Discussion of Experimental Proof for the Paradox of Einstein, Rosen, and Podolsky

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A brief review of the physical significance of the paradox of Einstein, Rosen, and Podolsky is given, and it is shown that it involves a kind of correlation of the properties of distant noninteracting systems, which is quite different from previously known kinds of correlation. An illustrative hypothesis is considered, which would avoid the paradox, and which would still be consistent with all experimental results that have been analyzed to date. It is shown, however, that there already is an experiment whose significance with regard to this problem has not yet been explicitly brought out, but which is able to prove that this suggested resolution of the paradox (as well as a very wide class of such resolutions) is not tenable. Thus, this experiment may be regarded as the first clear empirical proof that the aspects of the quantum theory discussed by Einstein, Rosen, and Podolsky represent real properties of matter.

1. INTRODUCTION

IN a well-known article,[1] Einstein, Rosen, and Podolsky have given an example of a hypothetical experiment capable of testing certain apparently paradoxical predictions of the current quantum theory. In order to illustrate this experiment we shall consider a special example which permits us to present the arguments of Einstein, Rosen, and Podolsky in a simplified form.[2]

We consider a molecule of total spin zero consisting of two atoms, each of spin one-half. The wave function of the system is therefore

$$\psi=\frac{1}{\sqrt{2}}[\psi_+(1)\psi_-(2)-\psi_-(1)\psi_+(2)], \tag{1}$$

where $\psi_+(1)$ refers to the wave function of the atomic state in which one particle (A) has spin $+\hbar/2$, etc. The two atoms are then separated by a method that does not influence the total spin. After they have separated enough so that they cease to interact, any desired component of the spin of the first particle (A) is measured. Then, because the total spin is still zero, it can immediately be concluded that the same component of the spin of the other particle (B) is opposite to that of A.

If this were a classical system, there would be no difficulty in interpreting the above results, because all components of the spin of each particle are well defined at each instant of time. Thus, in the molecule, each component of the spin of particle A has, from the very beginning, a value opposite to that of the same component of B; and this relationship does not change when the atom disintegrates. In other words, the two spin vectors are correlated. Hence, the measurement of any component of the spin of A permits us to conclude also that the same component of B is opposite in value. The possibility of obtaining knowledge of the spin of particle B in this way evidently does not imply any interaction of the apparatus with particle B or any interaction between A and B.

In quantum theory, a difficulty arises, in the interpretation of the above experiment, because only one component of the spin of each particle can have a definite value at a given time. Thus, if the x component is definite, then the y and z components are indeterminate and we may regard them more or less as in a kind of random fluctuation.

In spite of the effective fluctuation described above, however, the quantum theory still implies that no matter which component of the spin of A may be measured the same component of the spin of B will have a definite and opposite value when the measurement is over. Of course, the wave function then reduces to $\psi_+(1)\psi_-(2)$ or to $\psi_-(1)\psi_+(2)$, in accordance with the result of the measurement. Hence, there will then be no correlations between the remaining components of the spins of the two atoms. Nevertheless, before the measurement has taken place (even while the atoms are still in flight) we are free to choose *any* direction as the one in which the spin of particle A (and therefore of particle B) will become definite.

In order to bring out the difficulty of interpreting the result, let us recall that originally, the indeterminacy principle was regarded as representing the effects of the disturbance of the observed system by the indivisible quanta connecting it with the measuring apparatus. This interpretation leads to no serious difficulties for the case of a single particle. For example, we could say that on measuring the z component of the spin of particle A, we disturb the x and y components and make them fluctuate. This point of view more generally implies that the definiteness of any desired component of the spin is (along with the indefiniteness of the other two components) a potentiality[3] which can be realized with the aid of a suitably oriented spin-measuring apparatus.

In the case of complementary pairs of continuous variables, such as position and momentum, one obtains from this point of view the well known wave-particle

[1] Einstein, Rosen, and Podolsky, Phys. Rev. 47, 777 (1935), herafter referred to as ERP.
[2] See D. Bohm, *Quantum Theory* (Prentice-Hall, Inc., New York, 1951), Chap. XXII for a fuller discussion.
[3] D. Bohm, reference 2, Chaps. VI and XXII.

duality. In other words, the electron, for example, has potentialities for mutually incompatible wave-like and particle-like behavior, which are realized under suitable external conditions. In the laboratory those conditions are generally determined by the measuring apparatus although, more generally, they may be determined by any arrangement of matter with which the electron interacts. But in any case, it is essential that there must be an external interaction, which disturbs the observed system in such a way as to bring about the realization of one of its various mutually incompatible potentialities. As a result of this disturbance, when any one variable is made definite, other (noncommuting) variables must necessarily become indefinite and undergo fluctuation.

