difficult to conceive of a mechanism which would produce fewer particles at 50° than at both 90° and at 30°. Thus, the total intensity at 50° is probably at least as great as that at 90° and is very likely intermediate between that at 90° and at 30°. With this consideration, a comparison of the spectra in Fig. 5 shows that the angular distribution of pions with energies in the vicinity of 100 Mev is not different from isotropic by more than the experimental

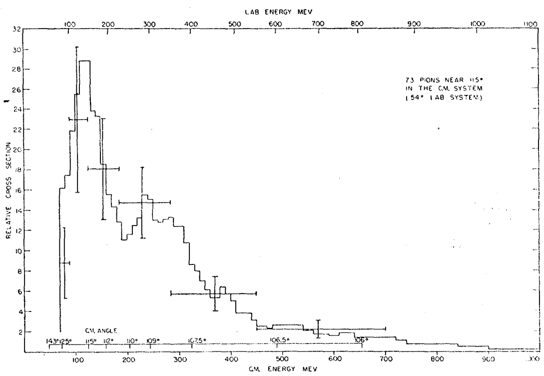

FIG. 6. Energy distribution of charged pions at 54° in the laboratory system and near 115° in the center-of-mass system.

error of about ±30%; whereas, the intensity of pions near 250 Mev increases by a factor of about three from 90° to 30°.

Another experimental limitation already discussed is the difficulty of distinguishing between pions and protons at the highest energies in the laboratory system. The omission of a number of high-energy pions probably accounts for the apparent sharp decrease of cross section with energy at 30° and 50° and the absence of the high-energy "tail" observed in the distributions near 90° and 115° where this limitation is much less important. Despite this limitation, the increase of intensity with decreasing angle in the vicinity of 250 Mev is still very evident. Without this limitation in the data taken at small angles, the anisotropy at the higher energies may be even more pronounced.

To summarize the results presented here: In the c.m. system,

1. All spectra from 30° to 115° have a maximum at about 100 Mev.
2. The angular distribution of pions in the vicinity of the maximum does not differ from isotropic by more than the experimental error of about ±30% from 30° to 90°.
3. The angular distribution of pions in the 200 to 300-Mev range is markedly anisotropic, the intensity varying by a factor of about 3 from 30° to 90°.

In addition to these results for the charged pions, some of the proton data have been plotted in the center-of-mass system, but the spread in angle introduced by the transformation makes interpretation difficult in this experiment. There is some evidence, however, that the protons, nearly all of which were inelastically scattered, tend to be emitted more in the polar directions than near 90°.

IV. DISCUSSION

Insofar as they can be compared, the results reported here are in agreement with those obtained in the Brookhaven counter experiments at 2.3 Bev[10] and the cloud chamber experiments at both 1.5 Bev and 2.75 Bev.[4,5] All c.m. pion spectra exhibit a maximum in the vicinity of 100 Mev and the majority of pions are found to have energies between 100 and 300 Mev. These earlier results have been compared extensively with the published theories.[6,14] Only a brief summary of these comparisons will be given here except for results concerning the angular dependence of these spectra. Since the emulsion experiment reported here permits a better comparison of the spectra at different c.m. angles than has previously been obtained, these results will be compared in more detail with recent theoretical calculations.

In general it is found that the pion spectra produced in nucleon-nucleon interactions at these energies contain considerably fewer high energy particles than predicted by calculations[15] based on Fermi's statistical theory.[16] It is also observed in the cloud chamber experiments, and has been inferred from the counter experiments, that the multiplicities of pion production are greater than predicted by calculations[15,17,18] based on the original Fermi theory.

However, two recent modifications of this theory have been made. One involves refinements in momentum and energy conservation[19] and the other takes into account the effects of final-state particle interactions,[20] particularly the pion-nucleon resonance interaction which is so evident from the results of pion scattering experiments. Both these modifications have led to results predicting lower energies and higher multiplicities in considerably better agreement with the experiments, although calculations, particularly of the expected energy distributions, are not yet sufficiently detailed to permit good quantitative comparisons with the experimental results obtained so far.

