(relative frequency of $\Lambda\rightarrow p+\pi^-$ to $\Lambda\rightarrow n+\pi^0)=2+\Delta$; A, B, C, and D are parameters proportional to the sines of differences of phase shifts and the real numbers v_i have to satisfy $\sum v_i^2=1$. Defining a vector v with components v_i, this condition can be written as $(v,v)=1$, and $\alpha(\Lambda\rightarrow p-)$ can be put in the form $f(\Delta)(v,mv)$ where m is the symmetric 4×4-matrix associated to the quadratic form in $\alpha(\Lambda\rightarrow p-)$. The maximum and the minimum of $\alpha(\Lambda\rightarrow p-)$ are therefore given by the maximum and minimum eigenvalues, respectively, of the matrix $f(\Delta)m$. Such two eigenvalues have the same magnitude, and by direct calculation one finds

$$|\alpha(\Lambda\rightarrow p-)|\leq(1/2\sqrt{2})f(\Delta)[S+(S^2-P^2)^{\frac{1}{2}}]^{\frac{1}{2}},$$

where

$$S=4\sin^2(\alpha_1-\alpha_{11})+2\sin^2(\alpha_3-\alpha_{11})+2\sin^2(\alpha_1-\alpha_{31})+\sin^2(\alpha_3-\alpha_{31}),$$

and $P^2=16\sin^2(\alpha_1-\alpha_3)\sin^2(\alpha_{11}-\alpha_{31})$. Taking the value 0.32 ± 0.05 reported by Steinberger's group for the fraction of Λ particles undergoing neutral decay,[4] and for the pion-nucleon phase shifts the values reported by Anderson,[5] we find $|\alpha(\Lambda\rightarrow p-)|\leq0.18\pm0.02$, if charge-conjugation invariance is satisfied. Similar limitations, under the hypothesis of conservation of C, can be given for the asymmetry parameters of Σ decays. We assume spin $\frac{1}{2}$ for Σ. The limitation $|\alpha(\Sigma^-\rightarrow n-)|\leq|\sin(\alpha_3-\alpha_{31})|$ for Σ^- decay, where only one final isotopic spin state can occur, is given in reference 3. The phase shifts are taken at an energy equal to the decay Q value. We find that $|\alpha(\Sigma^+\rightarrow p0)|\leq(1/2\sqrt{2})\times g(\Gamma)[R+(R^2-Q^3)^{\frac{1}{2}}]^{\frac{1}{2}}$, where $g(\Gamma)=\frac{4}{3}(1+\frac{1}{2}\Gamma)$, Γ being defined by (relative frequency of $\Sigma^+\rightarrow n+\pi^+$ to $\Sigma^+\rightarrow p+\pi^0)=1+\Gamma$,

$$R=\sin^2(\alpha_1-\alpha_{11})+2\sin^2(\alpha_3-\alpha_{11})+2\sin^2(\alpha_1-\alpha_{31})+4\sin^2(\alpha_3-\alpha_{31}),$$

$$Q^2=16\sin^2(\alpha_1-\alpha_3)\sin^2(\alpha_{11}-\alpha_{31});$$

$$|\alpha(\Sigma^+\rightarrow n+)|\leq(1/2\sqrt{2})h(\Gamma)[U+(U^2-Q^2)^{\frac{1}{2}}]^{\frac{1}{2}},$$

where

$$h(\Gamma)=\tfrac{4}{3}(1+\tfrac{1}{2}\Gamma)/(1+\Gamma),$$

and

$$U=4\sin^2(\alpha_1-\alpha_{11})+2\sin^2(\alpha_3-\alpha_{11})+2\sin^2(\alpha_1-\alpha_{31})+\sin^2(\alpha_3-\alpha_{31}).$$