Evidently, the foregoing interpretation is not satisfactory when applied to the experiment of ERP. It is of course acceptable for particle A alone (the particle whose spin is measured directly). But it does not explain why particle B (which does not interact with A or with the measuring apparatus) realizes its potentiality for a definite spin in precisely the same direction as that of A. Moreover, it cannot explain the fluctuations of the other two components of the spin of particle B as the result of disturbances due to the measuring apparatus.

One could perhaps suppose that there is some hidden interaction between B and A, or between B and the measuring apparatus, which explains the above behavior. Such an interaction would, at the very least, be outside the scope of the current quantum theory. Moreover, it would have to be instantaneous, because the orientation of the measuring apparatus could very quickly be changed, and the spin of B would have to respond immediately to the change. Such an immediate interaction between distant systems would not in general be consistent with the theory of relativity.

This result constitutes the essence of the paradox of Einstein, Rosen, and Podolsky.

2. POSSIBLE INTERPRETATIONS OF THE PARADOX OF EINSTEIN, ROSEN, AND PODOLSKY

It should be noted that the difficulties arising in connection with the ERP paradox are serious only for the case that particles are so far apart that: (a) the observing apparatus can influence only one particle at a time and (b) the two particles do not interact significantly. On the other hand, it was not possible previously to find an experiment which would test the many-body Schrödinger or Dirac equations under the conditions described above, in which this paradox can arise. For example, it is evident that the agreement with experiment of the energy levels of the many electron-atom cannot test for the essential points that we are discussing here. Moreover, as we shall see in the Appendix, it has not yet been possible to make such a test with the aid of scattering experiments.

At first sight it would seem then that there exists at present no experimental proof that the paradoxical behavior described by ERP will really occur. If this is so, then we are free to consider the assumption that perhaps the difficulty comes from the yet experimentally unverified extrapolation of the many-body Schrödinger and Dirac equations to the case where the particle's wave functions do not overlap and where the particles do not interact. In fact, Einstein has (in a private communication) actually proposed such an idea; namely, that the current formulation of the many-body problem in quantum mechanics may break down when particles are far enough apart.

The consequences of such an idea have already been discussed by Furry.[4] To illustrate Furry's conclusions in terms of our problem, we may consider the possibility that after the molecule of spin zero decomposes, the wave function for the system is eventually no longer given by Eq. (1), which implies the puzzling correlations of the spins of the two atoms. Instead, we suppose that in any individual case, the spin of each atom becomes definite in *some* direction, while that of the other atom is opposite. The wave function will be the product

$$\psi=\psi_{+\theta,\varphi}(1)\psi_{-\theta,\varphi}(2), \tag{2}$$

where $\psi_{+\theta,\varphi}(1)$ is a wave function of particle A whose spin is positive in the direction given by θ and φ. In other words, each particle goes into a definite spin state, while the fluctuations of the other two components of the spin are uncorrelated to the fluctuations of these components of the spin of the other particle. In order to retain spherical symmetry in the statistical sense, we shall further suppose that in a large aggregate of similar cases, there is a uniform probability for any direction of θ and φ.

It is true that in any single case, the total angular momentum will not be conserved (just because the fluctuations of the two particles are now uncorrelated). However, thus far, there has not been given an experimental demonstration of the detailed conservation of every component of the angular momentum, for particles that are far apart and not interacting. On the other hand, with the model that we have discussed here, the uniform probability of all directions will lead to the experimentally observed fact of conservation on the average. Thus, all evidence cited up to this point is equally consistent with either theory, but the model described above has the advantage of avoiding the paradox of ERP. For if this model should be correct, there will be no precise correlation of an arbitrary component of the angular momentum of each particle in every individual case, and our decision to choose a certain direction for measuring the spin of particle A will have no influence whatever on the state of particle B [since the wave function is just the product (2)].

