

Z Dependence of Positive Pion Production by Photons*

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The relative yield of positive pions from H, C, Al, Cu, Ag, and Pb produced by 335-Mev bremsstrahlung has been measured with a counter telescope at 5 angles to the beam line ranging from 45° to 150° and at 9 meson energies extending from 12 Mev to 125 Mev. The pion production cross sections at all angles and at all energies of 65 Mev or lower follow approximately a $Z/A^{\frac{1}{2}}$ dependence. Fair agreement with the optical model is obtained at the lower energies, provided account is taken of effects of the Coulomb potential and the potential arising from the pion-nucleon interaction. In the intermediate region from 22.5 to 65 Mev, the yields from the heavy nuclei are somewhat lower than predicted. At pion energies of 95 and 125 Mev, where the production cross sections follow more nearly a Z dependence, it is possible to get moderately good agreement with theory by considering the effects of the Coulomb potential on the production near the cutoff energy.

I. INTRODUCTION

THE efficiency of photoproduction of pions from complex nuclei has been shown to exhibit approximately an $A^{\frac{1}{2}}$ dependence.¹⁻⁶ This result has been explained in terms of an optical model in which the mean free path for a meson interaction is so short that mesons effectively come only from the surface of the nucleus. Such a short mean free path for meson interaction in nuclear matter, of the order of a meson Compton wavelength for pions of energy around 60 Mev, has been confirmed by meson-nucleon and meson-nucleus scattering experiments.⁷ But these same experiments predict a meson mean free path of at least 10 meson Compton wavelengths for pions of energy less than 30 Mev. At the lower energies, the optical model for the photoproduction efficiency would predict a yield per effective nucleon that would be nearly independent of A .

The A dependence of π^+ photomeson production has previously been measured only for pions of energy greater than 33 Mev and then only at one angle in each case. Furthermore, no data exist on the A dependence of the yield of positive pions having an energy greater than the cutoff value for production from hydrogen. In this case, the internal momentum distribution and

the Coulomb potential should have pronounced effects. It was therefore felt desirable to extend the measurements to lower and higher energies and other angles. In this experiment, C, CH₂, Al, Cu, Ag, and Pb targets were bombarded at the Berkeley 335-Mev synchrotron and positive pions of energies ranging from 12 Mev to 125 Mev were observed at a laboratory angle of 135°. Yields of 46- and 65-Mev pions were measured at laboratory angles varying from 45° to 150°.

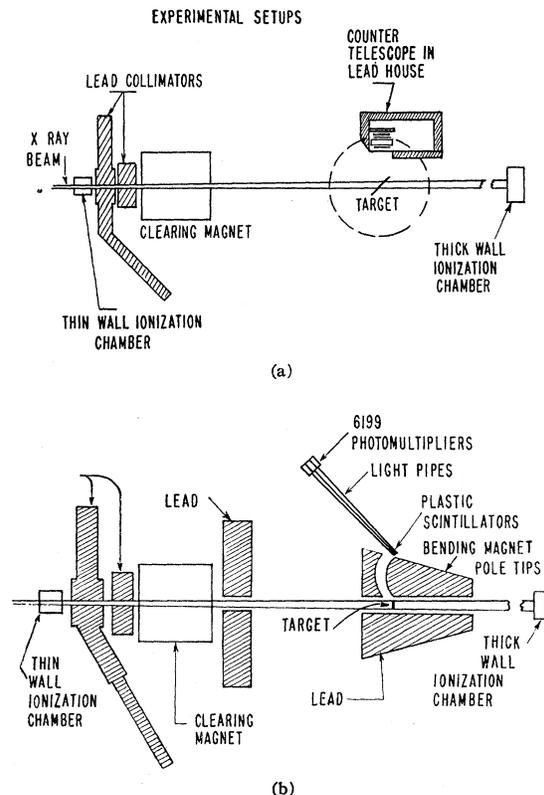


FIG. 1. Experimental layout showing relative positions of target, detector, and shielding, (a) without and (b) with a magnetic field for deflecting the pions.

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¹ R. F. Mozley, Phys. Rev. **80**, 493 (1950).

² R. M. Littauer and D. Walker, Phys. Rev. **86**, 838 (1952).

³ Panofsky, Steinberger, and Steller, Phys. Rev. **86**, 180 (1952).

⁴ Williams, Crowe, and Friedman, Phys. Rev. **105**, 1840 (1957).

⁵ J. D. Anderson, University of California Radiation Laboratory Report UCRL-3426 (unpublished).