Taking the value 0.45 ± 0.06 for the ratio $(\Sigma^+\rightarrow n+\pi^+)$ to the total Σ^+ decay rate,[6] and using the nucleon-pion phase shifts from reference 5, we find $|\alpha(\Sigma^-\rightarrow n-)|\leq0.14\pm0.06$, $|\alpha(\Sigma^+\rightarrow p0)|\leq0.27\pm0.04$, $|\alpha(\Sigma^+\rightarrow n+)|\leq0.37\pm0.06$, if C is conserved. For hyperons with spin $\frac{3}{2}$ the decay distributions will not in general be describable with a single parameter α. If C is conserved, the total asymmetry will still be severely limited for Λ decay, but presumably only weakly limited for Σ decay, because of the large α_{33}. One argument for Λ spin $\frac{1}{2}$, that based on the mesonic-decay to nonmesonic-decay ratio in hyperfragments,[7] may turn out to be inaccurate if a large p wave is observed in Λ decay. Because of the low final momentum, a large up-down asymmetry in Λ decay will be an important test for theories which predict the relative amount of parity-conserving and parity-nonconserving interactions on the basis of a universal interaction. The knowledge of the frequency ratio $(\Lambda\rightarrow p-)/(\Lambda\rightarrow n0)$, of $\alpha(\Lambda\rightarrow p-)$, and of $\alpha(\Lambda\rightarrow n0)$, together with the total decay rate, would constitute essential information on the Λ-decay matrix.[8] If the present value for the Λ branching ratio is taken as an evidence—in any case incomplete—in favor of $\Delta I=\frac{1}{2}$ [9] in Λ decay, then $\alpha(\Lambda\rightarrow n0)$ is predicted to be equal to $\alpha(\Lambda\rightarrow p-)$.

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Parity Nonconservation and the β Spectrum of RaE

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Since Wu *et al.*[1] showed that space inversion invariance is violated, the study of β decay entered into a new stage. At present we have not yet reached any

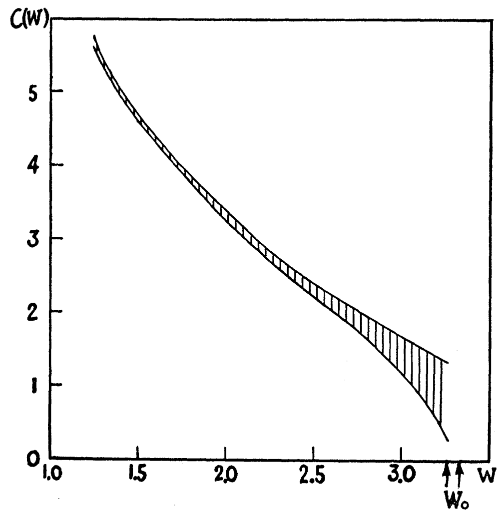

FIG. 1. Experimental correction factor for the RaE β spectrum obtained by Plassmann and Langer.[12] The error, $\pm 1\%$, of each count is taken into account in addition to the uncertainty, $\pm 1\%$, of the maximum electron energy.

definite conclusion about the type of the β-decay interaction. The aim of this short note is to point out that, if the Gamow-Teller component of the β-decay interaction is of the tensor type[2] and if $C_T=-C_T'$ [1,3] (C and C' are the parity conserving and nonconserving coupling constants respectively), it is very difficult to explain the shape of the β spectrum of RaE by the Fermi theory[4] unless all of the following conditions are met approximately: (*a*) the Fermi component is of the scalar type, (*b*) $C_S=-C_S'$, and (*c*) $C_S/C_T=$real.

The peculiar shape of the β spectrum of RaE (spin change 1^- to 0^+) has been understood as a special case where accidental cancellations among several nuclear matrix elements take place.[5] For these anlayses it is necessary to apply the finite nuclear size correction[6–8] (in the broad meaning) to the formulas by Konopinski and Uhlenbeck.[9] Recently, a development of the cancellation theory[10] showed that the main part of this correction can be included legitimately by merely replacing the ordinary nuclear radius in the Konopinski and Uhlenbeck formulas for $\alpha Z \ll 1$ by the "effective nuclear radii," which are expected to be about the same as or somewhat larger than the ordinary one. In the case of RaE this simplest version of the "effective radii theory" is certainly valid for cancellations down to 1/100 if each partial main term has the normal order of magnitude. This theory also suggests that the screening correction is negligible in this case.

