since the data are also in fair agreement with the optical model.

V. ACKNOWLEDGMENTS

We wish to thank Professor A. C. Helmholz for his encouragement and advice throughout the course of this experiment. The assistance in taking data given by Mr. John Caris and Mr. Walton Perkins is greatly appreciated. Finally, we would like to thank Mr. Rudin Johnson and the Synchrotron Crew for their help and cooperation in providing us with high beam intensities during the many phases of this experiment.

PHYSICAL REVIEW VOLUME 108, NUMBER 4 NOVEMBER 15, 1957

Bubble Density in a Propane Bubble Chamber*

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(Received August 6, 1957)

Measurements of the number of bubbles per centimeter in a propane bubble chamber are described. A method insensitive to the effects which cause inefficiency in direct bubble counting was used, based on the distribution of bubble spacings. The number of bubbles per centimeter measured by using this method is consistent with the rate of delta-ray formation.

THE results of counting the number of bubbles per centimeter on tracks in a propane bubble chamber of particles having known velocities were reported by Glaser *et al.*[1] We have observed that direct bubble counting underestimates the bubble density because of losses due to limited optical resolution and possible bubble coalescence. We have measured bubble densities for some of the tracks used by Glaser *et al.*,[1] using, however, a method which is insensitive to the principal effects which cause inefficiency in direct bubble counting.[2]

If the bubbles are distributed at random (i.e., giving a Poisson distribution) along the track, the distribution of the lengths of the spaces between bubbles is given by

$$f(x)=me^{-mx},$$

where m is the average number of bubbles per unit length. On a semilogarithmic plot, the slope of the distribution gives m. A typical example of an experimental distribution is shown in Fig. 1. The points follow a straight line down to a certain spacing, then fall off. The optical resolution and the bubble diameter of the tracks chosen are both approximately equal to the spacing at which the distribution fails to be random, so coalescence of the images or of the bubbles themselves is presumably responsible for the failure to observe small spacings.

The slope of the experimental distributions for large spacings was taken to give m. The temperature of the chamber varied appreciably in runs at different momenta. The tracks at minimum ionization (pions) which were in each set of pictures, were used to normalize the dense tracks (protons) to the temperature of one set of pictures by multiplying by the ratios m for pions.

The results, for two temperatures and several momenta, are shown in Fig. 2, along with the results of the

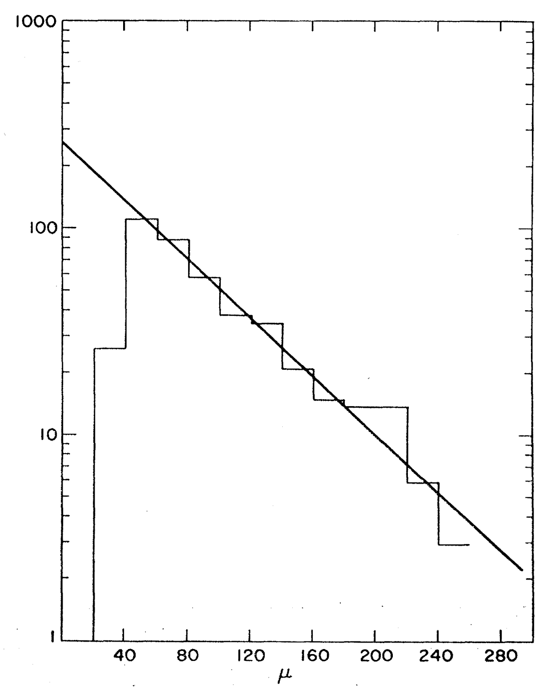

FIG 1. The distribution of the number of bubble spacings plotted against the length of the spacing on the film, for a track with ionization 2.3 times minimum in propane at 55.5°C.

* Research carried out under the auspices of the U. S. Atomic Energy Commission.
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[1] Glaser, Rahm, and Dodd, Phys. Rev. 102, 1653 (1956).
[2] Willis, Fowler, and Rahm, Bull. Am. Phys. Soc. Ser. II, 2, 6 (1957).

direct bubble-counting from Glaser *et al.*[1] The bubble densities obtained by this method are higher than those obtained by direct bubble-counting by almost a factor of three for very dense tracks.