⁶ Belousov, Popova, Semashko, Shitov, Tamm, Veksler, and Iagudina, *Proceedings of the CERN Symposium, Geneva, 1956* (European Organization of Nuclear Research, Geneva, 1956), Vol. II, p. 288.

⁷ G. Bernardini and J. Levy, Phys. Rev. **84**, 610 (1951); A. M. Shapiro, Phys. Rev. **84**, 1063 (1951); Byfield, Kessler, and Lederman, Phys. Rev. **86**, 17 (1952); J. F. Tracy, Phys. Rev. **91**, 960 (1953); F. H. Tenney and J. Tinlot, Phys. Rev. **92**, 974 (1953); D. Stork, Phys. Rev. **93**, 868 (1954).

TABLE I. Relative positive-pion yields per proton in the nucleus at 135° to the incident beam. At each pion energy the relative yields have been normalized to a carbon value of 1.00.

Pion energy (Mev)	Target nucleus					
	H	C	Al	Cu	Ag	Pb
12 ±6	2.61±0.76	1.00±0.09	0.77±0.12	0.68±0.10	0.45±0.08	0.31±0.06
16.5±4.5	2.46±0.36	1.00±0.06	0.98±0.09	0.76±0.08	0.45±0.05	0.28±0.04
22.5±6.0	1.26±0.27	1.00±0.04	0.77±0.04	0.67±0.04	0.51±0.06	0.30±0.04
30 ±4.5		1.00±0.05	0.92±0.05	0.79±0.05	0.60±0.06	0.31±0.09
36 ±4	1.73±0.36	1.00±0.03	0.89±0.03	0.74±0.03	0.66±0.03	0.43±0.06
46 ±4	1.92±0.16	1.00±0.04	0.89±0.04	0.76±0.04	0.65±0.04	0.49±0.03
65 ±4	1.95±0.42	1.00±0.03	0.86±0.03	0.64±0.03	0.61±0.03	0.42±0.04
95 ±3		1.00±0.08		1.03±0.09	1.15±0.10	0.80±0.12
125 ±3		1.00±0.14		1.75±0.30	1.96±0.28	1.70±0.30

II. EXPERIMENTAL METHOD

To minimize accidental counts, the spread-out synchrotron beam was used so that the duration of each pulse was about 4 msec. The beam was monitored with both a thin-walled ionization chamber and a thick-walled copper ionization chamber of the type used at Cornell, as shown in the experimental layout diagrams of Figs. 1(a) and 1(b). Positive pions emitted from the target were detected by their characteristic $\pi-\mu$ decay in a counter telescope, consisting in its most complete form of four plastic scintillators and one Plexiglas Čerenkov counter as shown in Fig. 2. The mesons are partially identified in the telescope by a prompt coincidence 1+3-2-5, where 2 is the Plexiglas Čerenkov counter which is insensitive to pions in the energy band selected by the telescope. This energy band is determined by the thickness of absorber necessary to slow down the pions and cause them to stop and decay in the fourth counter, where the identification is completed by making a delayed coincidence with the μ pulse. The block diagram of the electronics (Fig. 2) outlines the operation of the telescope and is essentially similar to what has been described in a previous paper.⁸

The Čerenkov counter was used only for the heavier elements at the forward angles where the electron background became excessive. In order to measure the meson yields at the lower energies it was necessary to remove the Čerenkov counter. For the lowest energy points, one of the front two scintillators was also removed. The deflecting magnet shown in Fig. 1(b) served as an aid in reducing the electron background so as to permit detection of low-energy mesons.

The targets were all considerably larger in cross-sectional area than the x-ray beam which was about 1.5 inches in diameter at the target. For meson energies above 30 Mev, the targets were of thickness between 0.5 and 1.75 g/cm² in the direction of the beam. For the lower energy points, the targets were all less than 1 g/cm² thick.

⁸ Imhof, Easterday, and Perez-Mendez, Phys. Rev. **105**, 1859 (1957).

III. RESULTS

Corrections were made for the absorption of the x-ray beam in each target, the nuclear interactions of the emitted pions in each target, and the variation with target of the energy resolution of the detector. None of these corrections amounted to more than 5%. Corrections which would merely change the detection efficiency by the same factor for all targets were not considered since only the Z dependence at each meson energy was of interest. However, for measurements of the energy spectrum from carbon, it was also necessary to consider nuclear absorption and multiple Coulomb scattering of the pions in the energy degrader and the energy dependence of the detection interval. Tables I-III give the final data after the corrections were applied to the raw data. The errors shown are the standard deviations due to the counting statistics.