[4] W. H. Furry, Phys. Rev. 49, 393, 476 (1936).

In Sec. 3, we shall describe and analyze an actual experiment which shows that the interpretation proposed above for the paradox of ERP is untenable. This experiment shows that we cannot avoid the paradox by assuming a breakdown of the quantum theory when particles are far apart and do not interact. For the case of the measurement of the spin of the two atoms which originally formed a molecule of total spin zero, the analogous result would be that there is definitely a precise correlation of the value of any component of the spin of atom A that we choose to measure with the same component of the spin of atom B, even in each individual measurement.

With this fact in mind, we return to the problem of interpreting the hypothetical experiment of ERP. Clearly for this case, we can no longer retain the notion that a definite value of a given variable is essentially realized through interaction with an appropriate apparatus, and that the indeterminacy principle represents only an uncontrollable fluctuation in complementary variables brought about by a disturbance originating in the apparatus.

Two general types of solutions have been offered for this problem.

First of all, Bohr[5] has proposed that the observing apparatus plus what is observed form a single indivisible combined system not capable at the quantum-mechanical level of being analyzed correctly into separate and distinct parts. Each particular kind of apparatus then forms with an electron for example, a different kind of combined system, not subject to comparison in detail with the system formed by the electron and some other kind of apparatus. Bohr then showed that one can consistently regard the quantum theory as nothing more than a means of calculating the probability of every observable result that can come out of the operation of all possible combined systems of different kinds of measuring apparatus with different kinds of entities that are to be observed. This notion is to be applied just as much to the observation of single-particle systems as to that of many-particle systems. Thus, our inability in principle to analyze in detail the motions of the spins of our two atoms is not basically different from our inability in principle to analyze in detail the motion of a single electron in an atom. In all cases we can only accept the total result that comes out of a measurement and calculate its probability.

It is clear that in Bohr's point of view, no paradox can arise in the hypothetical experiment of ERP. For the system of two atoms plus the apparatus which is used to observe their spins is, in any case, basically inseparable and unanalyzable, so that the questions of how the correlations come about simply has no meaning. We can show that there is no inconsistency in the quantum-mechanical conclusion that such correlations exist, but there is, in this point of view, no way even to raise the question of what is their origin.

The second general kind of idea which has been proposed for understanding the meaning of the paradox of ERP is along the lines of suggesting a deeper exploration of the quantum theory as a whole. In this kind of explanation, we agree with Bohr in treating the system consisting of apparatus plus what is observed as a single combined system; but we differ, in that we suppose that this combined system is at least conceptually analyzable into components which satisfy appropriate laws. Two possible ways of doing this have been suggested.

First, there is the so-called causal interpretation of the quantum theory.[6] This utilizes the idea already mentioned in Sec. 1 of a hidden interaction between distant particles. The hidden interaction is a new kind of so-called "quantum potential" which implies the possibility of a connection between distant particles even when their classical interaction potential is zero. It must be admitted, however, that this quantum potential seems rather artificial in form, besides being subject to the criticism of Sec. 1 that it implies instantaneous interactions between distant particles, so that it is not consistent with the theory of relativity.

Secondly, there has been developed a further new explanation of the quantum theory in terms of a deeper subquantum-mechanical level. The laws of this lower level are different from those of the quantum theory, but approach these latter laws as an approximation, much as the laws of atomic physics approach those of macroscopic physics when many atoms are involved. Explanations of this kind will be published later.[7] It will be seen with the aid of this theory that the paradox can be understood in a perfectly rational way, in terms of a new notion of coordinated fluctuations arising in the subquantum-mechanical level.

In sum, then, the quantum theory of the many-body problem implies the possibility of a rather strange kind of correlation in the properties of distant things. As we shall see in the next section, experiments proving the presence of this kind of correlation already exist. Any attempt to interpret the quantum mechanics and to understand its meaning must therefore take such correlations into account.

3. EXPERIMENT VERIFYING THE PARADOX OF ERP

While the paradox of ERP is most clearly expressed in terms of the correlations of spins of a pair of atoms, it is at present practicable to test it experimentally only in the study of the polarization properties of correlated photons. Such photons are produced in the annihilation radiation of a positron-electron pair. In

[5] N. Bohr, Phys. Rev. **48**, 696 (1935); also Chap. 7 in *Albert Einstein, Philosopher Scientist*, edited by P. A. Schilpp (The Library of Living Philosophers, Inc., Evanston, 1949).

