S_p, S_e represent the limit to which protons, electrons penetrate. The region between S_p and S_e contains only protons. Now in a cloud of somewhat higher density, considered at a time long enough after the beginning of the collision that a steady state has been established, the electric field of these protons will accelerate the electrons so that they can come nearer the earth. This same field will, of course, slow down the protons. Briefly then, the tendency of plasmas to keep themselves neutral, which has been mentioned by many authors, here operates to accelerate the electrons. The magnetic field of the earth forces some charge separation on the plasma, but this separation sets up an electric field which accelerates electrons to reduce the charge. The balance between these two tendencies, and hence the amount of acceleration, clearly depends on the initial density of the ion cloud.

An upper limit to the acceleration would be reached if the surfaces S_p and S_e came together. For particles moving in cylindrically symmetric fields we have

$$\frac{e}{M}\phi+\frac{1}{2}v^2+\frac{1}{2}\left(\frac{e}{Mc}A_\phi\right)^2=\text{const},$$

where e is charge and M is mass of the particle, ϕ is electrostatic potential, A_ϕ = azimuthal component of vector potential, and $v^2=v_r^2+v_\theta^2$ is the square of the velocity in the $r-\theta$ plane. If S_p, S_e are approximately characterized by $v^2=0$ and M, m are proton, electron masses, we have on $S=S_p=S_e$,

$$\tfrac{1}{2}v_0^2=\frac{e}{M}\phi+\left(\frac{e}{Mc}A_\phi\right)^2=-\frac{e}{m}\phi+\left(\frac{e}{mc}A_\phi\right)^2.$$

Eliminating A_ϕ, we have for the energy of the electrons at S

$$E_S=\tfrac{1}{2}mv_0^2+e\phi_S=\tfrac{1}{2}Mv_0^2,$$

and for the protons

$$\tfrac{1}{2}Mv_0^2-e\phi_S=\tfrac{1}{2}mv_0^2.$$

Electrons and protons have then exchanged energies. Ferraro's more detailed calculations[2] result in the same acceleration.

Time-of-flight from the sun and Meinel's[3] observation of Doppler shift indicate that 50–100 kev is a reasonable figure for the energy of protons in the ion clouds which are thought to cause auroras. If the mechanism under discussion accelerates the electrons to a good fraction of this energy, we might expect that x-rays, produced when the electrons strike the atmosphere, would be found in conjunction with auroras. We may make a rough estimate of the efficiency of this process by dividing the range of the electrons in air by the radiation length. For 50-kev electrons, about one electron in 10^4 will radiate by bremsstrahlung and the efficiency will increase rapidly with electron energy. The efficiency for production of x-rays by the incoming 50-kev protons would, of course, be many orders of magnitude smaller.

Radiations which are very likely x-rays have been observed in conjunction with auroras by Winckler and Peterson,[4] of this laboratory.

Calculations are in progress to relate the amount of acceleration of the electrons to the other parameters of the problem, using a hydromagnetic theory approach.

The author wishes to thank Professor E. P. Ney and Professor J. R. Winckler of the University of Minnesota, and Dr. W. H. Bennett of the Naval Research Laboratory for illuminating discussions.

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Superconducting Energy Gap from Ultrasonic Attenuation Measurements*

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THE fact that ultrasonic attenuation in very pure superconducting metals undergoes a rapid decrease as the temperature of the metal falls below the superconducting transition has been the subject of recent experimental study.[1–3] The attenuation due to electrons in a normal metal has been fairly well explained.[4,5] The details depend upon the value, relative to unity, of kl, where k is the propagation constant of the ultrasonic wave and l is the electron mean free path. When $kl<1$, which is the most usual case for ultrasonic waves, the attenuation is proportional to the square of the frequency and to the electron mean free path. When $kl>1$, the attenuation depends only upon basic parameters of the electron distribution and the electron-lattice interaction. Consequently such measurements can be expected to yield fundamental information in an especially direct way. Indeed, the case $kl>1$ is identically the situation contemplated in the basic problem of scattering of phonons by electrons. Thus one regards the ultrasonic wave as a coherent beam of phonons superimposed upon the equilibrium distribution of phonons, and one identifies the measured intensity attenuation as the reciprocal of the mean free path for lattice conduction limited by electrons.[6] The result for attenuation in the normal state, also given by Kittel,[7] is