IV. DISCUSSION

Curves for both a $1/A^{1/2}$ and a constant dependence of the meson production efficiency per proton on the atomic mass number are shown in Fig. 3 superimposed on the data taken in this experiment at 135°. Figure 4 is a similar plot of the data taken at other angles. The pion production cross sections at all angles and at all energies of 65 Mev or lower are seen to follow approximately a $1/A^{1/2}$ dependence.

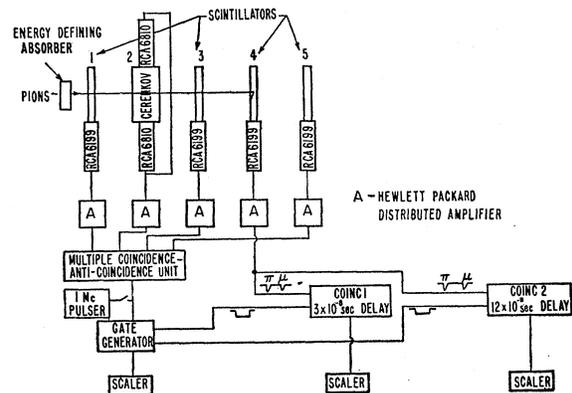


FIG. 2. Block diagram of electronic detecting apparatus.

TABLE II. Relative positive-pion yields per proton in the nucleus at 45°, 90°, 120°, 135°, and 150° to the incident beam. At each pion energy the relative yields have been normalized to a carbon value of 1.00 at 135°.

Pion energy (Mev)	Angle (deg.)	Target nucleus				
		H	C	Cu	Ag	Pb
46 ± 4	45	1.23 ± 0.19	0.75 ± 0.04	0.51 ± 0.04	0.44 ± 0.04	
	90	2.19 ± 0.24	1.05 ± 0.04	0.75 ± 0.04	0.71 ± 0.04	0.43 ± 0.07
	120	2.03 ± 0.26	1.05 ± 0.04	0.78 ± 0.04	0.59 ± 0.04	0.56 ± 0.07
	135	1.92 ± 0.16	1.00 ± 0.04	0.76 ± 0.04	0.65 ± 0.04	0.49 ± 0.03
	150	1.77 ± 0.23	0.95 ± 0.04	0.72 ± 0.05	0.67 ± 0.05	0.42 ± 0.05
65 ± 4	45	1.29 ± 0.21	0.84 ± 0.05	0.48 ± 0.04	0.41 ± 0.04	0.36 ± 0.05
	90	2.16 ± 0.24	1.16 ± 0.05	0.78 ± 0.05	0.60 ± 0.05	0.44 ± 0.04
	120	2.20 ± 0.21	1.09 ± 0.04	0.69 ± 0.04	0.62 ± 0.04	0.46 ± 0.04
	135	1.95 ± 0.42	1.00 ± 0.03	0.64 ± 0.03	0.61 ± 0.03	0.42 ± 0.04
	150	1.70 ± 0.23	0.91 ± 0.04	0.60 ± 0.04	0.60 ± 0.05	0.44 ± 0.05

These experimental results will now be compared with theoretical predictions based on the optical model. The efficiency of photomeson production from a nucleus was predicted by Brueckner, Serber, and Watson,⁹ according to the optical model, to be equal to

$$f \approx 3 \left[\frac{1}{2x} - \frac{1}{x^3} + \frac{1+x}{x^3} e^{-x} \right], \quad (1)$$

where $x = 2R/\lambda$, $R = r_0 A^{1/3}$ = radius of nucleus, and λ = mean free path for absorption of mesons in nuclear matter. In order to calculate the A dependence, it is also necessary to take into account effects of the Coulomb potential, the potential arising from the pion-nucleon interaction, the internal momentum distribution of the nucleus, and scattering of the pions inside the nucleus. Consideration will now be paid to each of these effects.