The spectrum has been measured precisely by many persons.[11,12] The experimental correction factor is shown in Fig. 1, in which we determined the width of the curve by taking into account the uncertainties; $\pm 1\%$ in the maximum energy W_0 and $\pm 1\%$ in each count.[12] In order to compare it with our "effective radii theory" we expand the experimental curve in a finite power series in W:

$$C(W)=a_{-1}/W+a_0+a_1W+a_2W^2. \tag{1}$$

From Fig. 1 we can prove that $-a_1/a_2>7$ if $a_2>0$. By neglecting the axial vector interaction[2] and assuming time reversal invariance[13] for nuclear interactions, the theoretical expressions for a_1 and a_2 are written simply as (for notation, see reference 9):

$$\begin{aligned} a_1=&\frac{4}{3}\left[\frac{\alpha Z}{2\rho_1(\beta\mathbf{r})}-\frac{2W_0}{3}\right](|C_S|^2+|C_S'|^2)|\textstyle\int\beta\mathbf{r}|^2 \\ &-\tfrac{4}{3}\alpha_{ST}(|C_T|^2+|C_T'|^2)(i\textstyle\int\beta\mathbf{r})^* \\ &\qquad\times\left[\int\beta\boldsymbol{\alpha}-\frac{\alpha Z\int\beta\boldsymbol{\sigma}\times\mathbf{r}}{2\rho_1(\beta\boldsymbol{\sigma}\times\mathbf{r})}\right] \\ &-\tfrac{4}{3}\alpha_{SV}(|C_V|^2+|C_V'|^2)(i\textstyle\int\beta\mathbf{r})^*(i\int\mathbf{r}) \\ &-(1/9)W_0(|C_T|^2+|C_T'|^2)|\textstyle\int\beta\boldsymbol{\sigma}\times\mathbf{r}|^2 \\ &\qquad-(4/9)W_0(|C_V|^2+|C_V'|^2)|\textstyle\int\mathbf{r}|^2, \end{aligned} \tag{2a}$$

$$\begin{aligned} a_2=&(8/9)(|C_S|^2+|C_S'|^2)|\textstyle\int\beta\mathbf{r}|^2 \\ &+(1/9)(|C_T|^2+|C_T'|^2)\textstyle\int\beta\boldsymbol{\sigma}\times\mathbf{r}|^2 \\ &\qquad+(4/9)(|C_V|^2+|C_V'|^2)|\textstyle\int\mathbf{r}|^2>0. \end{aligned} \tag{2b}$$

Here, $\rho_1(\beta\mathbf{r})$ and $\rho_1(\beta\boldsymbol{\sigma}\times\mathbf{r})$ are effective nuclear radii which may somewhat differ from each other, and

$$\alpha_{ST}=\mathrm{Re}(C_SC_T^*+C_S'C_T'^*)/(|C_T|^2+|C_T'|^2). \tag{3a}$$

If we use the Fierz condition, then

$$\alpha_{SV}=0. \tag{3b}$$

From (2) and (3b) we can easily see that the interference term between S and T (α_{ST} term) is important to make $-a_1/a_2$ large enough to fit the experimental results. In particular, the experimental data are incompatible with $\alpha_{ST}=0$, for which $-a_1/a_2<W_0$ unless we take an unreasonable negative value for $\rho_1(\beta\mathbf{r})$. It is interesting to note that $\alpha_{ST}=0$ for such cases as (*a*) the $VT(P)$ interaction, (*b*) $C_S/C_T=C_S'/C_T'=$pure imaginary, where the two-component neutrino theory[3] is valid but time-reversal invariance[13] is not, (*c*) $C_S=C_S'$ and $C_T=-C_T'$, where the two-component theory is not valid. However, if $|\alpha_{ST}|$ takes the maximum value, $\alpha_{\max}$, under the restriction $|C_S|^2+|C_S'|^2+|C_V|^2+|C_V'|^2=$constant, the situation is still the same as in the previous analyses[5] and there exists a region of parameters consistent with the experimental data.

In order to illustrate how the spectrum favors $|\alpha_{ST}|=\alpha_{\max}$ we also performed numerical calculations putting $C_V=C_V'=0$ and

$$\alpha Z/[2\rho_1(\beta\mathbf{r})]=\alpha Z/[2\rho_1(\beta\boldsymbol{\sigma}\times\mathbf{r})]=13.3,$$

which led to the conclusion that $|\alpha_{ST}|\leq\alpha_{\max}/\sqrt{2}$ is incompatible with the experimental data. This conclusion seems to be insensitive to the details of the values of the effective radii.