[6] D. Bohm, Phys. Rev. **85**, 166 (1952); **85**, 180 (1952).

[7] A general discussion of this problem is given in D. Bohm, *Causality and Chance in Modern Physics* (Routledg and Kegan-Paul, Ltd., London, 1957), see Chaps. III and IV.

this process, two photons are given off simultaneously, with opposite momentum $|p|=\hbar k$ (where k is the wave number). As a simple calculation based on the quantum theory of radiation shows,[8] each photon is always emitted in a state of polarization orthogonal to that of the other, no matter what may be the choice of axes with respect to which the state of polarization is expressed.[9]

The most general state for a photon of wave number k directed along the positive z axis is

$$\psi=rC_k^x\psi_0+sC_k^y\psi_0. \tag{3}$$

Here ψ_0 is the ground state of the radiation field, the C's are creation operators for photons polarized respectively along the x and y axes, and the amplitude factors r, s are normalized so that

$$|r|^2+|s|^2=1. \tag{4}$$

For circular polarization $r=2^{-\frac{1}{2}}$, $s=\pm 2^{-\frac{1}{2}}i$, where the $\pm$ sign is chosen in accordance with whether the polarization is right-handed or left-handed, ψ_+ or ψ_-. For a linear polarization along a direction n that makes an angle α with the x axis, we have $r=\cos\alpha$, $s=\sin\alpha$. Let a beam so polarized be analyzed by an apparatus that measures polarization along the x and y axes. The polarization will be found to lie in the x direction with the probability $\cos^2\alpha$, and in the y direction with the probability $\sin^2\alpha$.

This result has certain essential analogies to that of the spin measurement discussed in the previous section. For in both cases, we have a system that can be in one of two possible but mutually exclusive states. For the spin, these possibilities correspond to positive or negative spin in any chosen direction; and for the photon, they correspond to the two perpendicular directions in which the radiation oscillator can be excited. In both cases, when we analyze the wave function in terms of eigenfunctions corresponding to definite properties in some direction different from the original, we obtain a statistical fluctuation in the properties of interest.

Let us now go on to the problem of the two photons moving in opposite directions. For this case, we define the creation operators C_1^x, and C_1^y for the photon moving in the direction $+k$ and C_2^x and C_2^y for the photon moving in the opposite direction. The radiation field then has four possible wave functions

$$\psi_1=C_1^xC_2^y\psi_0, \qquad \psi_2=C_1^yC_2^x\psi_0, \tag{5a}$$

$$\psi_3=C_1^xC_2^x\psi_0, \qquad \psi_4=C_1^yC_2^y\psi_0. \tag{5b}$$

The wave functions ψ_1 and ψ_2 represent states of the combined system in which each photon is excited in a direction orthogonal to that of the other, while ψ_3 and ψ_4 represent states of the combined system in which each photon is excited in the same direction as that of the other. These relationships will, however, be valid only for the particular system of axes xy that has been chosen for the representation of the eigenstates of the excitation energy of a single photon.

If the polarization along another set of axes $(x'y')$ is measured, one will not in general obtain the same correlations in the directions of excitation of the photons that was obtained in the original set of axes. To show what actually happens for this case, we must express the wave functions of (5a) and (5b) in a rotated system of axes. We obtain

$$\begin{aligned}\psi_1&=(C_1^{x'}\cos\alpha+C_1^{y'}\sin\alpha)(-C_2^{x'}\sin\alpha+C_2^{y'}\cos\alpha)\psi_0\\ &=-\sin\alpha\cos\alpha\psi_3'+\sin\alpha\cos\alpha\psi_4'+\cos^2\alpha\psi_1'-\sin^2\alpha\psi_2'.\end{aligned} \tag{6}$$

with similar expression for ψ_2, ψ_3, and ψ_4 [for example, ψ_2 is obtained by interchanging the indices 1 and 2 in Eq. (6)].