The Coulomb potential influences the A dependence of the production cross section in three different ways. Two of these arise from the fact that even though the momenta outside the nuclei of the pions observed from

the various nuclei are identical, inside the nuclei the momenta of the mesons to be detected are different owing to the presence of the Coulomb potential. The meson production cross section depends on the kinetic energy inside the nucleus of the emitted pion, $T_\pi - V_R - V_C$, where T_π is the pion kinetic energy outside the nucleus, V_R is the real part of the optical model potential, and V_C is the Coulomb potential. Therefore, the relative yield of pions of a given energy at the detectors depends on the Z value of the nucleus. For calculation, V_C was assumed to be the same throughout the nucleus as at the nuclear boundary, and V_R was taken from the work of Frank, Gammel, and Watson.¹⁰ Since their values of V_R , at the pion energies considered here, can be fitted approximately by the expression $-0.40(T_\pi - V_R - V_C)$,⁴ the pion kinetic energy inside the nucleus is $(5/3)(T_\pi - V_C)$. The experimentally observed^{11,12} energy spectrum of pions produced from carbon with a 335-Mev bremsstrahlung beam was used to calculate the change in the relative production of pions from nuclei with different values of V_C . To a first approximation the effect of changing V_C is to shift the energy scale on this spectrum, the spectrum being shifted out in energy for the heavier nuclei.¹³ At low energies, the production from the heavy elements is reduced because of this effect; at intermediate energies, there is no effect; and at high energies, the yield from the heavy elements is increased. In order to make this calculation at 95 and 125 Mev it was necessary to meas-

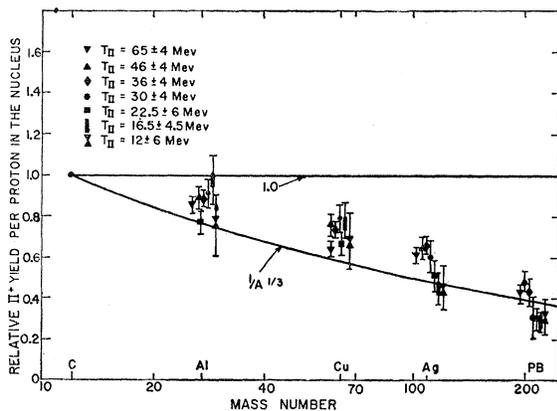


FIG. 3. Relative π^+ yields per proton in the nucleus at 135° normalized to a carbon value of 1.00. Superimposed on the data are curves for a constant production efficiency and for a $1/A^{1/3}$ production efficiency.

⁹ Brueckner, Serber, and Watson, Phys. Rev. 84, 258 (1951).

TABLE III. Relative yields of pions of various energies from carbon at 90° and 135°.

Mean pion energy (Mev)	90°	135°
45	94.2 ± 3.8	79.0 ± 2.9
65	82.4 ± 4.2	65.6 ± 3.0
85	55.1 ± 3.0	31.9 ± 2.0
105	29.9 ± 2.0	11.2 ± 1.4
125	8.6 ± 1.0	2.1 ± 0.6

¹⁰ Frank, Gammel, and Watson, Phys. Rev. 101, 891 (1956).

¹¹ Peterson, Gilbert, and White, Phys. Rev. 81, 1003 (1951).

¹² W. F. Dudziak (private communication).

¹³ T. Kinoshita, Phys. Rev. 94, 1331 (1954).

ure the pion energy spectrum from carbon in this region, as shown in Fig. 5. A second consequence of the different pion kinetic energies inside nuclei of various elements corresponding to a common energy outside is that the pion mean free paths, which are a function of the meson kinetic energy inside the nucleus, $(T_\pi - V_R - V_C)$, are different inside nuclei of different Z values. The appropriate mean free path in each case was taken from the work of Frank, Gammel, and Watson.

The third way in which the Coulomb potential influences the A dependence of the production cross section is that the production of low-energy mesons from heavy nuclei is affected by the Coulomb barrier. Transmission of the mesons through this barrier was calculated on the basis of the WKB approximation¹⁴ for S -wave mesons. Since it is possible that P , D , F , etc. wave mesons are also produced, their transmission through the centrifugal and the Coulomb barrier was calculated. A prediction of the effect of these transmission probabilities on the relative production of pions from the various nuclei requires knowledge of the fraction of pions produced in S , P , D , etc. states. A reasonable distribution in angular momenta of the pions was obtained by assuming that all pions are produced in an l state if they are produced at a position in the nucleus such that the perpendicular distance from the line of motion of the pion to the center of the nucleus r_\perp is given by $l\hbar/p \leq r_\perp < (l+1)\hbar/p$. The resulting transmission of pions through the centrifugal barrier, when averaged over the different angular momentum states, did not exhibit any pronounced dependence on the mass number. Since inclusion of pions in P , D , F , etc., states does not seem to affect the A dependence of the production very strongly, only the effect of the Coulomb barrier on the production of S -wave mesons was considered. The transmission for S -wave pions of energy greater than about 30 Mev is practically unity, so the Coulomb barrier does not influence the yields predicted for higher energy mesons.