$$\alpha_n=m^{*2}C^2k/(2\pi\rho v\hbar^3),\tag{1}$$

where m^* is the effective mass, C the electron-lattice

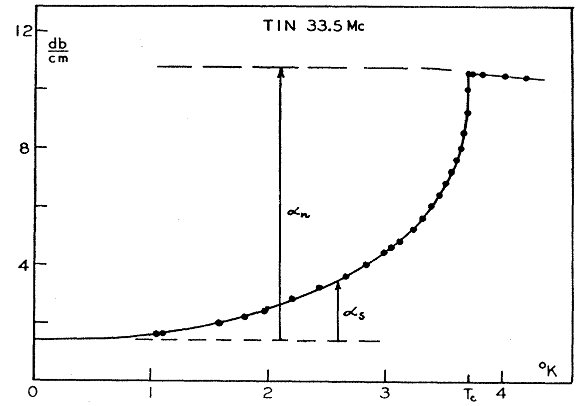

FIG. 1. Measured ultrasonic attenuation of longitudinal waves in a tin single crystal at a frequency of 33.5 Mc/sec.

interaction constant, ρ the metal density, and v the velocity of longitudinal waves.

It has been recognized that the falloff of the ultrasonic attenuation observed when a metal becomes superconducting is somehow connected with the diminishing number of normal electrons. However, it has not been observed to vary as T^4 as the simple two-fluid model would anticipate. Recently Bardeen, Cooper, and Schrieffer have proposed a theory of superconductivity which has shown a remarkable agreement with many of the observed facts.[8] An essential part of this theory is the appearance of an energy gap $2\epsilon_0(T)$ at the Fermi surface, which increases in width as the temperature falls below the transition point. The evaluation of the integral leading to Eq. (1) for the energy-gap model has been carried out by Bardeen and gives the following expression for the ratio of superconducting attenuation to that in the normal state[9]:

$$\alpha_s/\alpha_n = 2/(e^{\epsilon_0/kT}+1). \qquad (2)$$

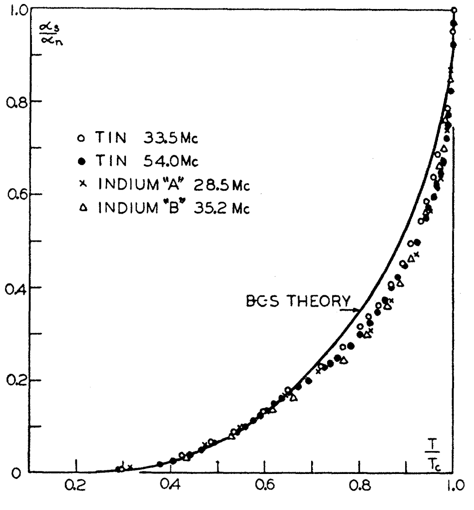

FIG. 2. Measured values of α_s/α_n compared with the theoretical variation of Bardeen, Cooper, and Schrieffer assuming $\epsilon_0(0) = 1.75kT_c$. For tin $T_c = 3.71$ °K and for indium $T_c = 3.40$ °K.

Thus the falloff of the attenuation previously observed should yield direct information about the energy gap. Conversely such measurements can serve as a specific validation of the theoretical model.

Some of the measurements that we have taken in polycrystalline indium and in a single-crystal tin specimen fall into the region where $kl>1$. The dependence of this attenuation on applied magnetic field, which will be reported at length in another paper, indicates that kl is at least 10 for all samples at a frequency of about 25 Mc/sec. A typical dependence of attenuation with temperature is shown in Fig. 1 where results are plotted for 33.5-Mc/sec waves in a tin single crystal oriented so that the propagation direction is along the (0,0,1) axis. The normal state attenuation (found by using a magnetic field) is essentially temperature-independent. There is, of course, a background ultrasonic attenuation which is nonelectronic in origin, and α_s and α_n have been determined by extrapolating the superconducting attenuation to absolute zero as shown.

Figure 2 shows measured ratios of α_s/α_n as a function of reduced temperature T/T_c for various frequencies in the tin sample, and in two different polycrystalline samples of indium. Comparison is made with the theory of Bardeen, Cooper, and Schrieffer by use of Eq. (2) and their calculated temperature variation of ϵ_0. The value of ϵ_0 at absolute zero chosen for the curve of Fig. 2 is $1.75kT_c$, which is the value predicted by the theory. This is also the value which gives about the best fit to the experimental points. Although the measured values of α_n/α_s decrease somewhat faster just below the transition temperature than the theory predicts, the agreement of the theory with experiment on the whole is remarkably good.

Figure 3 shows the energy-gap variation with T/T_c as calculated from Eq. (2) using the tin data. Comparison is made here with the theoretical energy-gap variation, and again the agreement is quite satisfactory.