It is clear from the above equation that in a rotated system of axes, the wave function ψ_1 no longer represents (as it did in the original system) a state in which the two photons have orthogonal directions of excitation. Rather, we see that it is possible for these directions either to be orthogonal or parallel.

As we pointed out in the beginning of this section, however, the correct wave function for the experiment under consideration must be such that the two photons are excited orthogonally, no matter what the choice of xy axes is. It is well known that such a function is obtained by taking a suitable linear combination of our starting functions. In this case, the correct linear combination is

$$\phi_1=\frac{1}{\sqrt{2}}(\psi_1-\psi_2)=\frac{1}{\sqrt{2}}(C_1^xC_2^y-C_1^yC_2^x)\psi_0. \tag{7}$$

The above function still evidently represents orthogonal directions of excitation for both photons. To see that this property holds in every coordinate system $(x'y')$, we need merely express ϕ_1, in a rotated system of axes. We obtain, after a simple calculation,

$$\phi_1=\frac{1}{\sqrt{2}}(C_1^{x'}C_2^{y'}-C_1^{y'}C_2^{x'})\psi_0. \tag{8}$$

Thus, the function, ϕ_1, has the required property, and Eq. (8) therefore constitutes the correct wave function for this case.

The other possible linear combination of ψ_1 and ψ_2 is the symmetric one

$$\phi_2=\frac{1}{\sqrt{2}}(\psi_1+\psi_2)=\frac{1}{\sqrt{2}}(C_1^xC_2^y+C_1^yC_2^x)\psi_0. \tag{9}$$

As is well known, ϕ_1 and ϕ_2 are not mixed in a rotation, because the rotation operates symmetrically on the

[8] See W. Heitler, *Quantum Theory of Radiation* (Oxford University Press, Oxford, 1954), third edition, p. 269.

[9] Snyder, Pasternack, and Hornbostel, Phys. Rev. **63**, 440 (1948).

wave functions of each photon. However, a similar calculation shows that on rotations, ϕ_2 leads to a linear combination of ϕ_2', ψ_3', and ψ_4'; so that a pair of photons in a state corresponding to ϕ_2 will not have orthogonal excitations in another system of axes $(x'y')$.

The wave function (7) for a pair of photons evidently resembles the wave function (1) for the spins of a pair of electrons. In both cases, we form a special linear combination of product wave functions, which guarantees that the two particles will be in opposite states, *in relation to a group of rotated coordinate frames*. In both cases, then, we have essentially the same puzzling kind of correlations in the properties of distant particles, in which the property of any one photon that is definite is determined by a measurement on a far-away photon. Thus, the paradox of ERP can equally well be tested by polarization properties of pairs of photons.

As in the case of spin, the definite phase relations with which ψ_1 and ψ_2 are combined lead, not only to correlations of the type described, but also to detailed conservation of the total angular momentum, for each individual case.

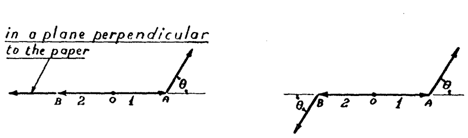
The experiment that we shall discuss here is aimed at testing whether there really is a correlation in polarization directions of the type described in the foregoing. The ideal way to test this point would be to measure the polarizations of each member of a statistical ensemble of pairs of photons produced by positron-electron annihilation; and to see whether the polarizations are always perpendicular in every system of axes, as predicted by the theory. But this is not yet possible in practice. Nevertheless, there has been done an experiment which, as we shall see, tests essentially for this point, but in a more indirect way. This experiment[10] consists in measuring the relative rate, R, of coincidences in the scattering of the two photons through some angle, θ, for the following two cases:

(1) When the planes π_1, and π_2 formed by the lines of motion of the scattered quantum and the original direction are perpendicular ($\varphi=90°$, where φ is the angle between the planes).

(2) When the planes are parallel ($\varphi=0$).

These cases are illustrated in Fig. 1. The photons originate at the point, 0.

In the first case, photon 1 is scattered by an electron in a block of solid matter at the point A, through some angle, θ which we take to be in the plane of the paper. Photon 2 is similarly scattered at the point, B, through the same angle, θ, but in a plane perpendicular to that of the paper. In the measurement, θ is fixed, and one counts the coincidences of photons. In case (2), the experiment is the same, except that both photons are scattered in the plane of the paper.