Another theoretical approach which has received considerable attention in recent years is that of the isobar model, in which it is assumed that in a nucleon-nucleon encounter one or both of the nucleons is excited to an isobaric state of angular momentum and isotopic spin 3/2 and subsequently decays by pion emission after separation of the interacting nucleons.[14,21] This model has led to predictions of multiplicities and charge distributions in better agreement with experimental results than obtained in most cases with the original Fermi theory, but not necessarily markedly better agreement than that obtained with the modified statistical theory.[22] On the basis of present evidence, it

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[17] E. Fermi, Phys. Rev. **92**, 452 (1953); **93**, 1434(E) (1954).
[18] M. M. Block, Phys. Rev. **101**, 796 (1956).
[19] J. V. Lepore and M. Neuman, Phys. Rev. **98**, 1484 (1955).
[20] J. S. Kovacs, Phys. Rev. **101**, 397 (1954).
[21] D. C. Peaslee, Phys. Rev. **94**, 1085 (1954).
[22] For example, both the isobar model[14] and the modified statistical theory[20] give values for the final-state ratio $(np+-):(pp-)$ well within the limits of error on the experimental value of

does not appear possible to choose between the modified statistical theory and the isobar model in terms of the ability of either to predict multiplicities and charge distributions observed in these events.

Recent calculations have been made of the pion spectra to be expected from the isobar model.[14] In these calculations it is assumed that the pions are emitted isotropically in the rest system of the isobar. Since it is also assumed that the isobars exist in the excited state sufficiently long after the interaction to separate and decay by pion emission as free particles, the resulting pion spectra should be strongly dependent upon the motion of the isobars. Two limiting cases are treated in which it is assumed that (1) the isobars are emitted isotropically in the c.m. system of the two interacting nucleons and (2) the isobars are emitted only in the polar directions.

The first of these assumptions leads to a predicted pion energy distribution in reasonably good agreement with that observed near a c.m. angle of 90° in the counter experiments at 2.3 Bev.[10] However, on the basis of this assumption, the model predicts both pion and nucleon spectra which are isotropic in the c.m. system of the incident nucleons. As discussed in the preceding section, it has been found in this emulsion experiment that while the lower energy portion of the pion spectrum is nearly isotropic, the higher energy part shows a strong angular dependence in the range from 30° to 90°.

It has also been observed in the cloud chamber experiments that at several different incident nucleon energies, for both *n-p* interactions[1] and *p-p* interactions,[3,5] the pion spectra are in most cases markedly anisotropic. Further, except at energies near the threshold for single pion production when most nucleons are emitted in an *S* state,[23] it is found that at all higher energies the nucleons tend to be emitted more in the polar directions.[1–6] Thus, it appears that the marked angular dependence of the spectra observed in most cases for the pions and in all cases for the nucleons in these interactions is not consistent with predictions of the isobar model using the assumption that the isobars are emitted isotropically.

For the case in which the isobars are assumed to be emitted in the polar directions only, the pion spectra at 90° and 0° in the c.m. system of the interacting nucleons have been calculated. The predicted energy distribution at 90° resembles that obtained near 70° in the counter experiment at an incident nucleon energy of 1.0 Bev.[10] This version of the model predicts quite anisotropic spectra for both pions and nucleons, more in agreement with those observed. However, it may be noted that although the angular distribution of pions of all energies is comparatively anisotropic at 0.8 Bev[3] and approximately isotropic at 1.5 Bev,[4] it is found to be markedly anisotropic at higher energies of 2.2 Bev, as reported here, and 2.75 Bev.[5] Further, as previously mentioned, the nucleon angular distributions are approximately isotropic at energies near the threshold for single pion production and become steadily more anisotropic with increasing incident nucleon energy.[3–5] Thus, the assumptions leading to the apparent agreement between the observed pion energy spectrum and that predicted by the isobar model at 1.0 Bev are more appropriate to both the pion and nucleon angular distributions observed at energies above 2 Bev; and conversely, the assumptions leading to the apparent agreement between the observed pion energy spectrum and that predicted by the isobar model at 2.3 Bev are more appropriate to both the pion and nucleon angular distributions observed at energies of 1.5 Bev or lower.

The inconsistencies between the particle angular distributions observed in these events and those which follow from the nature of the isobar model suggest that the assumptions concerning the lifetime of the isobaric state may be of questionable validity. As has been previously discussed,[24] if one takes the width at half-maximum in the pion-proton scattering resonance to be 150 Mev,[25] the uncertainty relation leads to a lifetime of the order of 10^{-23} second for this excited state in the rest system of the isobar. For the case in which both nucleons are excited at an incident nucleon energy of 2.2 Bev, the isobars move with a velocity of less than $0.5c$ in the c.m. system of the incident nucleons, so that the time dilation in transforming from the rest system of the isobars is small. Thus, the isobars can separate by a distance of only about 10^{-13} cm during the lifetime of the excited state, so that their existence as free particles is open to question.

The experimental results concerning the angular distributions of both the pion and the nucleon spectra observed in these events and the difficulties which they pose for the isobar model suggest that these distributions may be better accounted for by an appropriately modified version of the statistical model. Certain modifications of this model which lead to predicted pion energy distributions and multiplicities in considerably better agreement with experiment than obtained with the original Fermi model have already been mentioned.[19,20] Other modifications are required to account for the angular distributions observed.

