We have observed some additional weaker gamma transitions which may be associated with the decay of Re^{181}, but because of the complexity of the gamma spectra of the Re^{182} isomers with similar half-lives, the isotope assignment of the weaker transitions in our samples is somewhat uncertain.

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Energy and Angular Distributions of Neutrons from the Interaction of 14.1-Mev Neutrons with Zirconium*

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The distributions in energy and angle of the neutrons emitted from the interaction of 14.1-Mev neutrons with zirconium have been obtained by using nuclear emulsion plates as detectors. The energy distribution of the low-energy nonelastic neutrons is Maxwellian. The angular distribution of the low-energy nonelastic neutrons (0.5–4.0 Mev) is found to be isotropic; whereas, that of the high-energy nonelastic neutrons (4.0–12.0 Mev) is peaked in the forward direction. The significance of these results in terms of the statistical model of the nucleus is discussed.

I. INTRODUCTION

THE main object of this experiment is to test the validity of the compound-nucleus concept—and particularly the statistical model—by measuring the distributions in energy and angle of neutrons emitted in the nonelastic interaction of 14.1-Mev neutrons with natural zirconium. The differential cross sections at several angles for elastic scattering are also obtained in this experiment.

None of previous experiments agrees completely with any one of the present models of the nucleus. The results of this experiment indicate that the compound-nucleus model seems to apply to at least 80% of the nonelastic interactions. These results are quite similar to the results for Ta and Bi obtained by Rosen and Stewart.[1]

II. EXPERIMENTAL PROCEDURE

The experiment was performed by measuring the recoil proton tracks in nuclear emulsions. The tracks were obtained by exposing 200-micron Ilford C-2 emulsion plates to neutrons emitted from zirconium due to its interaction with the 14.1-Mev neutron beam obtained from the reaction $H^3(d,n)He^4$ in the Cockcroft-Walton generator at the Los Alamos Scientific Laboratory. The zirconium scatterer is of natural isotopic abundance and is in the form of a cylinder $1\frac{1}{4}$ inches long and $1\frac{1}{4}$ inches in diameter. The scatterer was oriented so that its axis of symmetry coincided with the collimator axis. The background was evaluated by making an irradiation with the scatterer removed. The iron collimator, which was developed by Rosen[1] and his group, was used. The plates were arranged around the periphery of a circle whose center coincided with the center of the scatterer. The details of tracks measurement and analysis are given by Rosen.[2]

III. ANALYSIS

The cross section $\sigma(E,\theta)$ for the emission of neutrons between energies E and $E+dE$ at angle θ is determined from:

$$F(E,\theta)dE=\frac{\bar{F}n_{Zr}\sigma(E,\theta)V_{Zr}}{\langle r^2\rangle_{Av}}dE, \tag{1}$$

where $F(E,\theta)dE$=neutron flux at the scattering angle θ, $\bar{F}$=average primary neutron flux on the scatterer allowing for attenuation, n_{Zr}=number of nuclei per cm³ in the zirconium scatterer, V_{Zr}=volume of zirconium scatterer, and $\langle r^2\rangle_{Av}$=average for the square of the distance from the center of the scatterer to the center of the scanned area. Here, the average primary neutron flux on the scatterer with length x is given by

$$\bar{F}=\frac{F_0}{x}\int_0^x e^{-\sigma' n_{Zr}x}dx=F_0\frac{1-e^{-\sigma' n_{Zr}x}}{\sigma' n_{Zr}x}, \tag{2}$$

where σ' is equated to the total cross section[3] in evaluat-

* Work supported by the U. S. Atomic Energy Commission through Argonne National Laboratory subcontract.
† On leave from Yonsei University, Seoul, Korea.
[1] L. Rosen and L. Stewart, Phys. Rev. **107**, 824 (1957).
[2] L. Rosen, Nucleonics **11**, No. 7, 32, and No. 8, 38 (1953).
[3] *Neutron Cross Sections*, compiled by D. J. Hughes and J. A. Harvey, Brookhaven National Laboratory Report BNL-325 (Superintentent of Documents, U. S. Government Printing Office, Washington, D. C., 1955).

ing the elastic scattering and to the transport cross section in evaluating the nonelastic scattering, and F_0 is the neutron flux incident on the front face of the scatterer which is determined by counting the alpha particles from the $H^3(d,n)He^4$ reaction.