The values of m obtained from the spacing distribution fit $m=C(T)/\beta^2$, which is proportional to the number of delta rays per centimeter, if those secondary electrons that are energetic enough to leave a recognizable track are excluded. This dependence has been predicted by Glaser *et al.*[1] and by Askarian.[3] In order to fit the present data it has not been necessary to introduce a term which is independent of the velocity of the incoming particle.[1,4] Only a portion of the tracks analyzed in reference 1 has been remeasured since further measurements did not seem indicated in view of the lack of reproducibility of the chamber sensitivity.

The $1/\beta^2$ dependence is obtained on the assumption that bubbles are formed only at the end of stopping delta rays. Another possibility is that, as in the cloud chamber, the number of bubbles per centimeter is proportional to the probable specific ionization (the total average ionization produced by the primary particle and all its secondary electrons having an energy less than η). This quantity is given by[5]

$$\left(\frac{dE}{dx}\right)_{Q<\eta}=\frac{2\pi n(Ze^2)^2}{mc^2\beta^2}\left[\ln\left(\frac{2mc^2\beta^2\eta}{(1-\beta^2)I^2}\right)-\beta^2\right],$$

where I is the average ionization potential, about 50 ev in propane. Delta rays in propane having energies $\eta>70$ kev would leave tracks visibly displaced from the primary (that is, of length greater than about 200 microns). Thus the expression in square brackets is

$$\left[\ln\left(2.8\times10^7\frac{\beta^2}{1-\beta^2}\right)-\beta^2\right],$$

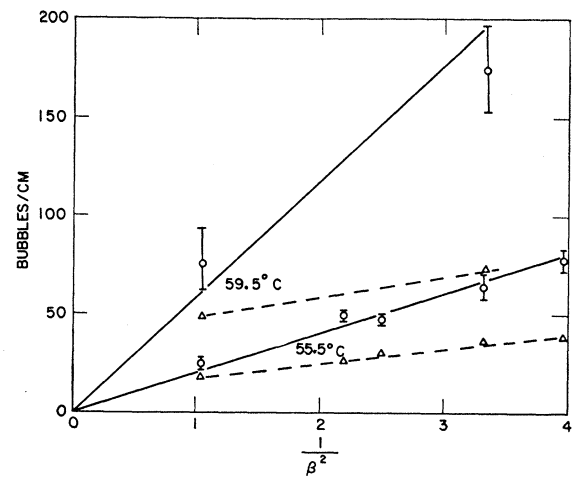

FIG. 2. Number of bubbles per cm along the track as a function of $1/\beta^2$. Results are plotted for two different temperatures. The circles are the results of the present measurements and the triangles those of Glaser *et al.*[1] Both measurements are on the same set of pictures.

which is a slowly varying function of β.

In fact, in the region of interest,

$$\left(\frac{dE}{dx}\right)_{Q<\eta}\simeq\frac{C}{\beta^{1.83}},$$

which is difficult to distinguish from $1/\beta^2$. Consequently, with the available data it is not possible to choose between the two hypotheses.

Blinov, Krestnikov, and Lomanov[6] have reported on a similar experiment using a chamber expanded to a controlled pressure, carrying out measurements to bubble densities of 80 times minimum. They used the method described here for intermediate densities and a gap-counting method at very high densities. They report a dependence of $1/\beta^2$ for the bubble density as a function of velocity.

[3] G. A. Askarian, J. Exptl. Theoret. Phys. U.S.S.R. **30**, 610 (1956) [translation: Soviet Phys. JETP **3**, 4 (1956)].
[4] Bertanza, Martelli, and Tallini, Nuovo cimento **5**, 940 (1957).
[5] E. A. Uehling, in *Annual Review of Nuclear Science* (Annual Reviews, Inc., Stanford, 1954), Vol. 4, pp. 315–350; B. Rossi, *High-Energy Particles* (Prentice-Hall, Inc., New York, 1952).

[6] Blinov, Krestnikov, and Lomanov, J Exptl. Theoret. Phys. U.S.S.R. **31**, 762 (1956) [translation: Soviet Phys. JETP **4**, 661 (1957)].