In the statistical theory as originally proposed, the conservation of angular momentum is neglected. Such a model, of course, can predict only isotropic particle distributions. The conservation of angular momentum and its effect on the predicted angular distributions of

4.0 ± 1.7 for *n-p* interactions at an average neutron energy of 1.7 Bev.[1] On the other hand, the final-state ratio $(pn+):(pp0)$ observed experimentally in *p-p* interactions at comparable energies is many times larger than that predicted by either the isobar model or the Fermi theory.[6]

[23] H. Rosenfeld, Phys. Rev. **96**, 139 (1954).

[24] F. L. Niemann, Ph.D. thesis, Harvard University, June, 1955 (unpublished).

[25] M. Gell-Mann and K. M. Watson, in *Annual Review of Nuclear Science* (Annual Reviews, Inc., Stanford, 1954), Vol. 4, p. 219.

the emitted particles have not yet been incorporated quantitatively into the statistical theory. However, Fermi has indicated from comparatively elementary considerations that both for energies near the threshold for single pion production[16] and for extremely relativistic energies,[26] taking into account the conservation of angular momentum leads to predicted angular distributions which are anisotropic, with particles tending to emerge in the polar directions if the total angular momentum of the system is different from zero.

At a given incident nucleon energy, then, the total angular momentum of the system and therefore the degree of anisotropy of the emerging particles increases with increasing impact parameter. If one makes the not unreasonable assumption that the multiplicity of pion production is also determined, at least in part, by the impact parameter, the resulting model is qualitatively capable of predicting angular distributions of the kind observed. This association of impact parameter with multiplicity is obvious for the extreme cases. In an elastic scattering, more probable for distant interactions, the multiplicity is zero and the emergent nucleon distributions are strongly peaked in the polar directions at energies above 1.0 Bev.[6] Conversely, if two nucleons have a total kinetic energy just sufficient to make n mesons, that number can be formed only when the system is in a S state. For any other state fewer pions will be formed; they will emerge with finite energies, and the particle angular distributions will be anisotropic in order to conserve angular momentum. The effect of increasing incident nucleon energy can also be seen qualitatively here by noting that if more than just sufficient kinetic energy to make n mesons is available, that number can be formed by a system with finite angular momentum and the emergent particle distributions will again be anisotropic.

Without further speculations as to the details of this approach, it can be seen qualitatively how such a model could predict angular distributions of the kind observed in these experiments. For example, the results of this emulsions experiment discussed in Sec. III may be interpreted as follows: Since the cross section for triple pion production is small compared to that for single or double production at these energies,[1,6] the lower energy portion of the pion spectrum around 100 Mev, which is roughly isotropic, as shown in Fig. 5, represents particles mostly from double pion events in which a considerable fraction of the available kinetic energy has gone into pion production so that such events are associated with a state of small total angular momentum and the resulting angular distribution tends to be isotropic. Conversely, the higher energy portion of the spectrum above 200 Mev represents particles from single pion events in which a larger total angular momentum must be conserved so that the emergent particle distribution is more anisotropic. One would also expect the same difference in the nucleon angular distributions between single and double pion events.

[26] E. Fermi, Phys. Rev. 81, 683 (1951).

It is interesting to note that, although the data are few, the results obtained in the cloud chamber experiment for p-p interactions at 2.75 Bev[5] are in every detail consistent with this picture. In this experiment energy and angular distributions of particles from single and double pion events are obtained separately. It is found that the spectrum of positive pions from single pion events has a maximum roughly in the vicinity of 200 Mev and is more anisotropic than the spectrum of positive pions from double pion events which has a maximum around 100 Mev. Likewise, protons from single pion events occur at higher energies and have a more anisotropic distribution than protons from double pion events.

Because of the qualitative nature of this interpretation, attempts to account for further details of the particle spectra observed in these events are not warranted. The purpose of the foregoing has been to suggest that the statistical model, appropriately modified to include the conservation of angular momentum as well as other effects such as final-state particle interactions, may be a more promising approach to a theory predicting the observed pion and nucleon spectra than that offered by the isobar model if one retains the assumption, basic to the model, that the isobars exist sufficiently long to separate and decay as free particles.