Case 1 Case 2

FIG. 1. Schematic representation of apparatus for observing correlations between photons.

We shall then consider the computation of the ratio, R, on the basis of two different hypotheses:

(A) The usual quantum theory is correct in all cases, so that as we have shown before the wave function is given by the antisymmetric combination $\phi_1=(1/\sqrt{2})\times(\psi_1-\psi_2)$. [See Eq. (7).]

(B) The usual quantum theory is correct only when the wave function of the photons overlap (or else when the photons interact appreciably). When the photons have separated sufficiently (as in the case of the experiment that we are considering) we suppose that the wave function is no longer a superposition such as (7), having definite phase relations of its components, which imply, as we have seen, the ERP type of correlations and a definite total angular momentum. Instead, we shall suppose that each photon goes into some definite state of polarization, which is definitely related to that of the other; and in order to obtain symmetry in the final statistical results, we shall suppose, wherever necessary, that there is a uniform statistical distribution over any direction that may be favored in each individual case. In order to bring out the consequences of such a hypothesis for the experiment under consideration, we shall consider here two extreme cases:

(1) Each photon becomes circularly polarized about its direction of motion, but the two photons are oppositely polarized.

(2) Each photon goes into a state of linear polarization in some direction, while the other goes into a state of perpendicular polarization. Over many cases, one obtains the same probability for an arbitrary direction of polarization of any one of the photons. (It is evident that for the hypothesis B, the combined angular momentum would not be conserved in individual processes, but as we have indicated in the previous section, the fact that it is conserved on the average would be all that is needed to fit the experiments that are available to date.)

To carry out the calculations needed to compare these suggested theories with the experiment, we shall cite the scattering cross section of a single photon from an electron; first for the case in which its initial polarization direction is parallel to the plane π (containing its initial direction of motion and its direction after scattering) and secondly, for the case in which its initial polarization direction is perpendicular to π. According to the Klein-Nishina formula, these probabilities[11] (which have been summed over the final polarization directions of the photon, and the final spin

[10] For a more detailed discussion, see C. S. Wu, Phys. Rev. **77**, 136 (1950).

[11] See W. Heitler, reference 8, p. 217.

TABLE I. Relative probability of coincidences in the Compton scattering of annihilation photons for the two geometries of Fig. 1.

Hypothesis	Scattering probability $d\Sigma$ divided by $(r_0^4/8)(d\Omega)^2(k^4/k_0^4)$ π_1 parallel π_2	π_1 perpendicular π_2	Ratio $R=d\Sigma_\perp/d\Sigma_\parallel$ For ideal angle 82°	Average for experimental solid angle
A (standard)	$2\gamma(\gamma-2\sin^2\theta)$	$(\gamma-2\sin^2\theta)^2+\gamma^2$	2.85	2.00
B_1 (product of states of opposite circular polarization)[a]	$d\Sigma$ unaffected by orientation of π_1 relative to π_2		1.00	1.00
B_2 (product of states of perpendicular linear polarization, randomized directions)	$(2\gamma^2-4\gamma\sin^2\theta+\sin^4\theta)$	$(2\gamma^2-4\gamma^2\sin^2\theta+3\sin^4\theta)$	<2	~ 1.5
B_{inter} (intermediate case of elliptic polarization)	Intermediate between B_1 and B_2		<2	<1.5
Observation (reference 10)				2.04 ± 0.08

[a] Equal probability for $\chi_1=\psi_+(1)\psi_-(2)$ and for $\chi_2=\psi_-(1)\psi_+(2)$. The correct wave function according to standard quantum theory is $2^{-\frac{1}{2}}(\chi_1-\chi_2)$.

directions of the electron, and averaged over the initial spin direction), are, respectively,

$$d\sigma_1=\tfrac{1}{2}r_0^2 d\Omega(k^2/k_0^2)(\gamma-2\sin^2\theta), \tag{10a}$$

$$d\sigma_2=\tfrac{1}{2}r_0^2 d\Omega(k^2/k_0^2)\gamma, \tag{10b}$$

where $\gamma=(k_0/k)+(k/k_0)$, k_0 is the wave number of the incident photon, k that of the final photon, r_0 is the classical electronic radius, and $d\Omega$ is the element of solid angle.