The efficiency of pion production from a complex nucleus relative to that from a free nucleon is also influenced by the momenta of the nucleons in the nucleus.¹⁵ The presence of internal momentum results in a wide spread in the gamma energy required for production of a pion at a particular energy and angle. Depending on the gamma energy required for pion production from a free nucleon, the internal momentum in a complex nucleus can thereby cause either an increase or a decrease in the production efficiency.

In interpreting the data of this experiment, one must therefore consider the effects of any internal momentum differences between carbon, aluminum, copper, silver, and lead. To get a quantitative measure of the relative internal momentum distributions of these elements, the pion productions were measured under extreme

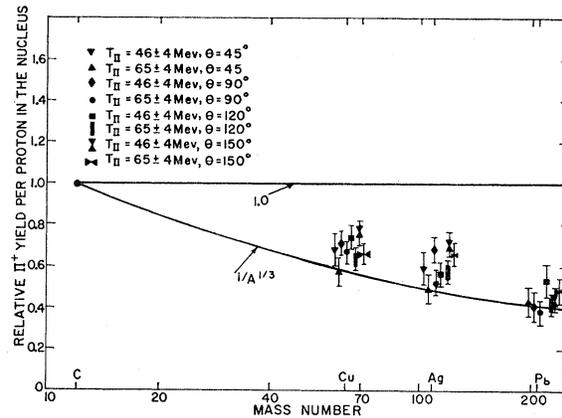


FIG. 4. Relative π^+ yields per proton in the nucleus at energies of 46 and 65 Mev and at angles of 45° , 90° , 120° , and 150° . The yields at each energy and angle are normalized to a carbon value of 1.00. Superimposed on the data are curves for a constant and for a $1/A^{1/3}$ production efficiency.

kinematical conditions. Pions of energy 125 Mev and 95 Mev were observed at 135° to the bremsstrahlung beam. The relative yields are shown in Fig. 10. No such pions could be produced from hydrogen since a 385-Mev photon would be required to produce a 95-Mev pion at this angle. For pions of this energy, the mean free path in the nucleus is known to be about $1r_0$, where $r_0 = 1.4 \times 10^{-13}$ cm. The predicted A dependence, when the shift in the energy spectrum due to the Coulomb potential is taken into account, is shown in Fig. 10. Since a fair agreement is obtained with the experimental data in the extreme cases, it is concluded that any internal momentum differences between carbon, aluminum, copper, silver, and lead should have small effect on the pion production efficiency at the lower energies.

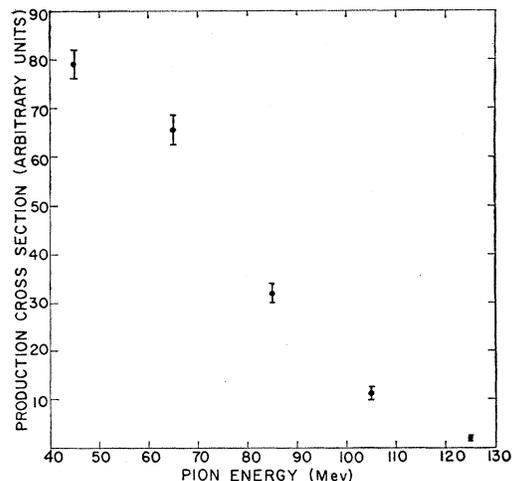


FIG. 5. Energy distribution of π^+ mesons from carbon at 135° . Only relative cross sections are given.

¹⁴ J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952), pp. 358-62.

¹⁵ M. Lax and H. Feshbach, *Phys. Rev.* **81**, 189 (1951).

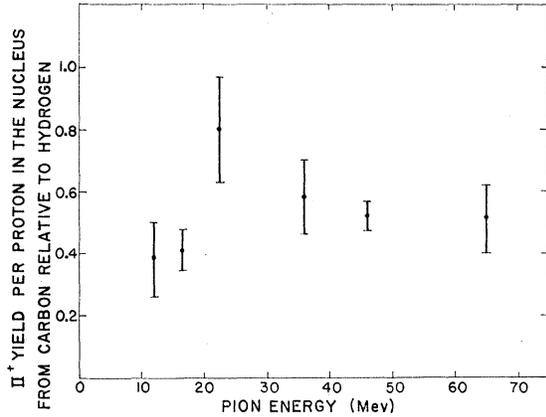


Fig. 6. Ratios of π^+ photomeson production efficiency from carbon relative to that from hydrogen at 135° .