The neutron flux at the detector is calculated from:

$$F(E,\theta)dE=\frac{4\pi}{\Omega}\frac{1}{\sigma_{n-p}(E)n_H 4\langle\cos\gamma\rangle_{Av}}\frac{1}{P(l)}\frac{N_p(E,\theta)}{V_e}dE, \quad (3)$$

where Ω=solid angle of acceptance of proton recoils, $\sigma_{n-p}(E)$=n-p scattering cross section at energy E, n_H=number of hydrogen atoms per cm³ in emulsion, $\langle\cos\gamma\rangle_{Av}$=average value of cosine of proton scattering angle, $P(l)$=probability that a track of length l will end within the emulsion layer, $N_p(E,\theta)$=number of proton recoils between energies E and $E+dE$ at angle θ, and V_e=emulsion volume scanned.

In addition to the calculation of Eq. (3), corrections such as attenuation, n-charged particle reactions, and multiple scattering were made. These corrections are discussed by Rosen and Stewart.[1] These corrections do not affect any qualitative property of neutron emission.

IV. RESULTS AND DISCUSSION

According to the statistical model of the nucleus,[4] the number of emitted neutrons having energies between E and $E+dE$, $N(E)$, may be represented by

$$N(E)dE=\text{const}\ E\exp(-E/T)dE, \quad (4)$$

where T represents a nuclear temperature expressed in a unit of energy. This Maxwellian energy distribution in the low-energy region for various elements was already obtained by several people such as Stelson and Goodman,[5] Whitmore and Dennis,[6] Graves and Rosen,[7] and O'Neill.[8] Figure 1 is the energy distribution for all energies, and Fig. 2 shows that the low-energy group from 0.5 to 4.0 Mev of the energy distribution satisfies the Maxwellian relation of Eq. (4), in our experiment. Here, the apparent nuclear temperature can be obtained (Table I). In fact, $(n,2n)$ reactions have a marked effect on the apparent nuclear temperature deduced from the energy distribution. In order to obtain the nuclear temperature for Zr corresponding to the evaporation of the first neutron from the compound nucleus, Lang and Le Couteur's method[9,10] was used. If one assumes that two neutrons can be emitted in succession while the nuclear temperature drops as the square root of excitation energy, the resultant energy distribution is

$$N(E)dE=\text{const}E^{5/11}\exp\left(-\frac{12}{11}\frac{E}{T_1}\right)dE. \quad (5)$$

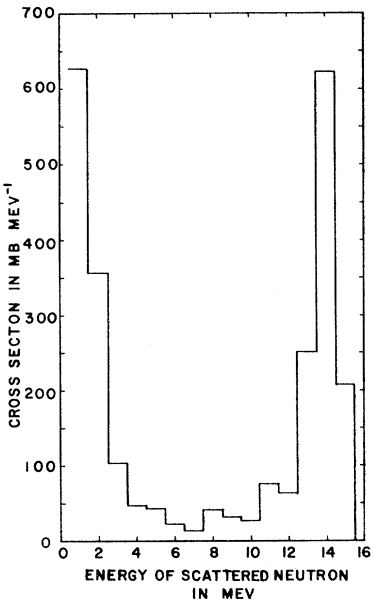

FIG. 1. Space-integrated energy distribution of the neutrons from the interaction of 14.1-Mev neutrons with Zr.