We can now apply these results to our problem, involving two photons going in opposite directions. For the general case, the wave function before scattering must, in such a problem, be a linear combination of the four possibilities,

$$\psi=\sum_{i=1}^{4} a_i\psi_i, \tag{11}$$

where the ψ_i are defined in Eqs. (5a) and (5b).

It is evident then that the scattering cross section for two photons will depend on the a_i. In general, we would expect that the probability of such scattering would contain cross terms such as a_ia_j where $i\neq j$. For the special case of the experiment that we are considering (i.e., the planes π_1 and π_2 are either parallel or perpendicular), it can be shown,[9] however, that if we choose the x axis in the plane π associated with either one of the photons (and the y axis perpendicular to this plane), then all such cross terms will drop out from the expression for the probability of scattering. With this choice of axes, then, we can compute the probability of scattering of two photons for an arbitrary state of the system in Eq. (11) by computing it separately for the four cases, ψ_i and multiplying the result of each computation by the probability of this case, $(|a_i|^2)$. For each case, ψ_i, however, this probability reduces to just the product of the probabilities of scattering of the single photons.

Calculating the scattering probabilities as just outlined, we obtain the results summarized in Table I.

The results in Table I show that this experiment is explained adequately by the current quantum theory which implies distant correlations, of the type leading to the paradox of ERP, but not by any reasonable hypotheses implying a breakdown of the quantum theory that could avoid the paradox of ERP.

APPENDIX. TREATMENT OF ERP PARADOX FOR CONTINUOUS VARIABLES

In this Appendix, we shall discuss the paradox of ERP, as applied to continuous variables. We shall see that with such variables, it is very difficult to obtain a clear experimental test for this paradox, thus showing that the best way of making such a test is with discrete quantities, such as spin of electrons or polarization of photons.

We may take as a typical example a case similar to one already discussed by Furry,[4] namely, an experiment in which one particle is scattered on another. To avoid questions of identity, we suppose that the two particles are different.

If particle 1 is initially at rest, and particle 2 has initially the definite momentum, P, the wave function for the system is then[12]

$$\exp\left(i\mathbf{P}\cdot\frac{\mathbf{r}_1+\mathbf{r}_2}{2}\right)F(\mathbf{r}_1-\mathbf{r}_2), \tag{12}$$

where

$$F(\mathbf{r})=\exp\left(i\frac{\mathbf{P}\cdot\mathbf{r}}{\hbar}\right)+\frac{e^{ikr}}{r}g(\theta).$$

In principle, we can measure $\mathbf{P}_2$ after scattering, and we know from conservation of momentum that $\mathbf{P}_1=\mathbf{P}-\mathbf{P}_2$. But we also have the alternative possibility of using a suitable lens to bring particle No. 2 to a focal point; and from this, we can deduce where the point of scattering was, and therefore where particle 1 was at the time of scattering. Thus, by measurements made solely on complementary properties of particle 2, we can determine the corresponding properties of particle 1, without any interaction between the particles or between the observing apparatus and particle 1.

In order to avoid for this case the paradox of ERP in a manner analogous to what was suggested for spin and polarization in Secs. 2 and 3 respectively, we could assume that after the particles which have scattered on

[12] D. Bohm, reference 3, Chap. 21, Sec. 24.

each other separate sufficiently, the wave function breaks up into narrow wave packets. There would be a small indeterminacy, $\Delta\theta$ in the angles of these packets and a corresponding indeterminacy, ΔP_θ in the momentum in the θ direction. The wave function would be $\psi(r_1,r_2)=f_A(r_1)f_B(r_2)$ where f_A and f_B represent packets whose centers move in accordance with the assumptions given above.

We then assume a statistical distribution in the mean directions of these packets, weighted in such a way as to give the usual probability of scattering as a function of angle. Thus, from measurements of the scattering cross section as a function of angle, one could not distinguish between the theory and the usual quantum theory.

In such a statistical distribution over pencils of directions, the total momentum is not conserved exactly [as is evident from the Fourier analysis of a function, such as $f(\mathbf{r}_1)f(\mathbf{r}_2)$]. But because we can choose $\Delta\theta$ small compared with macroscopic dimensions and yet so large that ΔP_θ is negligible, this very small failure of detailed conservation of momentum would be too small to have been detected in experiments that have been possible to date (of course momentum would still be conserved on the average). Thus, to test for the paradox of ERP in this case, one needs extremely accurate measurements of the momentum of both particles before and after scattering.