Another indication of the effect of the internal momentum distribution on production of low-energy pions was obtained by comparing the production cross section from hydrogen to that from carbon. As shown in Fig. 6, there is no evidence for a strong energy variation of the production efficiency from carbon relative to hydrogen. Since the difference in momentum distribution between carbon and hydrogen is much greater than that between carbon and the heavier nuclei, which by the above analysis seems to be quite small, it is unlikely that the relative production cross sections from C, Al, Cu, Ag,

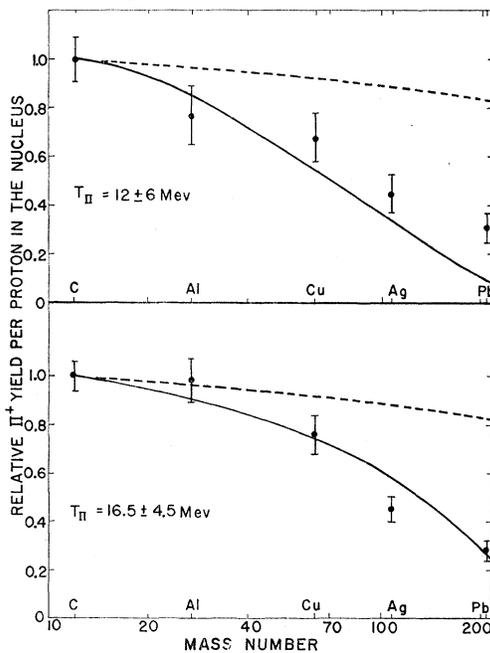


Fig. 7. Relative π^+ yields per proton in the nucleus at 135° at pion energies of 12 and 16.5 Mev. The yields at each energy are normalized to a carbon value of 1.00. The dashed curve represents the yield calculated on the basis of the optical model with neglect of the Coulomb potential. The solid curve represents the yield predicted when the effects of the Coulomb potential are included.

and Pb as a function of energy are very greatly affected by the momenta of the nucleons in the nucleus.

The optical model interpretation considered here is based on the assumption that the mesons are produced in the nucleus with an isotropic angular distribution. Although the angular distribution of photomesons from H at these energies is isotropic within about a factor of two,^{16,17} McVoy¹⁸ estimates that in a nucleus, under the conditions of this experiment, production in the forward hemisphere may be much more likely because of momentum-space requirements. Some evidence that this effect may not be too serious appears in the experimental observation, shown in Fig. 4, that the A dependence of the meson yields is approximately independent of angle. Although there are no experimental data on angular distributions for mesons of energy less than 46 Mev, any pronounced variations would seem rather unlikely.

The expression f defined in Eq. (1) assumes that all the meson interactions are of an absorption nature and that the pions are not scattered in the nucleus. Scattering would permit mesons, initially created at an energy too high for detection by the counters, to be degraded in energy, before leaving the nucleus, suffi-

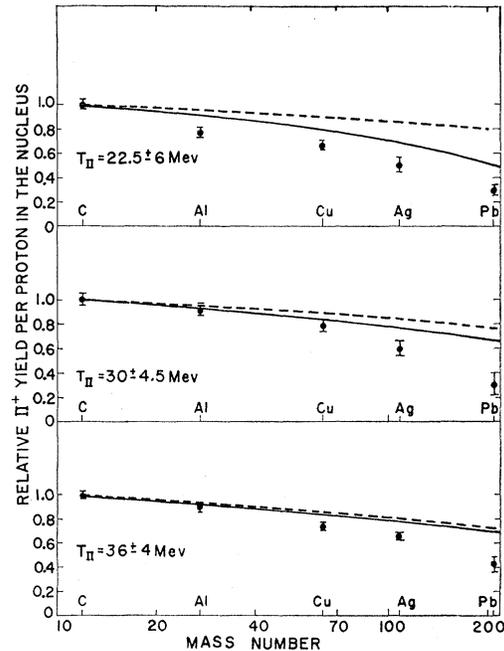


Fig. 8. Relative π^+ yields per proton in the nucleus at 135° at pion energies of 22.5, 30, and 36 Mev. The yields at each energy are normalized to a carbon value of 1.00. The dashed curve represents the yield calculated on the basis of the optical model with neglect of the Coulomb potential. The solid curve represents the yield predicted when the effects of Coulomb potential are included.

¹⁶ Walker, Teasdale, Peterson, and Vette, Phys. Rev. **99**, 210 (1955).