Thus our present analysis has made clear that the β spectrum of RaE definitely favors $|\alpha_{ST}|=\alpha_{\max}$. In this simplest case the $ST(P)$ interaction, the two-component neutrino theory,[3] and time-reversal invariance[13] are all valid. For the second forbidden β spectrum of Cs^{137} the situation is quite similar. A more detailed account taking into account the axial vector coupling will appear elsewhere.

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Considerations on Depolarization of Positive Muons in Gases; Effect of Molecular Ions*

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MUCH experimental information has been obtained recently on the depolarization of positive mesons in matter,[1] but as yet there appears to be no adequate quantitative understanding of the causes of depolarization. The primary purpose of this note is to suggest a mechanism for depolarization which does not appear to have been considered thus far. First a brief discussion of various causes of depolarization will be given with particular application to muons in gases.

Depolarization of an incident beam of polarized muons which is stopped in matter will occur if different muons are subjected during their lifetime to different magnetic fields of sufficient intensity so that their spins precess through different and appreciable angles. For a free muon in a gas the probability of reorientation in a collision with a gas atom or molecule can be estimated. Suppose the muon is at thermal energy for which its velocity is approximately 10^6 cm/sec (the conclusion will be the same for higher energy muons). A characteristic collision time will then be 10^{-14} sec, and, if the muon is subjected to a magnetic field of 7×10^4 gauss during the collision (which arises from a magnetic moment of 1 Bohr magneton at a distance of a_0, the Bohr radius of hydrogen, and which is as large a field as can be expected), the spin will precess through an angle of 6×10^{-5} radian. Hence because of the random direction of the precession from collision to collision some 3×10^8 collisions are necessary to produce an appreciable change in spin direction. For a gas at a pressure of 100 atmos, the collision rate is 10^{11} to 10^{12}/sec, and thus during the 2×10^{-6}-sec lifetime of the muon no appreciable reorientation of the spin will occur. The formation of muonium (μ^+e^-) and its depolarizing effects have been discussed.[2] At a gas pressure of 100 atmos, depolarization due to the capture and loss of electrons by the muon is negligible.

The additional depolarizing mechanism suggested here is associated with the possible formation of a molecular ion by the attachment of a μ^+ meson to a neutral gas atom or molecule. Since the muon mass is much greater than the electron mass $(m_\mu\simeq207\ m_e)$, the Born-Oppenheimer approximation for molecular theory[3] is applicable to a molecule containing a μ^+ meson. As a specific example, consider the molecular ion $He\mu^+$. In the Born-Oppenheimer approximation the electronic states of $He\mu^+$ will be the same as those of HeH^+. Hence the ground state of $He\mu^+$ will be a $^1\Sigma$ electronic state with a binding energy against dissociation into He and μ^+ of about 1.9 ev.[4] The fundamental vibrational frequency of $He\mu^+$ will be greater than that of HeH^+ by the factor $(M_1/M_2)^{\frac{1}{2}}$, in which M_1 is the reduced mass of HeH^+ and M_2 is the reduced mass of $He\mu^+$. It corresponds to an energy of about 1.2 ev, so the lowest two vibrational states should be stable. Many low rotational states will also be stable. In any but the $J=0$ rotational state there will be a magnetic field arising from the molecular rotation at the position of the muon, which causes a spin-rotation interaction. The Hamiltonian term $\mathcal{H}$ for this spin-rotation interaction energy is given by[5]

$$\mathcal{H}=-\frac{\mu_\mu g_\mu q}{crA}\mathbf{I}\cdot\mathbf{J}+2\mu_0\mu_\mu g_\mu\mathbf{I}\cdot\sum_j\frac{\mathbf{r}_{\mu j}\times\mathbf{P}_j}{r_{\mu j}{}^3},\tag{1}$$

in which $\mu_\mu=e\hbar/(2m_\mu c)$; $g_\mu=$gyromagnetic ratio for μ meson $(\simeq2)$; $I=$spin of μ meson $(=\frac{1}{2})$; $q=$charge of He nucleus $(=2e)$; $r=$internuclear distance between μ^+ meson and He nucleus; $A=$moment of inertia of molecular ion; $J=$rotational angular momentum of molecular ion; $\mu_0=$Bohr magneton; $\mathbf{r}_{\mu j}=$radius vector