At first sight, it might seem that one could distinguish between the two theories by trying to demonstrate interference of the scattered wave of a single particle, in order to show that the wave covers the whole range of angles without being broken into partial waves of width $\Delta\theta$. But this is not possible, because, as a simple calculation shows, interference phenomena will cancel out for a single particle (i.e., when one averages over the coordinates of the other particle). *Interference in space can be obtained only if the positions of both particles are observed with great precision;* and as in the case of testing the detailed conservation of momentum, the experiments available to date are not accurate enough to distinguish between the two theories.

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Parity Nonconservation and the Group-Space of the Proper Lorentz Group

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As a means of incorporating isotopic spin into the foundations of the theory of spin $\frac{1}{2}$ particles, a slight modification of the free-particle Dirac equations has previously been proposed which follows from a consideration of the group-space of the proper Lorentz group. Further considerations of the group-space suggest two additional modifications which may contribute, in a mutually complementary way, toward establishing a new symmetry principle applicable to parity-nonconserving interactions.

IN a recent note,[1] there was indicated a natural and irreducible way of incorporating isotopic spin into the foundations of the theory of spin $\frac{1}{2}$ particles. For that purpose, the solutions $\psi_\pm$ of the Weyl form of the free-particle Dirac equations,[2]

$$\Box_\mp\psi_\pm+i(mc/\hbar)\psi_\mp=0, \tag{1}$$

where

$$\Box_\mp=e_0\partial/\partial x^0\mp i(e_1\partial/\partial x^1+e_2\partial/\partial x^2+e_3\partial/\partial x^3), \tag{2}$$

and e_0, e_1, e_2, e_3 are the quaternion units in a 2×2 matrix representation, were considered to be a pair of general complex quaternions, rather than the usual pair of 2-component spinors. The resulting 8-component spinor wave function $\psi_\pm$ was then taken to represent the superposition of two different states of isotopic spin $\frac{1}{2}$, with the respective 4-component spinor wave functions

$$\psi_{\pm+}=\psi_\pm(e_0+ie_3)/2, \tag{3a}$$

$$\psi_{\pm-}=\psi_\pm(e_0-ie_3)/2. \tag{3b}$$

The present note deals with two further modifications of Eqs. (1), suggested, as the previous one was, by considerations of the *group-space* of the proper Lorentz group.[3] These modifications are: (A) that the original 2×2 matrix representation of the quaternion units e_0, e_1, e_2, e_3 be replaced by a regular 4×4 matrix representation[4]; and (B) that the differential operators $\Box_\mp$ be replaced by $e^{\mp2i\mu}\Box_\mp$, where μ is a real parameter.[5]

Through modification (A), which effects the simplification[6] $\Box_\pm{}^C=\Box_\mp$, the mutual symmetry of the operators $e^{\mp2i\mu}\Box_\mp$ becomes subsumed under the opera-

[1] E. J. Schremp, Phys. Rev. **99**, 1603 (1955).

[2] H. Weyl, *The Theory of Groups and Quantum Mechanics* (E. P. Dutton and Company, Inc., New York, 1931), p. 213, Eqs. (5.6).

[3] E. J. Schremp, Bull. Am. Phys. Soc. Ser. II, **2**, 191 (1957).

[4] E. J. Schremp, Naval Research Laboratory Quarterly Report, January 1956 (unpublished), pp. 6–13.

[5] E. J. Schremp, Naval Research Laboratory Quarterly Reports, July 1956 (unpublished), pp. 17–20, and January 1957 (unpublished), pp. 9–17. In these reports, as also in the present paper, μ is understood to be a function of the space-time coordinates x^0, x^1, x^2, x^3. The special case μ=constant is equivalent to a proposal just recently made, with a similar physical objective in view, by K. Nishijima, Nuovo cimento **5**, 1349 (1957).

[6] In taking the complex conjugate, denoted by the superscript C, it is here understood that the quaternion units e_0, e_1, e_2, e_3 are real, in accordance with modification (A).