¹⁷ Tollestrup, Keck, and Worlock, Phys. Rev. **99**, 220 (1955).

¹⁸ K. McVoy, thesis, Cornell University, 1956 (unpublished).

ciently to be detected. McVoy¹⁸ has shown that the contribution of such scattered pions to the experimentally measured cross sections would have a dependence on λ and A given by $(1-f)f$. The contribution of π^+ 's resulting from a charge exchange scattering of π^0 's would have a similar dependence on λ and R . For a given λ these scattered-in contributions have an efficiency, $(1-f)f$, which decreases less rapidly with increasing A than the direct contribution, f , does. Thus, the presence of a scattering-in contribution would, if anything, raise the predicted production efficiency at the heavy nuclei. So scattering effects would not improve the theoretical agreement with the data between 22.5 and 65 Mev.

Theoretical predictions of the A dependence of the pion production efficiency based on the optical model and the effects considered above are superimposed on the data in Figs. 7, 8, 9, and 10. Theoretical curves are given both with and without consideration of the effect of the Coulomb potential.

At pion energies of 12 ± 6 Mev and 16.5 ± 4.5 Mev, effects of the Coulomb potential are seen to be sufficient to bring the theoretical predictions into general agreement with the data. Similarly, consideration of the effects of the Coulomb potential on the optical model predictions at 95 ± 3 Mev and at 125 ± 3 Mev seems nearly sufficient to explain the data. The slight dis-

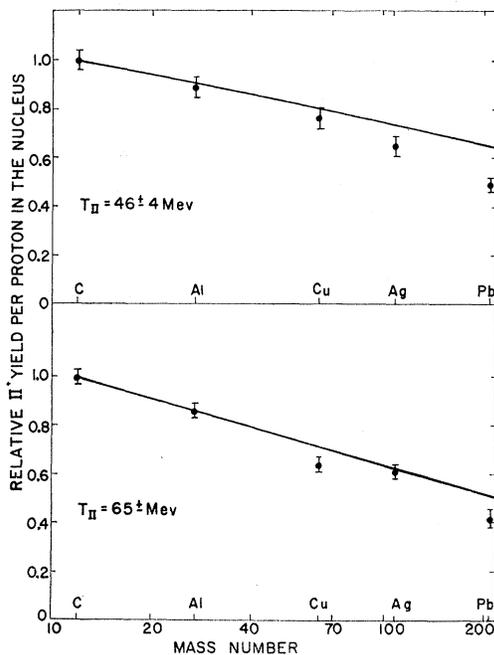


FIG. 9. Relative π^+ yields per proton in the nucleus at 135° at pion energies of 46 and 65 Mev. The yields at each energy are normalized to a carbon value of 1.00. The curve represents the yield calculated on the basis of the optical model with neglect of the Coulomb potential. Since at these energies the Coulomb barrier has little influence and the production cross sections can be considered independent of energy, effects of the Coulomb potential are not considered.

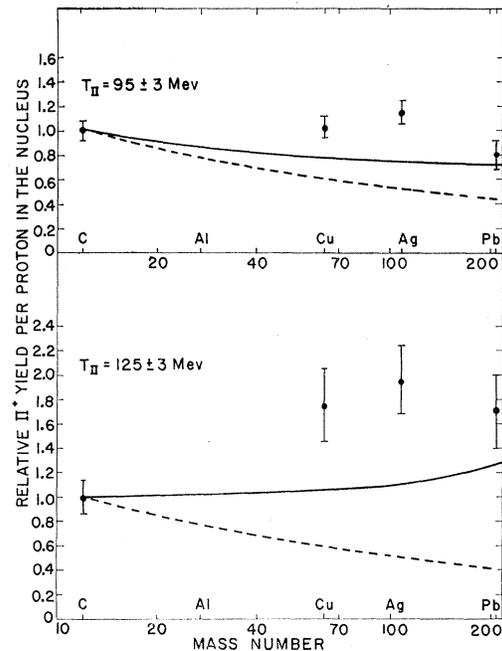


FIG. 10. Relative π^+ yields per proton in the nucleus at 135° at pion energies of 95 and 125 Mev. The yields at each energy are normalized to a carbon value of 1.00. The dashed curve represents the yield calculated on the basis of the optical model with neglect of the Coulomb potential. At these energies the Coulomb potential is important only because of the strong dependence of the production cross section on pion energy. The solid curve represents the yield predicted in the presence of a Coulomb potential when the dependence of the yield on pion energy shown in Fig. 5 is included.

crepancy that still exists could probably be explained by considering differences in the internal momentum distribution of the various nuclei. At the intermediate energies, 22.5 to 65 Mev, the data seem to be somewhat lower than predictions based on the effects considered above. However, these theoretical predictions are based on many approximations and might well be brought into agreement with the data if more detailed calculations are made.