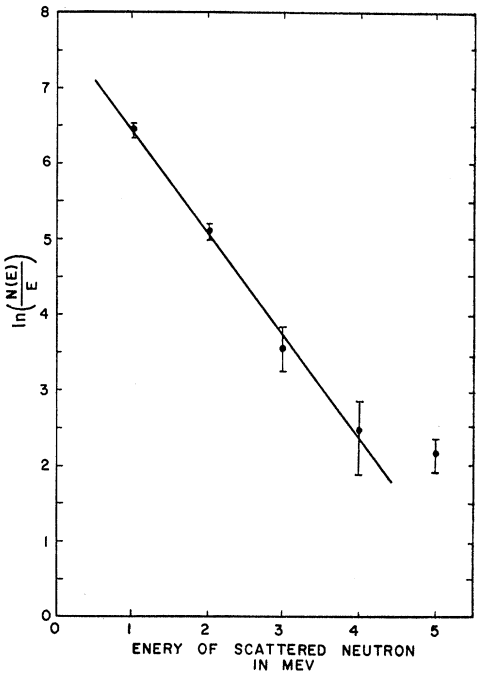

FIG. 2. Relation of $\ln[N(E)/E]$ *vs* E for Zr. A straight line is drawn according to a least-squares fit using data from 0.5 to 4.5 Mev.

TABLE I. Nuclear temperatures in Mev for natural zirconium.

Apparent, T	First neutron, T_1	Second neutron, T_2
0.74±0.09	0.99±0.12	0.57±0.07

[4] V. Weisskopf, Phys. Rev. **52**, 295 (1937).

[5] P. H. Stelson and C. Goodman, Phys. Rev. **82**, 69 (1951).
[6] B. G. Whitmore and G. E. Dennis, Phys. Rev. **84**, 296 (1951).
[7] E. R. Graves and L. Rosen, Phys. Rev. **89**, 343 (1953).
[8] G. K. O'Neill, Phys. Rev. **95**, 1235 (1954).
[9] K. L. Le Couteur, Proc. Phys. Soc. (London) **A65**, 718 (1952).
[10] J. M. B. Lang and K. J. Le Couteur, Proc. Phys. Soc. (London) **A67**, 586 (1954).

TABLE II. Cross sections in barns for 14.1-Mev neutrons on zirconium.

$\sigma_{n,n'}$	$\sigma_{n,2n}$	σ_{non}	σ_{emi}
0.41±0.09	0.61±0.10	1.0±0.2	1.6±0.3

Here, the value of nuclear temperature T_1 corresponding to the evaporation of the first neutron can be obtained (Table I). The nuclear temperature T_2 corresponding to the evaporation of the second neutron can be obtained, after the first neutron contribution has been subtracted, by assuming the energy distribution is represented by Eq. (4) (Table I).

Let the nonelastic cross section be defined by

$$\sigma_{\text{non}}=\sigma_{n,n'}+\sigma_{n,2n};$$

then

$$\sigma_{\text{emi}}=\sigma_{n,n'}+2\sigma_{n,2n}$$

will give the cross section for emission of nonelastic neutrons. These cross sections, given in Table II, are obtained under the assumptions that the Maxwellian energy distribution can be extended to zero energy, and that an $(n,2n)$-reaction is realized only when the energy of the first neutron is low enough to make the emission of the second neutron possible.

In comparing our results for Zr with the results for Ta and Bi obtained by Rosen and Stewart.[1] $\sigma(n,n')$ is relatively large and $\sigma(n,2n)$ is small. These can be explained by the fact that Zr includes an isotope Zr^{90} which has a neutron number corresponding to a magic number and an especially high binding energy. Paul and Clarke[11] obtained $\sigma(n,2n)=478$ mb for Zr^{90}, which is especially smaller than the corresponding cross sections for the neighboring elements, by a calculation using the formula for $\sigma(n,2n)$ developed by Blatt and Weisskopf.[12] Their experimental value[10] was, however, $\sigma(n,2n)\geqslant 79.8$ mb for Zr^{90}.

Since the energy distribution is Maxwellian up to 4 Mev and attributable to elastic scattering above 12 Mev, the scattered neutrons have been divided into three groups in describing the angular distribution of scattered neutrons. They are (1) the group from 0.5 to 4.0 Mev, (2) the group from 4.0 to 12.0 Mev, and (3) the group higher than 12.0 Mev.