V. SUMMARY AND CONCLUSIONS

A study of the charged pion spectrum produced by 2.2-Bev protons on beryllium has been made. If one assumes a simple proton-nucleon interaction in the target, this spectrum is found to have a maximum around 100 Mev and to be roughly isotropic in this energy range and markedly anisotropic around 300 Mev over a 30° to 90° angular range in the c.m. system of the two interacting nucleons. A review of the experimental evidence available indicates that the results obtained here are consistent with those of similar experiments in the Bev range of incident nucleon energies. It also appears from a number of experiments that the inelastically scattered nucleons in these events emerge with angular distributions whose anisotropy increases with incident nucleon energy.

Examination of recent calculations of pion energy distributions predicted theoretically by the isobar model in these events shows that assumptions concerning the angular distributions of the emerging particles, which are necessary to obtain agreement with the experimental pion energy distributions, are quite inconsistent with the observed particle angular distributions and the energy dependence of these distributions. This suggests the conclusion that the assumption, basic to the isobar model, that the isobars, formed at the time of the interaction, exist sufficiently long to separate and decay as free particles may not be warranted.

Elementary qualitative considerations suggest that some model which takes into account the conservation of angular momentum may be capable of accounting for both pion and nucleon angular distributions and the energy dependence of these distributions if it is assumed that

(1) the pions are emitted during the interaction time while the radiating system has no linear motion in the c.m. system of the interacting nucleons;

(2) the more isotropically distributed lower energy pions are associated with the lower total angular momentum states of the system and the higher multiplicities of pion production and, conversely, the more anisotropically distributed higher energy pions are associated with the higher total angular momentum states and the lower pion multiplicities.

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Capture of μ^- Mesons by Protons

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By use of a two-component theory of the neutrino the capture process and the radiative capture process of a μ^- by a proton is analyzed in some detail.

WITH the establishment[1] of the nonconservation of parity, many experiments have been done and will be done which will greatly increase our knowledge concerning the nature of the coupling in β decay, in π decay, and in μ decay. The purpose of the present note is to point out that it may be possible to do the same with the phenomenon of μ^- capture:

$$\mu^- + p \rightarrow n + \nu. \tag{1}$$

One notices, first of all, that the two main questions concerning μ^- capture are

(a) whether the μ^- capture process is given by (1), or by

$$\mu^- + p \rightarrow n + \bar{\nu}, \tag{1'}$$

or by a combination of (1) and (1′);

(b) what is the coupling that is responsible for μ^- capture.

To make the analysis definite we shall assume that the neutrino is described by a two-component theory with zero mass.[2,3] Furthermore, we assume first that process (1) prevails and write the interaction Hamiltonian as

$$H = \sum_i C_i(\psi_n^\dagger O_i \psi_p)(\psi_\nu^\dagger O_i \psi_\mu), \tag{2}$$

where i runs over S, V, T, P, and A, and O_i are the corresponding 4×4 matrices.[4]

We list the following results for the capture of μ^- in hydrogen.

1. The rate for process (1) in hydrogen is

$$(1/\tau)_{\text{cap}} = p_\nu^2 \xi / (2\pi^2 a^3), \tag{3}$$

where

$$\xi = |C_S + C_V|^2 + 3|C_A + C_T|^2, \tag{4}$$

p_ν is the momentum of the neutrino, and a is the Bohr radius of the μ^- mesonic atom.

2. In process (1) with a *completely* polarized μ^-, the angular distribution of the neutron is of the form[5]

$$1 + \alpha \cos\theta_1, \tag{5}$$

where

$$\theta_1 = \angle(\boldsymbol{\sigma}_\mu, \mathbf{p}_n),$$

representing the angle between the spin direction of μ^-, and the momentum of the neutron. The asymmetry parameter α is

$$\alpha\xi = -|C_S + C_V|^2 + |C_A + C_T|^2. \tag{6}$$

[1] Wu, Ambler, Hayward, Hoppes, and Hudson, Phys. Rev. **105**, 1413 (1957); Garwin *et al.*, Phys. Rev. **105**, 1415 (1957); J. I. Friedman and V. L. Telegdi, Phys Rev. **105**, 1681 (1957). For other work on nonconservation of parity, see, e.g., *Proceedings of the Seventh Annual Rochester Conference on High-Energy Physics* (Interscience Publishers, Inc., New York, 1957).

[2] T. D. Lee and C. N. Yang, Phys. Rev. **105**, 1671 (1957). We adopt here the convention of that paper: that by ν is meant the state for which the angular momentum is parallel to the momentum.

[3] A. Salam, Nuovo cimento **5**, 299 (1957); L. Landau, Nuclear Phys. **3**, 127 (1957).

[4] The O_i are defined explicitly in Eq. (11) of reference 2.

[5] The angular distribution of the neutron [Eq. (5)] has also been independently considered by W. Panofsky (private communication).