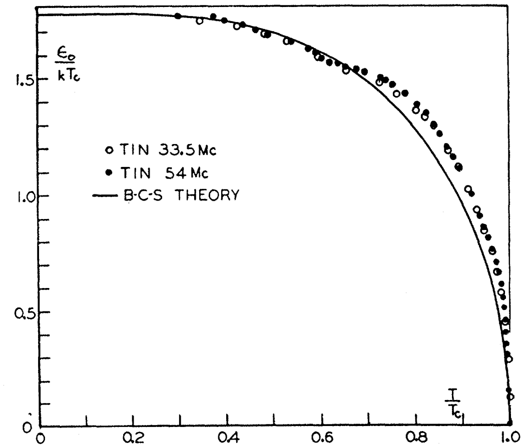

FIG. 3. Variation of $\epsilon_0(T)$ from tin data using Eq. (2) compared with theoretical variation assuming $\epsilon_0(0) = 1.77kT_c$.

(It will be noted that there is a slight dip in the experimental points near $T/T_c=0.7$. This is not felt to be a real effect; it is, however, being investigated further.)

It seems, therefore, that this preliminary comparison gives good support to the new theory of superconductivity. It would also appear as if one of the most direct ways of measuring the superconducting energy gap as a function of temperature is by means of ultrasonic attenuation.

Presumably one could also deduce similar information from measurements for the region $kl<1$. The theory here, however, is essentially more complicated because the electron mean free path enters, and this will also depend upon the energy gap. One would expect that the electron mean free path would increase as the gap increases with decreasing temperature until eventually kl became greater than unity, and the attenuation would become independent of electron mean free path. Experimentally one gains support for this picture since for $kl<1$, the attenuation near the transition temperature varies as the square of the frequency but becomes dependent on the first power of the frequency sufficiently far below the transition temperature.

The authors would like to thank Professor Bardeen for pointing out the exact connection between his theory and our measurements and for sending a table of values for the energy gap. They would also like to thank Mr. David Gavenda for his invaluable assistance in making the measurements.

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New V-Center Spin Resonance in LiF

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In previous work[1,2] it was shown that by x-irradiation of LiF crystals at or below liquid nitrogen temperature an electron deficiency center is created which can be described as an F_2^- molecule-ion and which probably is not associated with vacancies or other imperfections. The original F_2^- centers disappear irreversibly and two new kinds of center appear if the LiF crystal is subsequently warmed up to about 130°K. One of these is also an F_2^- molecule-ion but probably associated with a vacancy pair; the other center, to be discussed in this letter, consists of an Fe_3^{--} group presumably associated with a vacancy aggregate. The spin resonance of the $F_3{}^{--}$center is readily separable from the spin resonance of the F_2^- centers because of the much shorter relaxation time of the former.

We can deduce the atomic structure of the center directly from the gross features of the resonance spectrum. The over-all spread of the hyperfine spectrum for the new center is almost the same as for the F_2^- centers, indicating that the resonance is also due to a hole on fluoride ions. The basic pattern consists of 8 hyperfine lines of equal intensity, implying that the hole interacts with *three* fluorine nuclei. For special orientations of the dc magnetic field the eight-line pattern degenerates into a six-line pattern with the intensity ratio 1:2:1:1:2:1, indicating that two of the three nuclei are equivalent, i.e., the nuclear configuration is an isosceles triangle. Two to six basic patterns are observed simultaneously, depending upon the orientation of the crystal in the dc magnetic field. The analysis of this observation leads to the conclusion that the base of the triangle is a [110] axis and its plane a (001) plane. All six [110] axes are equally populated. The triangle is flat, i.e., the nuclei are not far from collinearity. The simplest vacancy configuration that makes the center uncharged and complies with the observed symmetry is the one proposed by Seitz[3] for the V_4 center (Fig. 1).

For most orientations the basic eight-line pattern contains four independent line separations, which can be consistently described by three parameters, each parameter giving the contribution to the hyperfine interaction of one nucleus α ($\alpha=1$, 2, 3). Thus, first-order perturbation theory is a good approximation for

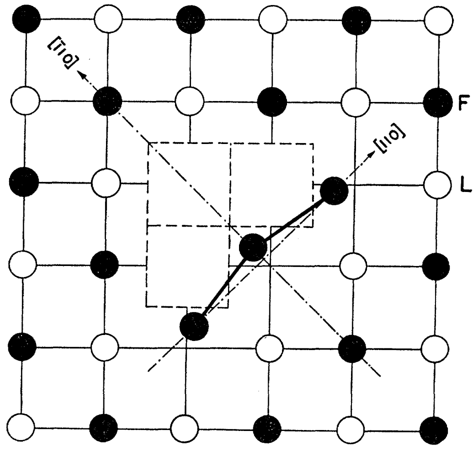

FIG. 1. Symmetry and orientation of the triatomic V center as deduced from the spin resonance spectra. The vacancy configuration and the exact positions of the nuclei cannot be derived rigorously from the spectra, but the model proposed here is consistent with our observations.