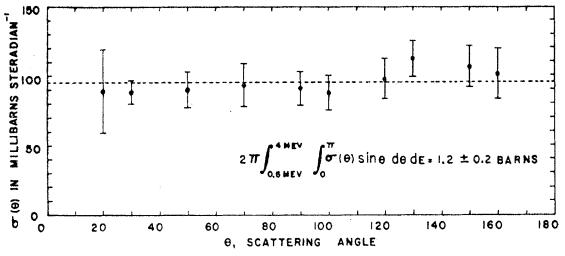

FIG. 3. Angular distribution of the nonelastic neutrons from the interaction of 14.1-Mev neutrons with Zr in the low-energy region from 0.5 to 4.0 Mev. θ is the scattering angle in the center-of-mass system.

The angular distribution of neutrons from 0.5 to 4.0 Mev is isotropic within the statistical accuracy of the experiment (Fig. 3). O'Neill showed that the angular distribution of low-energy neutrons emitted from the interaction of neutrons as a result of the reaction $H^3(d,n)He^4$ with Pb, by time-of-flight measurements at two angles, is isotropic. Rosen and Stewart[1] recently obtained an isotropic angular distribution in the low-energy region for Ta and Bi by the nuclear emulsion method.

Thomas[13] pointed out that it is a common misconception that the theory of the compound nucleus alone can explain the isotropic angular distribution. Wolfenstein,[14] and Hauser and Feshbach[15] proved that the theory of the compound nucleus leads only to an angular distribution symmetric with respect to a plane perpendicular to beam direction when averaged over a sufficient number of resonance levels. These authors also showed that for isotropy it must be assumed that the energy levels of the compound nucleus and residual nucleus are sufficiently dense and have a dependence on their respective spin J which is proportional to $2J+1$ for the range of J values that can participate. The fact that the angular distribution from 0.5 to 4.0 Mev is not only symmetric about 90° but also isotropic shows that these theoretical conditions for isotropy are justifiable.

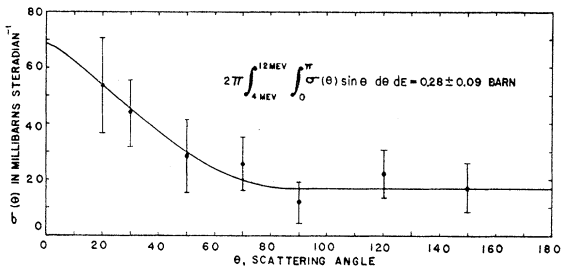

FIG. 4. Angular distribution of the nonelastic neutrons from the interaction of 1.41-Mev neutrons with Zr in the high-energy region from 4.0 to 12.0 Mev. θ is the scattering angle in the center-of-mass system.

The angular distribution of neutrons from 0.4 to 12.0 Mev is peaked in the forward direction (Fig. 4). Of course, the angular distribution of this group which is neither isotropic nor symmetric about 90° cannot be attributed to compound-nucleus formation, but must instead be related to direct or nucleon-nucleon interaction. The portion of isotropic distribution in the group from 4.0 to 12.0 Mev may belong to the tail of the Maxwellian

[11] E. B. Paul and R. L. Clarke, Can. J. Phys. **31**, 267 (1953).
[12] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952), Chap. 8, Sec. 6.
[13] R. G. Thomas, Phys. Rev. **97**, 224 (1955).
[14] L. Wolfenstein, Phys. Rev. **82**, 690 (1951).
[15] W. Hauser and H. Feshbach, Phys. Rev. **87**, 366 (1952).

energy distribution which describes the low energy. The result of this experiment indicates that compound-nucleus formation applies to at least 80% of the nonelastic interactions.

Recently Weisskopf[16] proposed a more general scheme for the description of nuclear reactions. Instead of the two-stage Bohr description, he divided the nuclear reaction in three successive stages, the independent stage, the compound-system stage, and the final stage. He also pointed out that we face a varied range of phenomena in the second stage of his model, which can be grouped in the two extremes, direct interaction and formation of real compound nucleus.