In addition to the optical model as considered above, another mechanism which would affect the photopion production efficiency has been postulated by Butler.¹⁹ He suggested that photomesons are produced preferentially on the surface of nuclei, production in the nuclear core being suppressed. A mechanism for such suppression, in which the emitted meson is reabsorbed by the parent nucleon and its nearest neighbor, was proposed by Wilson.²⁰ This surface production model predicts a $1/A^{1/3}$ production efficiency regardless of the magnitude of the mean free path for meson absorption. Although the results of this experiment are consistent with such a surface production model, it does not appear to be necessary to invoke its additional assumptions

¹⁹ S. T. Butler, Phys. Rev. **87**, 1117 (1952).

²⁰ R. Wilson, Phys. Rev. **86**, 125 (1952); **104**, 218 (1956).

since the data are also in fair agreement with the optical model.

V. ACKNOWLEDGMENTS

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Bubble Density in a Propane Bubble Chamber*

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Measurements of the number of bubbles per centimeter in a propane bubble chamber are described. A method insensitive to the effects which cause inefficiency in direct bubble counting was used, based on the distribution of bubble spacings. The number of bubbles per centimeter measured by using this method is consistent with the rate of delta-ray formation.

THE results of counting the number of bubbles per centimeter on tracks in a propane bubble chamber of particles having known velocities were reported by Glaser *et al.*¹ We have observed that direct bubble counting underestimates the bubble density because of losses due to limited optical resolution and possible bubble coalescence. We have measured bubble densities for some of the tracks used by Glaser *et al.*,¹ using, however, a method which is insensitive to the principal effects which cause inefficiency in direct bubble counting.²

If the bubbles are distributed at random (i.e., giving a Poisson distribution) along the track, the distribution of the lengths of the spaces between bubbles is given by

$$f(x) = me^{-mx},$$

where m is the average number of bubbles per unit length. On a semilogarithmic plot, the slope of the distribution gives m . A typical example of an experimental distribution is shown in Fig. 1. The points follow a straight line down to a certain spacing, then fall off. The optical resolution and the bubble diameter of the tracks chosen are both approximately equal to the spacing at which the distribution fails to be random, so coalescence of the images or of the bubbles themselves is presumably responsible for the failure to observe small spacings.

The slope of the experimental distributions for large spacings was taken to give m . The temperature of the

chamber varied appreciably in runs at different momenta. The tracks at minimum ionization (pions) which were in each set of pictures, were used to normalize the dense tracks (protons) to the temperature of one set of pictures by multiplying by the ratios m for pions.

The results, for two temperatures and several momenta, are shown in Fig. 2, along with the results of the

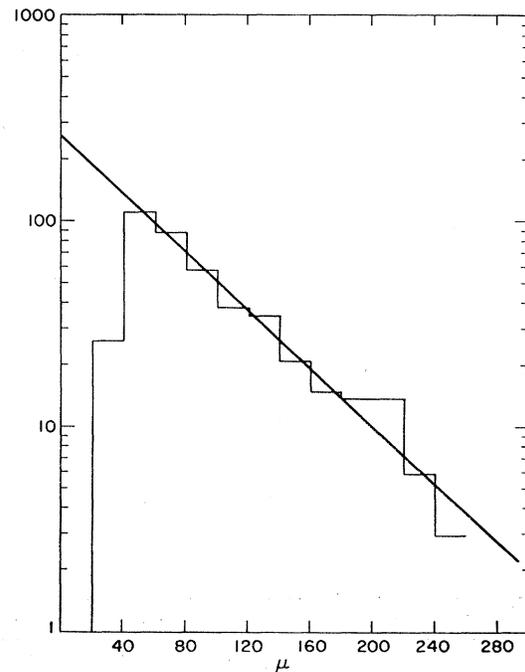


FIG. 1. The distribution of the number of bubble spacings plotted against the length of the spacing on the film, for a track with ionization 2.3 times minimum in propane at 55.5°C.

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† National Science Foundation Predoctoral Fellow.

¹ Glaser, Rahm, and Dodd, *Phys. Rev.* **102**, 1653 (1956).

² Willis, Fowler, and Rahm, *Bull. Am. Phys. Soc. Ser. II*, **2**, 6 (1957).