Table III shows the elastic differential cross sections at several angles. The angular resolution for each point is ±8°.

[16] V. F. Weisskopf, Revs. Modern Phys. **29**, 174 (1957).

TABLE III. Elastic differential cross sections in barns per steradian at several angles in the center-of-mass system.

θ	20°	30°	50°
$\sigma(\theta)$	1.25±0.09	0.35±0.04	0.025±0.008
θ	70°	90°	120°
$\sigma(\theta)$	0.018±0.007	0.014±0.006	0.010±0.006

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Rotational Spectrum of Tm^{171}†

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The gamma rays emitted from Tm^{171} following the Er^{171} beta decay have been measured with the two-meter curved-crystal spectrometer and a semicircular beta-ray spectrometer, enabling a comparison of the nuclear levels and rotational parameters for the Tm^{171} ground-state band with the corresponding levels and parameters of Tm^{169}. The measured gamma rays have the following energies in kev: 5.06±0.05, 12.40 ±0.05, 111.63±0.02, 116.69±0.03, 124.03±0.03, 210.62±0.15, 284.9±0.7, 295.97±0.15, 308.37±0.15. In comparing the rotational parameters, a slight increase in the deformation of Tm^{171} over that of Tm^{169} is noted. The relatively large change in the decoupling parameter ($a=-0.8563$) for Tm^{171} compared with that of Tm^{169} cannot be accounted for entirely by the small change in nuclear deformation.

RECENT experimental studies[1] of the gamma-ray spectrum of Tm^{171} following the 7.8-hour Er^{171} beta decay have shown that the Tm^{171} nuclear level structure has striking similarity to that of Tm^{169}. From the curves of odd-proton orbitals in a deformed nuclear potential given by Mottelson and Nilsson,[2] both Tm^{169} and Tm^{171} could be expected to have $K=\frac{1}{2}$ ground states since both of these odd-A nuclei contain 69 protons. The anomalous rotational spectrum, characteristic of a $K=\frac{1}{2}$ ground state, has indeed been observed in the studies of nuclear levels in both isotopes.[1,3]

Since a precise determination of the energy levels in Tm^{169} had been carried out with the two-meter curved-crystal spectrometer at Caltech,[4] it was of interest to determine the energies of the corresponding levels in Tm^{171} with the same instrument. In addition, two low-energy transitions of 5 and 12 kev were implied from the proposed Tm^{171} level scheme[1] but had not been observed so that a study of the low-energy internal conversion spectrum was also desirable.

Approximately 10 mg of high-purity Er_2O_3 (supplied by Johnson, Matthey and Company, Ltd., London), enclosed in a 0.015 in.×1 in. quartz capillary, was irradiated for 24 hours in the Materials Testing Reactor at Arco, Idaho,[5] to serve as the gamma-ray source for the curved-crystal spectrometer. A thin layer of the radioactive Er_2O_3 was evaporated in vacuum on an aluminized mica backing and represented the source for the low-energy beta-ray spectrometer. The semicircular

† Supported by the U. S. Atomic Energy Commission.
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[1] S. D. Koički and A. M. Koički, Bull. Inst. Nuclear Sci. Boris Kidrich **6**, 1 (1956); Cranston, Bunker, Mize, and Starner, Bull. Am. Phys. Soc. Ser. II, **1**, 389 (1956); S. A. E. Johansson, Phys. Rev. **105**, 189 (1957).
[2] B. R. Mottelson and S. G. Nilsson, Phys. Rev. **99**, 1615 (1955).
[3] *Nuclear Data Cards*, edited by C. L. McGinnis (National Research Council, Washington, D. C.).
[4] Hatch, Boehm, Marmier, and DuMond, Phys. Rev. **104**, 745 (1956).
[5] We are grateful to the staff of the MTR at Arco for their efficient cooperation in connection with the irradiation.